FORTIFYING FRANCE: LES VILLES DE VAUBAN

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ABSTRACT

This workshop will focus on work done with many students aged between 12 and 14 over the past 4 years. It will be presented by the Marquis de Vauban (aka Peter Ransom), a 17th century Marshal of France in period costume.

The workshop deals with various topics: the geometry of fortification, based on plates from an 18th century French mathematics book by Du Chatelard, P. (1749) and associated works; 3D transformation geometry; enlargement (dilation); ratio and quadratic formulae leading to the study of projectiles. It stresses the fact that mathematics is an integrated subject and one in which students should appreciate the interconnectedness of the various topics.

Accompanied by images (stills and videos) of French towns and cities fortified by Vauban, participants will have many opportunities for creative work. They will make a pair of proportional dividers and experience how these have been used in the mathematics classroom. It will be explained how the work has expanded to include the use of siege engines.

Handheld wireless technology will be used and the impact this has on students' learning will be evidenced. Participants will be given a CD-ROM of all materials used over the past 4 years.

KEYWORDS

3D transformation geometry, cross-curricular, enlargement (dilation), fortification, geometry, graph transformation, projectiles, quadratics, ratio, Vauban.

1 The impetus of this work

I incorporate events from history into mathematics lessons because I find it very interesting to see the practical applications of mathematics set into the period when it was used.

Using Jankvist's and Grattan-Guiness' previous works (Jankvist 2009, Grattan-Guiness 2004), Tzanakis & Thomaidis (2011) classify the arguments and methodological schemes for integrating history in mathematics education and this episode fits into the two-way table mainly as *History-as-a-tool* and *Heritage* though there are overlaps into the *History-as-a-tool* and *History* cell (*op.cit.* section 4, Table 1). The over-riding concept in my work is *History-as-a-tool*.

The reliability of my work in the sense of reproducibility by someone else is impossible to quantify, since teachers use such episodes in different ways with different students and probably not in costume! Every session I do with students is different according to local conditions and the knowledge students bring to the sessions, so any classroom outcomes will, of course, vary.

2 Vauban and his fortifications

It was Frédéric Metin who first introduced me to French fortifications. His paper (Metin 2002) deals with how fortification was integrated into the curriculum at the Jesuit colleges. Then and now this is an incredibly rich area for geometry. Marshal Sébastian le Prestre de Vauban, or Vauban (1633-1707) as he is better known, spent years under Louis XIV, the Sun King, fortifying towns and cities in France. He rose to prominence as an engineer during campaigns of the 1650s then set to work to reconnoitre the defences of France. After the War of Devolution (1667-68) he took the lead in planning the sieges and fortress building on the Belgian frontier. I was fortunate to acquire an old mathematics text (Du Chatelard, 1749) that contained a treatise on fortification (as well as gnomoniques (sundials) - another interest of mine) and the plates intrigued me so much I developed a series of lessons based on them.

The number of fortresses that Vauban designed is unknown. Estimates vary between 60 and 330 - it all depends on how much work Vauban put into the fortresses before one counts them. There are considerable differences between the number of complete new fortresses built from scratch such as Neuf Brisach (normally quoted as 8 or 9), the number of improvements to existing fortresses, and the number of ideas for future work that he laid out for others to build.

3 The school-based work

Four of the five plates on which the initial work was based are shown in the appendix together with the questions asked about each one. The other one with which I start the work is shown here.

I use this plate with students asking them about the symmetries of the shapes and how they would draw them accurately using dynamic geometry software or otherwise. For this we use the TI-nspire

Fig 1: The author (left) with Frédéric Metin below the Vauban statue in his natal town of Saint Léger Vauban

CX hand-held technology since it allows them to use appropriate software within their own classroom without having to move to an IT room. By using the wireless Navigator software I can capture their screens and see how students are progressing.

I give the following information to students about the above plate. It refers to campaign forts which are used when armies are on the move. 1 toise was exactly 6 pieds (feet), i.e. about 1.949 metres in France before 10 December 1799, so I ask them to take 1 toise to be 2 metres. I ask them to describe the symmetries of shapes f.23 to f.30. There is a short discussion about f.25 and we realise that the printer has probably deliberately omitted part of the fort at corner b to get the whole plate onto one page. The construction marks on one side of f.28 allow students to see how each side is divided to

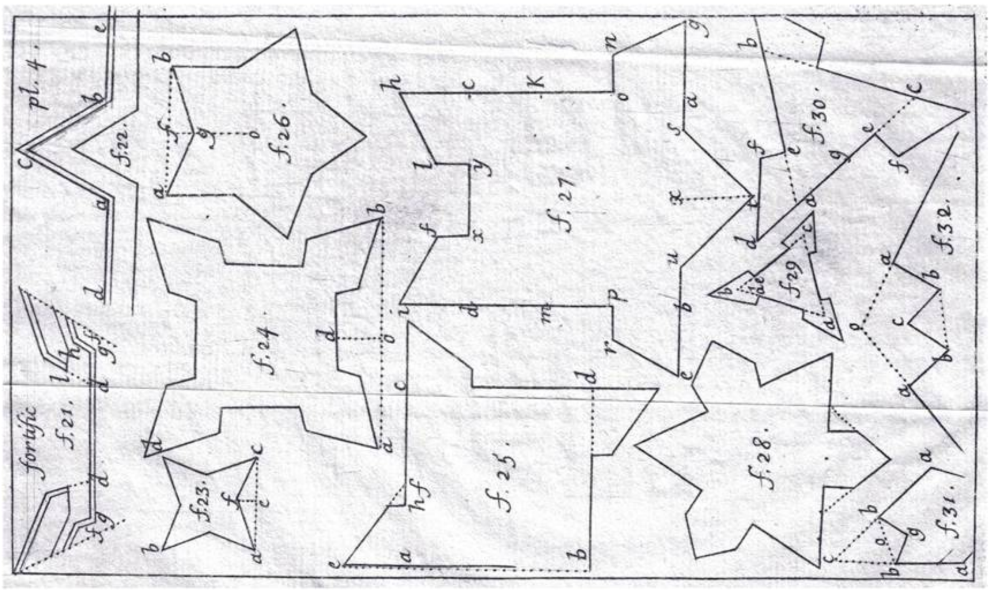

Fig 2: Plate of campaign forts, rotated for convenience. Note construction marks on f28

obtain the bastions. Students are asked to describe how this fort is constructed since mathematical communication is important. I ask students to draw the 'four star' fort of f.23, given that ac is 12 toises, and *ef* is 2 toises (Vauban's work states that the indent should be 1/6 of the side length), then to calculate the area of the fort in m^2.

3.1 Developing mental geometry

This is a neglected part of school mathematics. To encourage students to develop their mental geometrical skills I show them the following simple figure based on a 5 by 5 square array of dots, drawn large enough for the whole class to see.

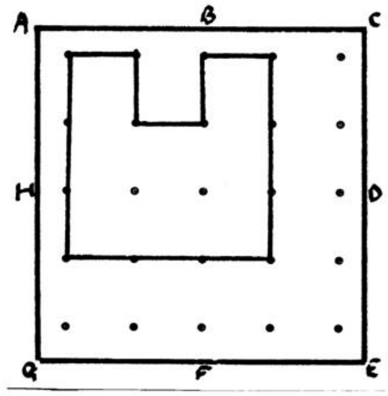

Fig 3: A simple figure ready to be rotated about an axis of symmetry

I hold the board at B and F and give it a half turn so that the class now sees the back of the board. They then have to draw exactly what I can see. Students can use IT and wireless technology or draw the result on a piece of dotty paper - both systems work well. After they have drawn their attempt, I turn round so that they can see the result. This is repeated with hands at HD, then at CG. For a challenge rotate the board 180° with hands at AE then follow this immediately by a 90° rotation with the picture still facing you!

Initial success with this task is rare - however repeating this every month or so soon sees increasing success with mental geometry and students need to appreciate that since we live in a 3D world things can appear differently depending on our viewpoint.

3.2 Proportional dividers, enlargement (dilation) and ratio

Few mathematics teachers have heard of proportional dividers (and even fewer used them), yet they date back many years. Heron of Alexandria (1st century AD) has been credited with a device that was probably an early version of a fixed proportional compass. In 1565 Jacques Besson in Lyons describes and illustrates a slotted variety with legs engraved with linear scales.

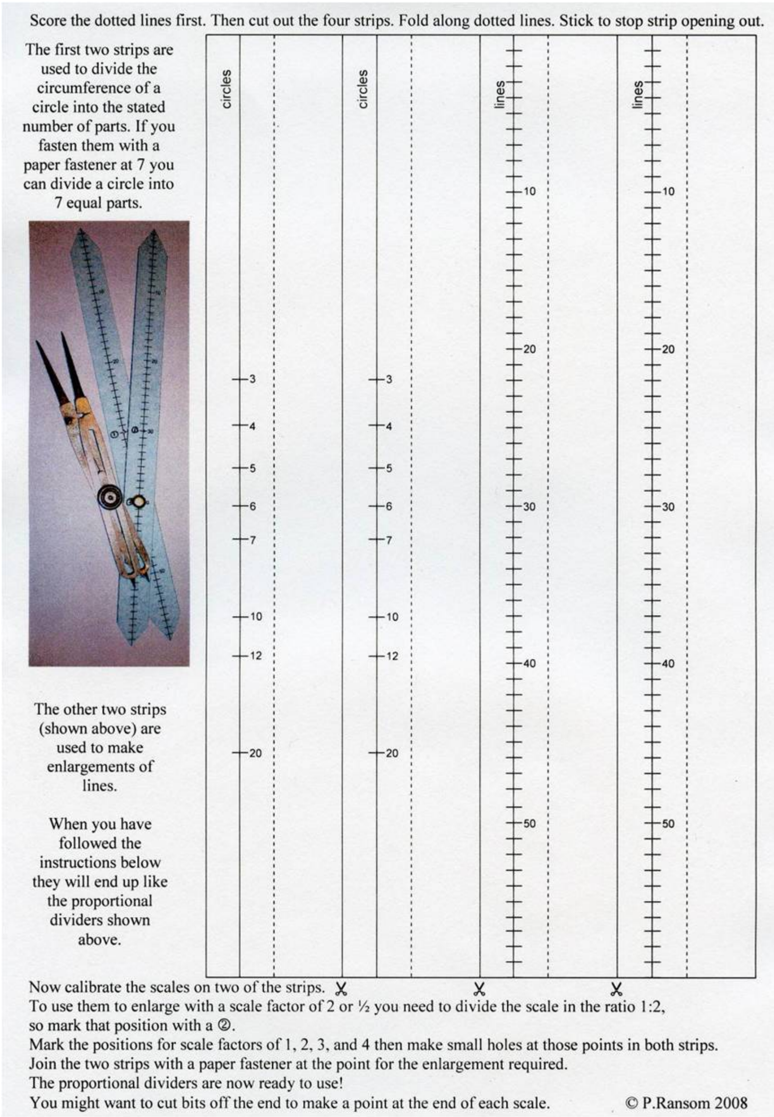

Fig 4: 9 inch brass proportional dividers and student made version from card with instructions for making one from card

I have a set of brass proportional dividers that students use to divide a circle into seven equal parts. The proportional dividers have four scales: *lines* that enlarge lines with a given factor; *circles* that divide circles into a given number of equal parts; *planes* that enlarge areas with a given factor and *solids* that enlarge volumes with a given factor. Once students have divided their circle into seven parts they then construct a seven pointed star fort based on Vauban's indent of one sixth of the side and the use of the brass proportional dividers. Here are the instructions the students follow.

Today you will draw a plan of a fort in the style of Vauban, the French engineer who fortified many French towns in the 17*th* *century.*

You will use an old mathematical instrument called 'proportional dividers' to construct a regular heptagon (a 7-sided shape with equal sides and angles).

BE CAREFUL!

These instruments are sharp and expensive so take care not to damage them or yourselves!

1 Loosen the screw

2 Move the slider so that the line on moveable part lines up with the 7 on the CIRCLES scale.

3 Tighten the screw - not too tight!

4 Open the dividers so that one of the long points touches the centre of the circle and the other touches the circumference of the circle.

5 Place one of the small points on the circumference so it makes a small mark.

6 Keeping that point fixed rotate the dividers and make a small mark where the other point meets the circle.

7 Keeping that new point fixed rotate the dividers again and continue until you are back where you started.

8 Using a ruler and pencil join up the marks you made.

9 Find the perimeter of the heptagon (that's the distance all the way around)

10 Now close the dividers.

11 Loosen the screw and set the slider so that it lines up with the 2 on the LINES scale.

12 Tighten the screw - not too tight!

13 Open the dividers so that the long points touch the ends of one side of the heptagon.

14 Now place one of the small points on a vertex and make a mark with the other point on a side.

15 Repeat with all the sides.

16 Join these mid-points to the centre of the circle with a feint line.

17 Now close the dividers.

18 Loosen the screw and set the slider so that it lines up with the 6 on the LINES scale.

19 Tighten the screw - not too tight!

20 Open the dividers so that the long points touch the end of one side of the heptagon.

21 Now place one of the small points on the mid-point of a side and make a mark with the other point on the line that goes to the centre.

22 Repeat with all the sides.

23 Join these new points to the vertices of the heptagon.

24 Find the perimeter of this 14 sided shape.

They then make their own pair of proportional dividers for enlarging lines from a sheet of A4 card. This introduces the concepts of dividing a scale in a given ratio and similar triangles. To get the proportional dividers to enlarge with a factor of 2, the scale needs to be divided in the ratio 1:2. Since the scale divides the length into 60 parts it is equivalent to dividing 60 in the ratio of 1:2 and so students add the ratio parts and divide 60 by that sum and so can pin the two legs at either 20 or 40 to give the desired enlargement. They then work out where to put the pin to enlarge with a factor of 3, 4, 5 etc. and as an extension with a factor of $1\frac{1}{2}$. There is the 'wow!' factor when they check that it actually does work with the scale factor of 2 - the number of times one hears students say 'It works!' never ceases to please me. Part of the fact that is does work is that it does not matter about how accurately the student cuts out the pieces because the central scale is not affected - the other reason it does work is because it is mathematics!

3.3 Projectiles, quadratic graphs and their transformation

The first plate in the appendix gives a scaled cross section of a fortification and the final lesson revolves around using IT to get a projectile from a given point outside the defences to a specific point inside using a quadratic graph. The document used is described in a series of screen shots shown below and students work through these dynamic pages answering questions about the graphs and their transformations. It finishes with a worksheet for them to identify the graphs of various quadratics.

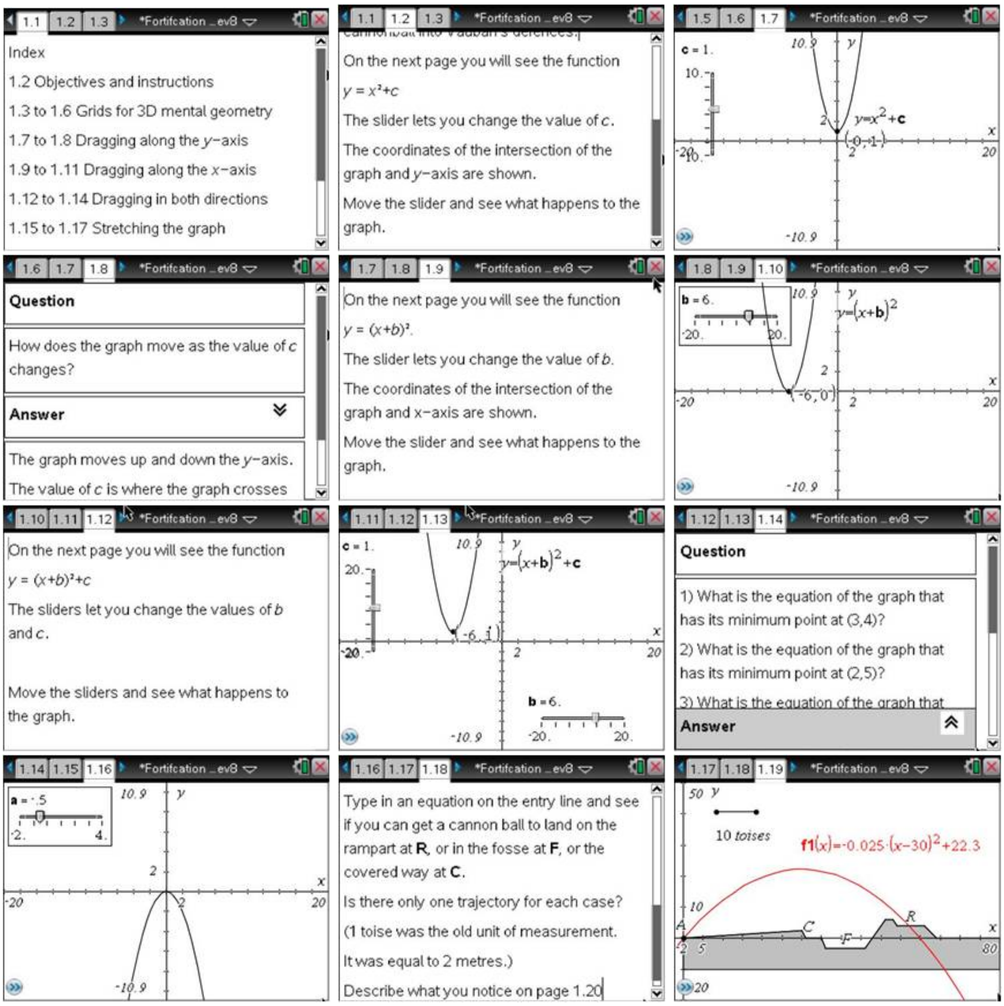

Fig 5: A series of screenshots from a TI-nspire CX

4 Conclusion

When used in the classroom with 12/13 year old students there was a surprise result. Students worked in groups of three on these activities and I had asked them to produce a plan for defending the mathematics area in school from invaders. Three groups went one better than just a plan - they also built a scale model over a week's half term. I was amazed at how much time and effort went into their work since one group had met up and worked for a total of 20 hours together. Imagine the uproar if I had asked them to spend that amount of time on mathematics homework.

In fact unknown to them Louis XIV ordered similar things, with Louvois (his minister of war) producing maquettes for each town that Vauban fortified. There is a collection of 15 of these in the basement of Le Palais des Beaux-Arts in Lille.

Following on from this my 14/15 year old students explored the mathematics of siege engines such as the *ballista* (a large Roman crossbow), *onager* (large Roman catapult) and *trebuchet* (a mediaeval counterweight catapult). They experienced small scale models in the classroom to see how they worked, then explored the effect of how far they could throw a ball when extending their arm length. This was achieved by throwing a ball outside with and without using a dog throw (a piece of plastic with a scoop at one end which in effect provides a longer arm). The data was then analysed using IT and students wrote a report on their findings.

The workshop at HPM will focus on the mental geometry, use of IT and proportional dividers exercises with participants receiving a CD-ROM with all the materials used.

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Appendix - the plates and questions

All the questions about the plates start with the following information: echelle is French for 'scale'

1 toise was exactly 6 pieds (feet) i.e. about 1.949 meters in France before 10 December 1799.

For the purpose of this sheet, take 1 toise to be 2 metres.

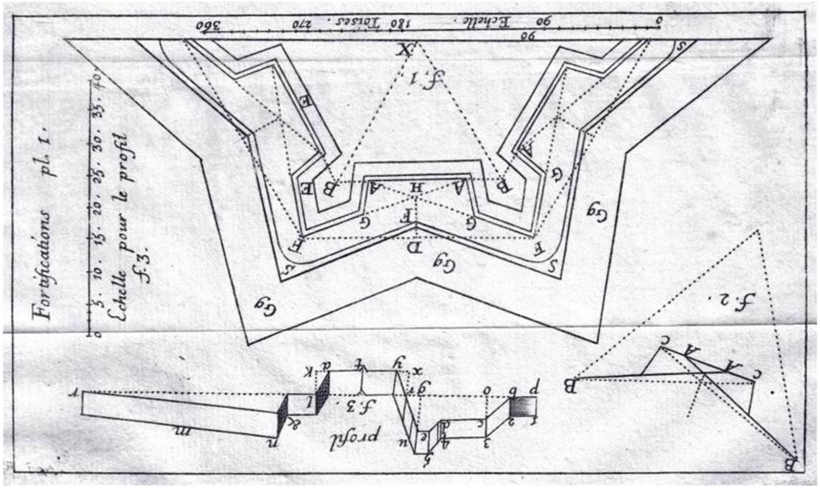

Plate 1 1. On what basic shape is this fort based? 2. What kind of triangle is triangle BXB?

3. What kind of triangle is triangle FHF?

4. Draw an enlargement of triangle FHF with scale factor 2. Measure $\angle FFH$.

5. Use the information you have gathered to calculate $\angle GFG$.

6. Now draw the full fort (you can omit the glacis Gg)

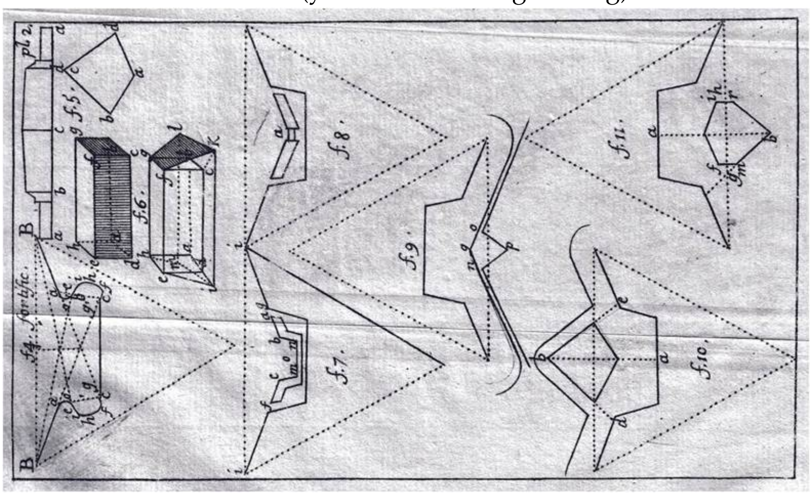

Plate 2 1. What is the name of the solid above f.6? 2. On what basic shape is f.10 based?

Concentrate on f.10 - all the following questions refer to this figure.

3. On what type of triangle is this figure based? 4. What is the (mathematical) name of shape b?

(Its defensive name is a *ravelin*.)

5. What are the properties of the diagonals of the shape that touches b?

6. Construct f.10, taking the side of the triangle to be 12cm.

You need to know that the distance ab is half the side of the triangle and the short diagonal of the shape at b is one-third the side of the triangle.

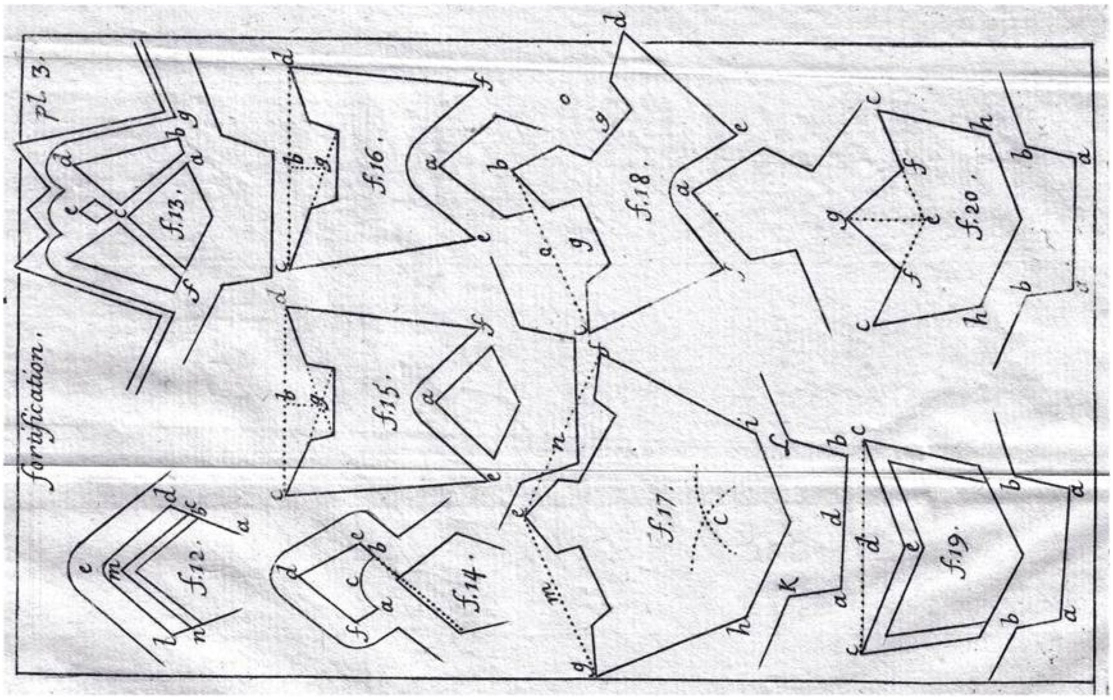

Plate 3 This plate refers to the construction of *ravelins* (f.17-20) and *hornwork* (f.15 & 16)

Here are the instructions on how to draw the horn in f.15.

Draw diagonal of the ravelin from a then from a on the ravelin continue the line for 85 toises to b.

At b draw a perpendicular line cd such that cb and bd are each 60 toises.

From b measure 20 toises to get g.

Join cg and dg to get the angles of the contrascarp. The faces are 38 toises long.

Use a scale of 1cm to 5 toises and draw the five edges of the front face of the horn.

You have two cannons to put on this horn.

Where do you put them to defend as much of this face as possible?

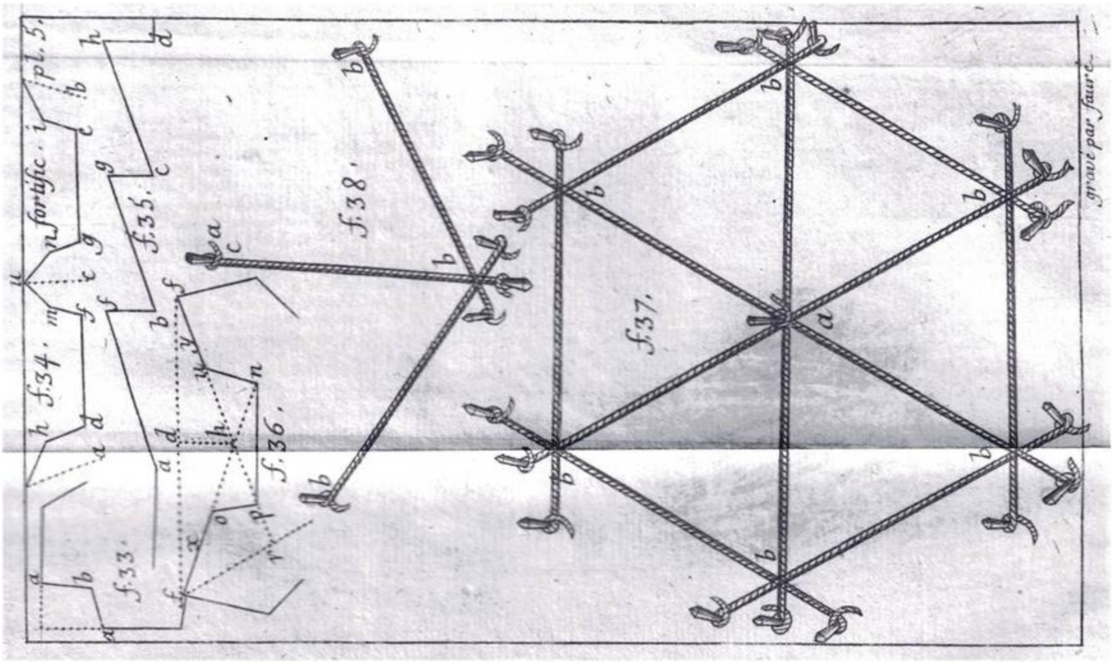

Plate 5 This plate gives more details on bastions (f.33, f.34, f.36), redans (f.35) and laying out equilateral triangles. You need to work with f.36 since you are trying to calculate all the angles in this figure, which

is based on a square fort with bastions. You are given the following.
fr passes through the centre of the square. $rphuf$ is a straight line perpendicular to fr.
fhn is a straight line.
$\angle opn$ is $100°$.
Sketch f.36 and work out the size of all the angles that you can find. Give reasons for your answers.