# TEACHING HISTORY OF MATHEMATICS TO TEACHER STUDENTS

## Examples from a short intervention

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**ABSTRACT**

Two years in a row, I have developed six-hour teaching sequences on history of mathematics for prospective teachers for pupils age 11-16. The development was based on the research literature of the HPM group as well as on a questionnaire given to the students before the teaching. But at the same time, the opportunity to teach these students was used to do further research on the teacher students' conceptions of history of mathematics, by means of pre- and post-teaching questionnaires.

Based on the research literature and on the questionnaire results, I set up the following goals for the teaching sequence: It should give the students examples of different ways of teaching with history of mathematics (more than just "telling stories"), it should be connected to the students' curriculum, it should give ideas that are suitable for different age levels in the 11-16 bracket, it should show both how mathematics has been developed and how it has been used and it should give pointers to further studies for interested students.

In this workshop, I will show how I tried to meet these goals and give examples of activities included in the teaching.

More detail and background to the development of the teaching sequence is included in my talk at the TSG20 at ICME the week before this conference.

## 1 Background

The 2010 teacher education reform in Norway created a new course: A one-year (60 ECTS) course in mathematics education for students who want to teach mathematics in grades 5-10 (ages 11-16). The course as it is taught in my institution does not have much of a historical perspective, but I was asked to give a short (six hour) course on history of mathematics for these students. I decided to try to combine the task of developing a meaningful six-hour "taster" of history of mathematics with trying to develop an understanding of teacher students' conceptions of history of mathematics. This workshop will give examples of what I decided to include. A paper at the TSG20 at ICME12 details both the development part and the research part of the project. (Smestad, 2012b)

The six hour course has so far been held twice, for the first year students in academic years 2010/11 and 2011/12. This workshop will be based on both iterations. I will first state the goals and give a short description of the intervention. The main part of the paper will be concrete examples of what was done in the classroom.

## 2 Information on the interventions

### 2.1 Goals for the first iteration

The first year, my goal was to create a teaching sequence of six hours which

1. fits into the context in the particular course, in particular the course curriculum and the target group: pupils in the 11-16 age bracket.
2. gives the students examples of different ways of teaching with history of mathematics (more than just "telling stories").
3. stresses issues that are important in the literature on HPM
4. takes into account students' conceptions of history of mathematics before the course, and thereby
5. gives students an introduction to history of mathematics that motivates them to search for more knowledge

### 2.2 Outline of the first iteration

I decided to organize the teaching around the second goal above, and to give the students examples of history of mathematics based on different methods of working with pupils. I still kept a little bit of chronology but leapt from topic to topic. The plan turned out like this (the references are to papers where I have written about this before):

- Using old techniques: Russian peasant multiplication, Gelosia method of multiplication, casting out nines (Smestad & Nikolantonakis, 2010)
- Using old concrete materials: Navigation
- A play in the classroom: Plato's Meno
- Exercises based on history: Pascal's triangle, the Pascal-Fermat correspondence, unit fractions
- Etymology: Etymological crossword
- Working on original sources: Babylonian clay tablets (Smestad, 2012a)
- Multi-curricular: Perspective drawings (Smestad, 2011)
- Biography: Niels Henrik Abel and Florence Nightingale

Of course, this is far too much for a six-hour lecture, thus giving opportunities for choice as the teaching evolved.

### 2.3 Outcome of the first iteration

There was no test at the end of the teaching, only an informal evaluation. In this, the students made several points. I had made the error of having most of the „talking" in the first three hours with much more group work in the last three. A better balance is easy to achieve. More importantly, the students found it difficult to take in so many different topics, and would have preferred to concentrate on a smaller number of topics.

Surprisingly, many of the students insisted that the etymological crossword had been the most entertaining part. Navigation, on the other hand, became more of a lecture than a discussion or a hands-on-activity, partly due to lack of time.

### 2.4 Goals for the second iteration

In the next academic year, circumstances dictated that I did my teaching not long before the exam, at a time when the students were working on combinatorics and probability. Based on the feedback from the students the year before, I felt it would be a good idea to concentrate on one topic, and this had to be the history of probability. But I didn't want to let go of the other goals, for instance I still wanted to introduce the students to „different ways of teaching with history of mathematics". Thus, I had to find mathematically interesting materials on probability with a variation of methods.

### 2.5 Outline of the second iteration

I included parts where I lectured and discussed with students (l), group work on exercises based on history (e), a play (p), work on original sources (os) and a game (g). These were the contents:

- Introductory remarks on the history of probability (l)
- Remarks on the history of statistics (l)
- A story of St. Olaf, King of Norway (Smestad, 2011) (l)
- The two problems of de Méré (The Pascal-Fermat correspondence) (e/os)
- Expected value, insurance, the Archbishop of Bremen (e)
- The St. Petersburg Problem (Smestad, 2011) (e)
- Buffon (e)
- The Monty Hall problem (p)
- Combinatorics: Varahamihira, Mahavira's formula, Bhaskara (l)
- Pascal's triangle (os)
- Pepys and Newton (e/os)
- Bertrand's chord paradox (Smestad, 2011) (l/e)
- D'Alembert's misconception (e)
- Leibniz and Galileo (e)
- Montmort's paradox (e)
- Etymological crossword (g)

Again, this is far more than could fit into six hours, leaving me with choices to make in the classroom.

### 2.6 Outcome of the second iteration

This time, the mathematical learning was more obvious to the students. Actually, at times, the students got so involved in the mathematics that they may have forgotten that they were working on history of mathematics. The meta-issues of different ways history of mathematics can be included in teaching was probably lost on most of the students.

## 3 Examples from the work

Here, I will give some examples of what I did with the students. The examples are chosen to give an overall idea of the teaching I did. At the same time, I try to avoid repetition of examples that I've published elsewhere.

### 3.1 Navigation

The history of navigation is a rich source for teachers, as navigation was essential knowledge for a long time and involved many different kinds of mathematics. In this short introduction, I chose to start out by asking students how they would find their way from A to B at sea, given a map but no GPS. There are at least four answers which can come out of such a question:

1. Keep to the shore until you see your destination.
   (But if there is no shore connecting A and B, you could)
2. find the right direction from A and try to stick to that.
   (Sadly, you tend to drift away from the set direction, so you should also)
3. try to keep track of where you have sailed.
   (Which happens to be quite difficult, so it would be better to)
4. try to figure out where you are at any given moment.

There's not much mathematics in the first point, but it is still worth discussing that keeping close to the shore has its problems, as there may well be people there that you don't want to see you. The second point gives opportunities to discuss when the compass was invented (and the difference between magnetic North and geographical North), as well as other ways of telling directions at sea.

To keep track of where you have sailed, you need to know both your direction and your speed. In the 1500s, the "chip log" was developed. This consisted of a line with knots on it which was attached to a piece of wood and dropped in the water. As the line went out, the number of knots would tell the speed. This is also the origin of the measuring unit *knot*. (Jeans, 2004) Obviously, in this way you could only measure the speed relative to the water, which could be quite a serious problem.

Seamen also used their local knowledge to tell if they were at the right place. A "lead line" was a nice tool - basically just something heavy that you sunk to the bottom, covered in wax so you could find out what was on the bottom. At the same time, you also got to know the depth. Eventually, whole books were published describing the conditions of the seabed, including descriptions such as this: "Betwixt latitude $3^\circ\ 40'$ S. and $1^\circ\ 40'$ S. the soundings are from 22 to 18 fathoms, sand, with some few cafts, muddy soundings, extremely regular; but when you come to pass this latitude, namely, $1^\circ\ 40'$ S. you will meet with soundings 18 fathoms, fine red sand." (Wright & Herbert, 1804, p. 452)

But the mathematically most interesting part appears when we start discussing how to get to know where you are at a certain moment. You need only two pieces of information; the longitude and the latitude. Moreover, if you travel at the right latitude, you can just stay on it and you will eventually find your destination. The geometry of longitude and latitude is not all that well-known to students, and bears repetition at this point. I will not go into that in this article.

The simplest way to measure the latitude is to measure the angle between Polaris and the horizon. At day, you could measure the solar altitude - but this would only give the latitude if you had tables of solar altitudes for different latitudes. Astronomical almanacs including solar altitudes were standard equipment for any navigator for centuries.

Explaining why the angle between Polaris and the horizon equals the latitude includes a bit of geometry and is all very well. But it is something else to stand on a ship at night and actually do the measurements. Over time, instruments were devised to make these measurements as accurate as possible.

The khashabat, developed in the 9th century and later developed into the kamāl, were such instruments. The kamāl consisted of a wooden rectangle with a hole in the middle, as well as a string. With one part of the string in his mouth, the seaman would place the rectangle so that the top just covered Polaris while the bottom was even with the horizon. Measuring the length of the string from the seaman's teeth to the board gave a measurement of the latitude. The kamāl was not really useful at higher latitudes, though, as it would end up very close to the face, which would lead to inaccurate measurements. (McGrail, 2004) Thus, other instruments were devised here, for instance the quadrant, the cross-staff, the Davis' quadrant (a backstaff), the reflecting octant, the sextant and so on. For our purpose, it may be enough to see pictures of the different inventions to understand how important these measurements were. But it would also be possible to do experiments with pupils trying to find the latitude by means of the kamāl or other instruments. These experiments could also be done, or at least discussed, with the students.

Finding the longitude is more difficult. The theory is simple: when the sun is at its highest, you know that the local time is 12, and if you know the local time of some other place, you can calculate how far east or west you are from that place. The problem, though, was to know the local time of the other place. Until the beginning of the 1700s, even the best clocks had a margin of error of about 10 minutes per day, which would translate into an error of 278 km a day. In 1764 John Harrison invented a clock that was usable at sea, which was used by James Cook as he sailed around the world in 1779. (Library, 2004)

In connection with the mathematics discussed while working on navigation, it makes sense to include Eratosthenes' calculation of the circumference of the Earth. That is a simple calculation giving impressive information about the globe we live on. It is too well-known to be included in this article.

After all this talk about different ways to navigate, we discussed why these issues are relevant for teaching. Mathematics is obviously included, and the topic also includes knowledge of the world we live in that every child should be aware of. The topic gives clear examples of how mathematics is (and has been) important, and how people have been using mathematics to expand human knowledge in other areas.

When doing this kind of classroom discussion, the teacher should preferably have quite broad knowledge of the field. In my case, I had to answer some of the questions by promising to check things and give the answer later. Probably, most other ways of working on history of mathematics with students demand less of the teacher on the spot than whole-class discussions like this.

### 3.2 Dramatization: Two plays

In both the iterations, I included some dramatization. The first time, the students and I „performed" (without any rehearsals) part of Plato's dialogue Meno, wherein there is a discussion of what happens to the area of a square when the side is doubled. The second time, I wrote my own „play", based closely on the story of the Monty Hall problem and the letters sent to Marilyn von Savant.

Plays are beneficial in that they may make history of mathematics „come alive", as the reasonings from history are given by people with flesh, blood and voices. See for instance Hitchcock (1992) for more on theatre in mathematics teaching.

The Monty Hall play had the added benefit of including 18 roles, making more students active in the performance. Moreover, many of the students had recently debated the problem heatedly, which contributed to their interest. The way the attitude towards Marilyn von Savant turned around as more and more people experimented on their own, was a useful reminder of the role of experiments and simulation in the teaching of probability.

### 3.3 St. Olaf, King of Norway

(Artist: Erik Werenskiold)

In *Heimskringla*, there is the following story about St Olaf, King of Norway:

> Thorstein Frode relates of this meeting, that there was an inhabited district in Hising which had sometimes belonged to Norway, and sometimes to Gautland. The kings came to the agreement between themselves that they would cast lots by the dice to determine who should have this property, and that he who threw the highest should have the district. The Swedish king threw two sixes, and said King Olaf need scarcely throw. He replied, while shaking the dice in his hand, "Although there be two sixes on the dice, it would be easy, sire, for God Almighty to let them turn up in my favour." Then he threw, and had sixes also. Now the Swedish king threw again, and had again two sixes. Olaf king of Norway then threw, and had six upon one dice, and the other split in two, so as to make seven eyes in all upon it; and the district was adjudged to the king of Norway. We have heard nothing else of any interest that took place at this meeting; and the kings separated the dearest of friends with each other. (Sturlason)

I take this story as a vehicle to discuss the concept of probability. Just as the kings did not consider the dice to be decided by chance, but rather by God, the students may also encounter pupils who consider events to be decided by fate, by gods or by who "deserves" to win. This is a complication in teaching probability that the students need to be aware of and ready for.

### 3.4 Pacioli's Problem of Points

The Pascal-Fermat correspondence, which many regard as the beginning of probability theory, provides interesting questions for pupils. I have developed a few exercises based on the history, which leads students through some of the problems. The example here is on what is known as the problem of points.

**Exercise 1**
De Méré's second problem was as follows:
Two persons are playing a game consisting of a series of rounds, and in each round each player has the same chance of winning. The winner of the game is the one to first win six rounds. But suddenly the game is (for some reason) stopped. At that time, A has won four rounds and B has won three.

a) How should the prize money be divided? Try to find a solution to the problem. (Maybe you have to look at a simpler problem first to get going - for instance the numbers can be changed to make it easier.)
b) Check if your way of solving the problem gives reasonable solutions when you use different numbers in the exercise (for instance if the number of rounds they have to win is a hundred or only two, or if A has a big lead or a small one···)

Luca Pacioli also looked at this problem, in his book Summa, and the problem is therefore sometimes called "Pacioli's problem of division". Pacioli solved the problem like this: as A has won four rounds of the seven they have played, he should have 4/7 of the prize money.

c) This solution was criticized 60 years later by Niccolo Tartaglia, who referred to what would happen if the game was stopped after only one round. Can you explain what Tartaglia meant?

**Exercise 2** Pacioli's solution was criticized by Tartaglia, who felt it was unfair that A would get the whole prize if he had a 1-0 lead as the game was stopped. This would also make A very eager to stop the game if he was leading 1-0, and this would leave B feeling unhappy, as he would otherwise have a fair chance of overtaking A's lead and get the whole prize.
Tartaglia solved the problem like this: A leads by one point, which is one sixth of the number needed to win. Therefore, he should have one sixth of B's stake, which would give A 7/12 and B 5/12 of the prize.

a) Do you find this reasonable? Do you find it fair that a 1-0 and a 5-4 lead should be treated the same way?

Tartaglia himself commented this: "The resolution of such a question must be judicial rather than mathematical, so that in whatever way the division is made there will be cause for litigation". (Hacking, 1975, p. 51)

Later, it was agreed that it was necessary to look at what the chance each would have of winning the game if it had continued.

b) What is A's probability of winning the game (at the point when the score is 4-3)? (Or: what is A's expected prize at the score 4-3)? Remember that each player is supposed to have the same chance of winning each round.

Here are some of the proposed solutions:

- Pascal argued like this:[1] Suppose the score was 5-4 and the prize 64 dollars. If A wins the next round, he has won, so he gets all the 64 dollars. If B wins the round, they are even, so they can split the prize with 32 dollars each. So A can say to B: "I am sure to get the 32 dollars even if I lose this round, and as far as the other 32 dollars go, maybe I will get them and maybe you, the chance is the same for both. So let us share the 32 dollars equally and give me also the 32 dollars which I'm sure of winning". Thus, A has a right to 48 dollars and B 16 dollars.
  Suppose now that the score was 5-3. Either A wins the next round and will get all the 64 dollars, or he loses, and is entitled to 48 dollars as we just established. Thus, he should have 56 dollars.
  Now suppose that the score was 4-3. Either A wins the next round and should have the 56 dollars by the argument above (as the score is then 5-3), or he loses, and the score is then 4-4 and they have a right to 32 dollars each. Thus A should have 44 dollars.
- Pascal sent this solution to another mathematician, Pierre de Fermat. We don't have Fermat's answer, but based on Pascal's answer back, we can reconstruct Fermat's solution: Fermat saw that the game would necessarily have to stop by the time four more rounds had been played. Therefore, we could easily list all possible outcomes of those four rounds:

  **aaaa aaab aaba aabb**
  **abaa abab abba** abbb
  **baaa baab baba** babb
  **bbaa** bbab bbba bbbb

  Of these 16 possibilities, there are 11 where A wins. He thus has 11/16 chance of winning, and should have 11/16 of the prize. (This is the same answer as Pascal's solution.)
  This method of solution was criticized by several mathematicians, Roberval and d'Alembert among others.[2] d'Alembert pointed out that the cases (aaaa, aaab, aaba, aabb, abaa, abab, baaa, baab in this example) would never happen, as the game is finished when A has won another two rounds. The only real possibilities are therefore aa, aba, abba, abbb, baa, baba, babb, bbaa, bbab, bbb. Of these, A wins in 6 cases, which means that his chance is 6/10. Roberval's argument was similar: "It is wrong to base the division method on the supposition that there will be four more games; when the one needs two points and the other three, they will not necessarily play four rounds since they may happen to play only two or

[1] I have tried to keep the solution method (after Todhunter, I. (1865). *A history of the mathematical theory of probability from the time of Pascal to that of Laplace*.: Chelsea Publ. Co.), but both the numbers and the wording is changed.
[2] I know only d'Alembert's solution of similar problems, I don't have access to what he did on this problem in particular.

three.” (Based on Pascal’s description of Roberval’s reasoning in his answer to Fermat.)

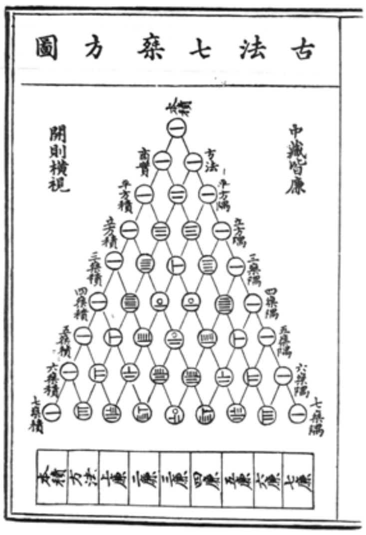
c) Which solution do you agree with?

Pascal’s answer (August 24th, 1654) was as follows: “It is not clear that the same gamblers, not being constrained to play the four throws, but wishing to quit the game before one of them has attained his score, can without loss or gain be obliged to play the whole four plays, and that this agreement in no way changes their condition? For if the first gains the two first points of four. will he who has won refuse to play two throws more, seeing that if he wins he will not win more and if he loses he will not win less? For the two points which the other wins are not sufficient for him since he lacks three, and there are not enough [points] in four throws for each to make the number which he lacks.” (Smith, 1959, pp. 556-557)

In this example, the exercises are based on history, but the students do not work on original sources directly. Working on a condensed summary like this may be more effective and also simpler for the students, but of course also risks introducing inaccuaracies and does not give the students the „direct“ connection to history that original sources do. See for instance Glaubitz (2007) for more on using original sources.

### 3.5 Pascal’s Triangle

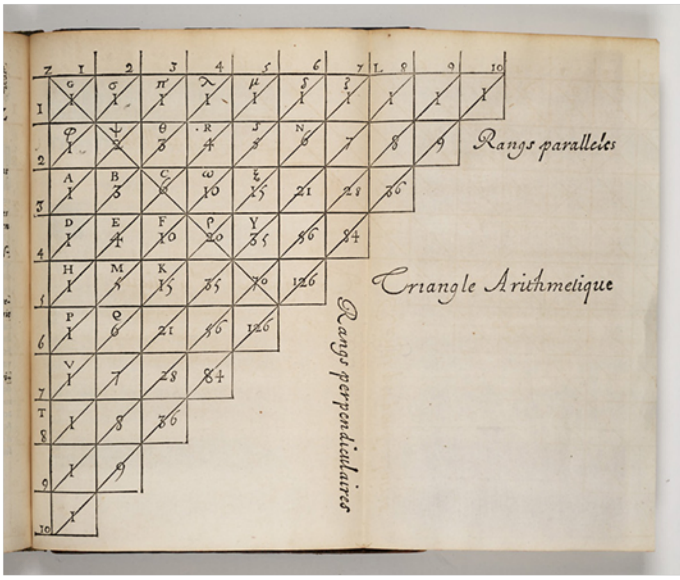
The work on Pascal’s triangle includes work on a Chinese table as well as Pascal’s treatise. We start by looking at a Chinese original source:




The numbers in the circles are written in Chinese numerals. Try to find out what they can mean. Also try to find out what the pattern in the table is. There is at least one error in the table. Find it.

The table in this exercise was writted in 1303 in China. It is known from many places and times. Today it is known as "Pascal's triangle". It is a well-known phenomenon that a little too much is named after famous (and Western) mathematicians instead of unknown mathematicians, but in this case there are reasons for this. Pascal wrote a whole book on the triangle - *Traité sur le triangle arithmétique* - which went further than anyone before him in the study of the triangle.

Here is the triangle as given in Pascal's treatise. Before we continue our work on the treatise, please work out the connection between this table and the Chinese table above.




This rest of my work on Pascal's triangle part is heavily inspired by David Pengelley and Janet Heine Barnett's workshop at the HPM2008 (Barnett, Lodder, Pengelley, Pivkina, & Ranjan, 2012), see also Pengelley (2009). In this work, students work on the original text from Pascal (although translated into English). By working on the original sources in this way, there may be a risk that the students ' overwhelming idea is that today's notation is much better than the one Pascal used. However, I will discuss with them whether the modern notation, which is very compact, is always a better one, and whether we could also be inspired by these original texts to use more words in our teaching with children.

### 3.6 Pepys and Newton

Another exercise is based on the correspondence between diarist Samuel Pepys and Sir Isaac Newton, which is entertainingly summarized in McBride (2007).

In 1693, Isaac Newton was a living legend in England. His main work *Principia* had been published six years earlier, and had revolutionized how the laws of physics were viewed. At the same time, he used mathematics in new ways. Samuel Pepys was a far less important person, even though he belonged to the upper classes and was a naval administrator. For posterity, he is mainly known for his diary, which described his life in a wealth of detail through nine years (1660-1669). His diaries were not published until after his death.

In 1693, Pepys sent a letter to Newton about a problem he had: What is most probable: to get at least one six when you throw six dice, to get at least two sixes when you throw twelve dice, or to get at least three sixes when you throw eighteen dice. He asked Sir Isaac Newton about this, and after several letters to and fro, Newton managed to convince Pepys of the answer. What do you think it was?
(A hint: To calculate the probability of at least one six when you throw six dice, it is enough to calculate the probability of not getting any sixes. To calculate the probability of getting at least two sixes when throwing twelve dice, it is enough to calculate the probability of not getting any sixes and of getting exactly one six. And so on.)
Newton answered thus:

> What is ye expectation or hope of A to throw every time one six at least wth six dyes?
> What is ye expectation or hope of B to throw every time two sixes at least wth 12 dyes?
> What is ye expectation or hope of C to throw every time three sixes at least wth 18 dyes?
> [...]
> If the Question be thus stated, it appears by an easy computation that the expectation of A is greater then that of B or C, that is, the task of A is the easiest. And the reason is because A has all the chances of sixes on his dyes for his expectation but B & C have not all the chances on theirs. For when B throws a single six or C but one or two sixes they miss of their expectations.

Pepys was not convinced, and asked to see the calculations. Newton wrote a long letter where he calculated that the probability for A was 31031/46656, the probability of B was 1346704211/2176782336, and he was satisfied to say that the probability for C was even less. When Pepys again asked for further explanations, Newton returned to the start:

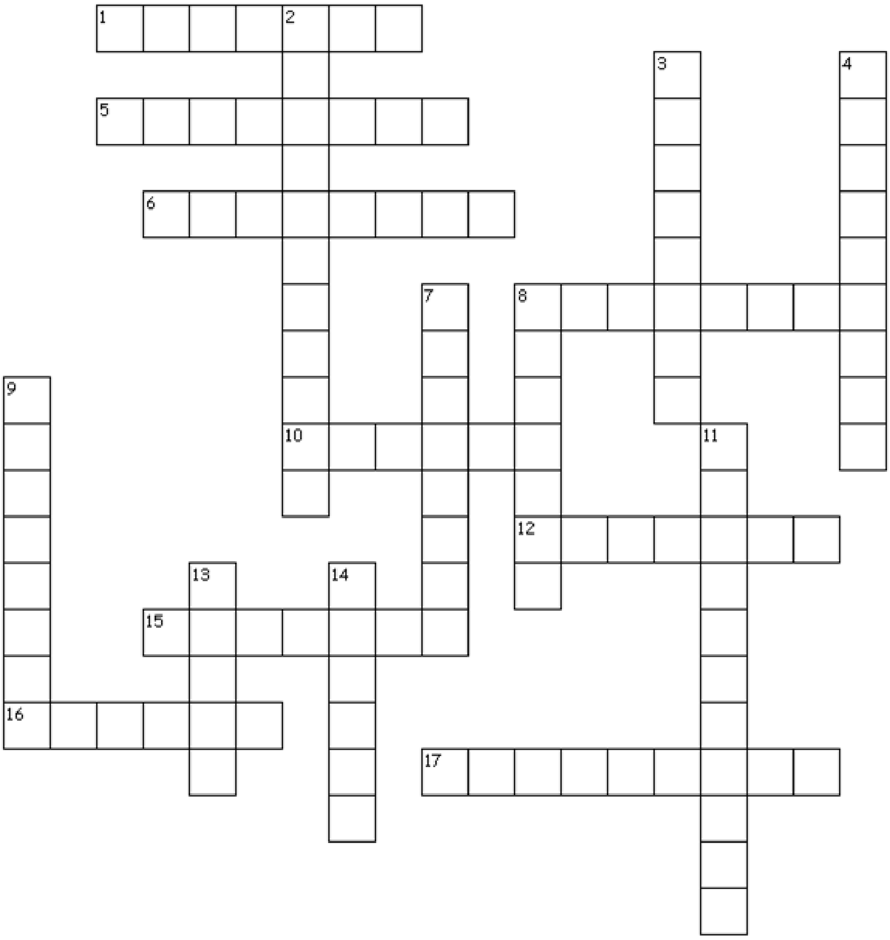
> As the wager is stated Peter [A] must win as often as he throws a six but James [B] may often throw a six & yet win nothing because he can never win upon one six alone. If Peter flings a six (for instance) four times in eight throws he must certainly win four times, but James upon equal luck may throw a six eight times in sixteen throws & yet win nothing.

Newton's reasoning has a logic flaw - can you find it? (A hint, possibly: Newton's reasoning works just as well (or badly) in this situation: A is to have at least one six in 12 dice, while B is to have at least two sixes in 24 dice. And so on.)

The subtle error in Newton's reasoning is detailed in Stigler (2007). This correspondence is a (perhaps unnecessary) warning to the students that basing their decisions on what seems reasonable instead of on calculations may at times be a bad idea. On the other hand, finding Newton's error may give the students confidence in their own abilities.

### 3.7 Etymological crossword

As mentioned above, a favorite part of the course has been the etymological crossword. Here is an English version made especially for this article:




**Across**

1. from Latin "tenth"
5. from Latin "standing firm, stable, steadfast, faithful"
6. from French "space between palisades or ramparts"
8. from French "increase, augmentation"
10. from Latin "small ring"
12. from Greek "spinning top"
15. from French "two"+"million"
16. from Latin "staff, spoke of a wheel, beam of light"
17. from Latin form of Al-Khwārizmī

**Down**

2. from Greek, "knowledge, study, learning"
3. from Latin "broken"
4. from French "right"+"angle"

7. from Latin "perform, execute, discharge"
8. from Arabic "reunion of broken parts"
9. from Greek "across"+"measure"
11. from Latin "from under"+"to pull, draw"
13. from Latin "fold in a garment, bend, curve." (from Sanskrit "bowstring")
14. from Latin "to separate"

While etymological crosswords should certainly not be at the core of mathematics teaching, I do believe that teachers should reflect on the fact that most words used in mathematics have interesting etymologies. The etymologies sometimes are connected to the concept itself, at other times they point to the history of the concept's development. In both cases, the etymology may be used to demystify the mathematical words.

## 4 Concluding remarks

The point of this workshop was to share some ideas and to give opportunities for discussion on how (very) short courses on history of mathematics for teachers could be structured. As mentioned before, I still hope that even six hours may be enough to broaden students' view of what history of mathematics is and how it could be included in teaching. Learning enough to develop their own materials, however, is left to their own enthusiasm.

Looking back at the goals set up for the first iteration, I think I succeeded in combining both traditional and less traditional ways of teaching with history of mathematics, while at the same time keeping close to important parts of the curriculum for the teacher students. However, it would be interesting to look more into the tendency I thought I noticed: that a closer focus on the mathematics may make the students less aware of the history, while organizing the teaching around different ways of using history of mathematics may make the students more aware of the pedagogical questions, but less aware of the mathematics.

Some readers will have noticed that there are ways of working on history of mathematics that are missing in this article. For instance, music is not found here, neither is project work. And even working with concrete materials, which I do mention, is not really included in the examples. It is a continuous challenge for the community to keep publishing a variety of examples and to discuss them, to bring the art of teaching with history of mathematics forwards.

## 5 Acknowledgements

Teaching of and with history of mathematics, like any other teaching, benefits from inspiration from other educators. Most of the ideas in the teaching mentioned here, leans on ideas from the HPM literature. I have included many references to articles and books where I have borrowed ideas, but there may well be other influencers that are not mentioned, because I've heard the ideas in passing many years ago. My thanks to them all.

**REFERENCES**

- Barnett, J. H., Lodder, J., Pengelley, D., Pivkina, I., & Ranjan, D. (2012). ”Designing Student Projects for Teaching and Learning Discrete Mathematics and Computer Science via Primary Historical Sources”. In V. J. Katz & C. Tzanakis (Eds.), *Recent Developments on Introducing a Historical Dimension in Mathematics Education* (pp. 187-200). Washington, D. C.: The Mathematical Association of America.
- Glaubitz, M. R. (2007). *The Use of Original Sources in the Classroom: Theoretical Perspectives and Empirical Evidence*. Paper presented at the European Summer University 5, Prague, Czech Republic.
- Hacking, I. (1975). *The emergence of probability : a philosophical study of early ideas about probability, induction and statistical inference*. Cambridge: Cambridge University Press.
- Hitchcock, G. (1992). Dramatizing the birth and adventures of mathematical concepts: two dialogues. In R. Calinger (Ed.), *Vita mathematica: historical research and integration with teaching* (pp. 27-41). Washington: Mathematical Association of America.
- Jeans, P. D. (2004). *Seafaring lore and legend*: International Marine/McGraw-Hill.
- Library, R. N. M. (2004). John Harrison and the finding of longitude. Retrieved from `http://www.royalnavalmuseum.org/info\_sheets\_john\_harrison.htm`
- McBride, J. M. (2007). Samuel Pepys & Isaac Newton: How Might They Do in Chemistry 125? Retrieved from `https://webspace.yale.edu/chem125/125/history99/2Pre1800/SPepysINewton/PepysStudent.htm`
- McGrail, S. (2004). *Boats of the world: from the Stone Age to medieval times*: Oxford University Press.
- Pengelley, D. (2009). “Pascal’s Treatise on the Arithmetical Triangle: Mathematical Induction, Combinations, the Binomial Theorem and Fermat’s Theorem” In B. Hopkins (Ed.), *Resources for teaching discrete mathematics: classroom projects, history modules, and articles*. Washington, D.C.: Mathematical Association of America.
- Smestad, B. (2011). “History of mathematics for primary school teacher education. Or: Can you do something even if you can’t do much?” In V. J. Katz & C. Tzanakis (Eds.), *Recent Developments on Introducing a Historical Dimension in Mathematics Education* (pp. 201-210): Mathematical Association of America.
- Smestad, B. (2012a). *Examples of “Good” Use of History of Mathematics in School*. Paper presented at the International Conference on Mathematics Education 2012 (ICME12).
- Smestad, B. (2012b). *Not just “telling stories”. History of mathematics for teacher students - what is it and how to teach it?* Paper presented at the International Conference on Mathematics Education (ICME12).
- Smestad, B., & Nikolantonakis, K. (2010). “Historical Methods for Multiplication.” In E. Barbin, M. Kronfeller & C. Tzanakis (Eds.), *ESU 6* (pp. 235-244). Wien: Holzhausen.
- Smith, D. E. (1959). *A source book in mathematics*. New York: Dover Publications.
- Stigler, S. M. (2007). Isaac Newton as a Probabilist. *Statistical Science, 21*(3), 400-403.
- Sturlason, S. Heimskringla Available from `http://omacl.org/Heimskringla/haraldson3.html`
- Todhunter, I. (1865). *A history of the mathematical theory of probability from the time of Pascal to that of Laplace.*: Chelsea Publ. Co.
- Wright, G., & Herbert, W. (1804). *A new nautical directory for the East-India and China navigation*. London: W. Gilbert.