USING HISTORY FOR *MATHS IN EFL* TEACHING
Original texts and cultural approach in the classroom

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ABSTRACT

A new kind of mathematics course in France is a part of "European studies", including one or two hours a week of math in English as a Foreign Language (EFL). The major aims are to provide the students with a solid basic knowledge of words, methods and practices of mathematics that are studied in other European countries and/or Anglophone countries, and to allow a cultural approach to mathematics.

The use of original English texts is perfectly adapted to this purpose, as parts of the general culture and history of England; it's a good opportunity of working together with teachers of English civilization and History teachers.

In this paper we show several examples of original texts that we have used in the classroom with students from 10th to 12th grade (aged 15 to 18), as well as the use of this kind of texts in final exams.

1 English as a foreign language (EFL) and mathematics teaching

As it may be the case in all other countries, teaching foreign languages in France is a specialty of foreign language teachers. However, a recent change gave high school students the opportunity of taking a part of their mathematics course in foreign language (namely English, which is the most important, but also German or, more rarely, Spanish); this part is not necessarily linked with the ordinary curriculum, because it was mostly intended to improve the students' abilities in speaking and writing English.

The "European sections" are not widely spread, as only several high schools in every school district are allowed to create one, despite the constant increase of the students' and families' interest. Even if the original intentions of the Government were generous (and then rather costly), the high schools were not free to open European sections, and consequently, their number is desperately small.

Indeed, the abilities in foreign language speaking have traditionally also been rather inadequate in France (the fact is well-known) and several attempts to correct this have been made, including the creation of European sections; but the families tend to sign their children up for these classes to allow them to be part of the elite classes in the school⋯

1.1 Mathematics in the European section

The main problem (and the most interesting challenge!) for math teachers in "Euro" sections is their freedom to choose the contents they will teach. The exam is part of the Baccalaureate pupils take at the end of their third year at the *lycée* ("Terminale") when they are 18, and the special euro exam is created by the group of teachers themselves, which is not ordinary in France.

So, the *euromaths* teachers do not have to follow any curriculum, and they tend to favour "good old mathematics", like geometry, as well as useful (discrete maths, graph theory ⋯) or "foreign" ones (exotic calculations, matrices ⋯). The Baccalaureate is also a good opportunity to let the students face everyday problems using probability or investigating some kind of recreational mathematics, as flight charges or shoe lacing.

In my practice, I take a cultural approach to foreign civilizations trough historical texts, books or papers on the history of mathematics. Inspired by previous works by Leo Rogers and Peter Ransom[1], I use different texts from the 16th century England in the classroom, especially practical texts allowing students to apply ancient techniques in computations and measurements as well.

1.2 The choice of original texts

There are many ways to link mathematical contents to the English-speaking cultures, when you choose your documents amongst the huge quantity of books you can find on *Google Books* for instance, or in National Libraries websites[2]

One of the tricks I use to get the students involved is to choose the books according to the destination of our school trips: John Napier's *Rabdologia* when we sailed to Edinburgh, Leonard Digges' *Tectonicon* because we need to know the height of the main tower in Doune Castle (in order to attack it as a tribute to Monty Python's film about King Arthur's quest[3]), Voster's *Arithmetic* published in Ireland, and so on.

But why do we need original texts when present day textbooks would be enough to introduce all the notions we use? Well, the problem with our textbooks is that they do not focus on language and thus it is not easy to gain knowledge of vocabulary and idiomatic expressions through a mere reading. Moreover, you hardly learn anything about Anglo-Saxon cultures through contemporary literature on maths!

2 Getting to know the vocabulary: John Kersey's *Algebra*

The first year students know but few words about mathematics: the first hundred numbers, the classical shapes; but in geometry as well as in arithmetic or algebra, we have to know the right words, and use them properly. A simple French-English dictionary would be inappropriate, because we want our students to progressively become able to think in English naturally, and not to think in French first and then translate their ideas into English.

This objective makes it necessary to practise the language in situation, reading and analysing texts without exactly knowing what the vocabulary is. A cover/discover/uncover strategy is used with success with extracts of the *Algebra* of John Kersey; at the beginning of this book, Kersey stresses the relationship between formulae and expressions in ordinary language. For the classroom, a slideshow

[1]See bibliography: Rogers (2006) and Ransom (2006).

[2]example, the website of the Royal Society of London offers a large part its Philosophical transactions online. You can find a variety of references on the website of the (virtual) European Library too.

[3]*Monty Python and the Holy Grail* (1974) is also about mathematics: a famous scene deals with logic and burning witches⋯

is made using pages of the original book, covering the answers; every student of the class has to fill in one of the blanks by telling the answer aloud. Everyone can try and prepare their answer.

In the example below, we use chapter 10 (*A collection of easie questions*, that is a set of exercises), in which the author the author begins with the definition of two quantities, *whereof the greater is a (or 3), the lesser is e (or 2)*, and follows with several questions, namely English expressions that must be translated into formulae (*by Letters*) or numerical results (*by Numbers*). The columns on the right are covered first and uncovered gradually to check the correctness of the answers (or give them when no answer comes).

Anſwers	*by Letters,*	*by Numbers.*
1. The Sum of the two Quantities propoſed is	$a+e$	5
2. Their Difference, or the exceſs of the greater above the leſs, is	$a-e$	1
3. The Product of their Multiplication is	ae	6
4. The Quotient of the greater divided by the leſs is	$\frac{a}{e}$	$\frac{3}{2}$

As the chapter contains a variety of exercises, from very basic algebraic expression to more complex ones, we can browse the complete vocabulary for beginners, and sometimes students can discuss the answers.

3 Old English texts and practical mathematics

One of the first activities I propose to my second year students consists in analysing an old text about geometry, focusing on the language and trying to find the present equivalents of some weird words.

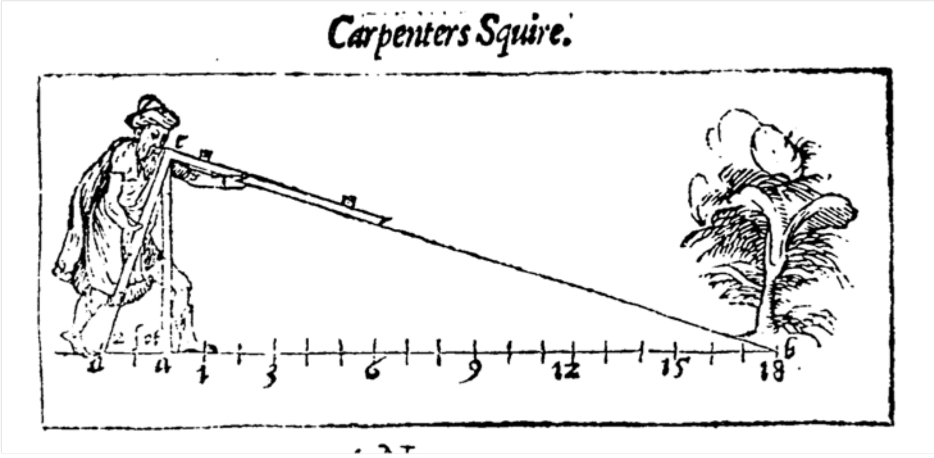
The first contact is always a sort of amazement, both about the typographic issues and the expressions that are used. Indeed, it is not that easy to decipher Digges on measurements of the towers height! This is the object of the first activity sheet: can you correct the vocabulary and make sense of the maths?

3.1 Leonard Digges' *Tectonicon* and *Stratioticos*

In his book *Tectonicon*, Leonard Digges gives general definitions of the geometric shapes and concepts, along with several methods to measure lengths, heights, widths (because these different lines are not considered the same) and then areas and volumes. The text is to be read aloud in order to understand the words···

How lengths in plaine Ground are searched by the Carpenters or Masons Squire. The staffe .a.c. in this fygure is imagined 6 fote, & *the space .a.d. 2. Foote. Consideryng nowe that .6. the lengthe of the staffe conteyneth .2. thrise, therfore the longitude desiered .a.b. of force muste conteyne thre tymes the staffe (whiche staffe is .6. foote) that maketh .18. foote. As this is proved true by a small grounde in the fygure folowynge: so the Arte fayleth not in a greater space, whiche the good speculator and diligent practiser by anye waye canne not denye.*

Does the Art really prevent failure? The practice of geometry on the field is highly recommended as a sequel to the study of texts in the classroom; and it is a good way to test the accuracy of actual measures of distant lengths or heights. I organised several out-of-classroom workshops for 20 years, always pretending we had an urgent need of measures in order to attacks castles or fortified cities. In



February 2012, during our school trip to Scotland already mentioned, the challenge was to evaluate the height of the main tower of the Castle of Doune: the students had studied a part of *Tectonicon* before, as well as other books about practical geometry, and they knew the principal techniques they could use: proportionality of triangles or use of the mirror. Actually, we discovered they used a lot of different tricks to meet with the challenge: as they had no stick, no astrolabe nor any mathematical instrument, they used their cameras (the height of a pupil they know, compared to the height of the tower), and their own bodies (someone standing on a bench, someone lying on the ground and searching alignment between the top of the tower and the top of the other one). Of course they found different values (from 20 to 30 meters), and we will have to draw conclusions about this; fortunately, we were able to get entrance to the Museum of the University of Saint Andrews and have a look at showcases presenting astronomical instruments; the students then could see by themselves that more and more preciseness was a constantly growing concern for instrument makers.

For the same purpose of interpreting old words, the other book by Digges that we use in the classroom is *Stratioticos* (1579), which deals with military matters, including problem solving from an arithmetical point of view. Students find it more difficult to read because of the lack of pictures (!), and the old units of measure they have to translate into modern ones:

Certaine Questions touching the Office of the High Marshall and Campe Maister

The firste question: Admit I finde by experience that 3000 footemen may commodiously be encamped in a plat of grounde 300 pace Square. I demaund how many pace the grounde shold be square that shall receyve 1[0]000. Footemen. [Answer: between 549 and 550]

In both cases, the teacher has to help students find their way through the old problems, old units and old fashion in general; who cares about the depth of a well nowadays? Who needs to know the space occupied by the infantry on the battlefield?

3.2 Robert Record's and John Napier's calculations

It is interesting to have the students read original texts in their original form, especially when they are printed in Old English font: just allowing the teacher to keep the contents at a certain distance and let the students cross the language door to reach the hidden mathematics: turn familiar into uncanny.

For instance, hereafter is the detailed multiplication of 2036 by 23 *by a checker table*, as it is explained by Robert Record in *The Ground of Arts*, which would be currently published in the same period as Shakespeare's *Romeo and Juliet*, but with greater success!

First I consider that my greatest nomber hath .iiii.fygures or places, and therefore I make so many roumes betwene lynes, thus. Then I see that of my multyplyers there are .ii. Wherefore I drawe so many lynes a crosse the other, that there may be 2 roumes betwene them. Then draw a crosse barre thorough every close square, so that it may reche down to the lowest overth warte lyne, as in this fourme.

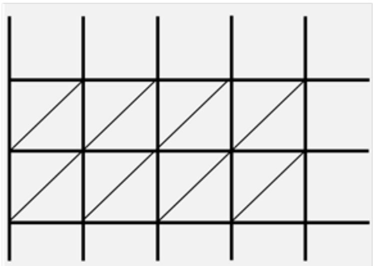


Figure 1: *

The empty checker table

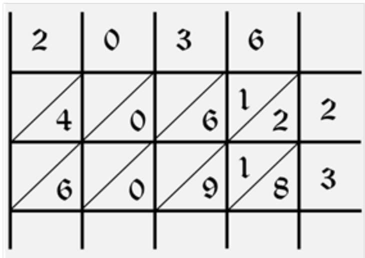
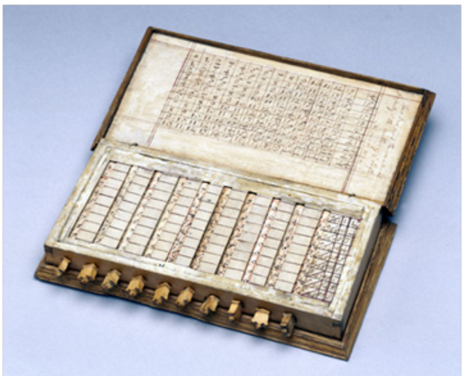


Figure 2: *

The complete one

As you can see, this is the well-known multiplication *per gelosia*; you can verify the partial products and the final result that is given by adding the numbers in the oblique sets of cells [NB: Record gives result 46,828]

This activity leads us naturally to the study of Napier's bones; logarithms should be a logical sequel, but it has to be left for the third year (and we haven't had time to undertake this activity before writing this paper). Actually, it was a part of the project of the same school trip to Edinburgh: the students saw two copies of Napier's bones sets in The National Museum of Scotland (see pictures below), and they could note that the artefact they had learned the use of was really in use.



4 Strange mathematics

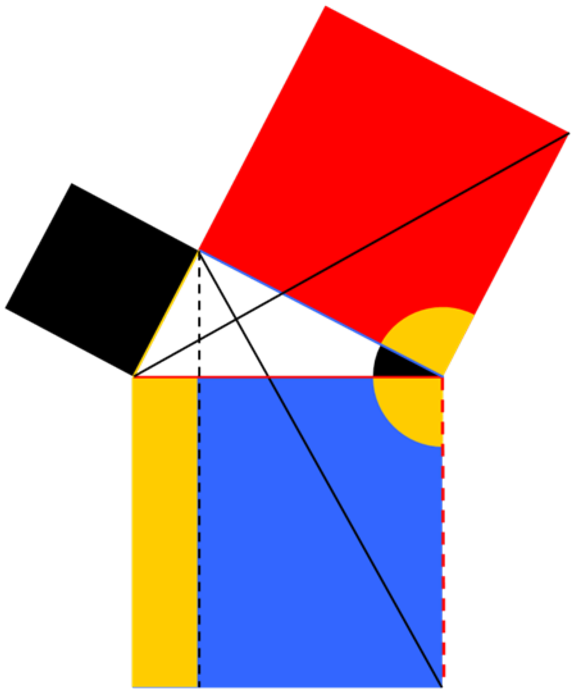
Some of the matters we study in the euroclass are different from the mathematics in the main course in French. We pick subjects in newspapers, on YouTube or in recreational or serious books, but none of them has the special taste of the 19th century England inventions or strange and forgotten ways of thinking the usefulness of mathematics.

Oliver Byrne's edition of Euclid's *Elements* was printed in colours, which was quite rare at the time; moreover the book was written in favour of the girls, who were illiterate then and consequently not able to read mathematics; that is the reason why Byrne wrote but a few words, letting the figures speak for themselves. In the same period, Thomas Fowler invented a system to perform calculations rapidly for Poor law Unions (Parish charity), but this ternary system has been completely forgotten, after the success of binary system in computers. In both cases, the context of 19th century England is worth the study, in order to link maths to social studies in a historical point of view.

4.1 Oliver Byrne's Euclid

Studying this book with can show students how they should use colours in their drawings or think differently the relation between the text and the illustrations. An interesting follow-up is the redesigning of usual geometry theorems, using only strictly necessary words, without naming the points but showing the shapes in the text.

The picture on the right is used to sketch the proof of Pythagoras's theorem, as done in the 47th proposition of Euclid's *Elements*, book I. The points are not labelled and the shapes are quoted as they are: for instance, the theorem would be expressed in terms of red square, black square and blue/yellow square. The text following the picture is full of shapes: the segment lines according to their colours; the squares, with respect to their colours and positions (but not sizes); the yellow angles, each one paired with the black one; the similar triangles that are used in the proof.



The main idea in the proof is expressed very simply: the (area of) the yellow rectangle is equal to the (area of) the black square, and similarly the (area of) the blue rectangle is equal to the (area of) the red square.

Finally, as the (area of) square on the hypotenuse is the sum of (the area of) the two rectangles, the theorem follows. Isn't that an interesting way of simplifying the text?

4.2 Fowler's *Balanced Ternary*

The main part of the lesson was about bases in general, with examples in binary and hexadecimal systems; it didn't use original texts except the famous paper by Leibniz about binary (Leibniz, 1703). In a search of topics for Baccalaureate subjects, I read a paper in Scientific American, it was about "the most efficient integer base", which appeared to be base 3; a remark was made about Fowler's machine and allowed me to close my slideshow on bases with a historical presentation.

Thomas Fowler (1777-1843) was a self-taught mathematician, who lived in Great Torrington, Devon, UK. As he explains in the preface of his *Tables for Facilitating Arithmetical Calculations* (Fowler, 1838), he published them *chiefly for the purpose of facilitating the very troublesome Calculations, which occur every Quarter in making up the Accounts of Poor Law Unions. Having [him]self been employed in the Torrington Union, to make up the Accounts, at the commencements, [he] found the most troublesome part of the business [$\cdots$].* The Balanced Ternary system doesn't use the digits 0, 1 and 2 as does the usual ternary system, but 0 (which is denoted by $\odot$), $+$ and $-$, provided that every integer can be written in a unique manner as the sum or difference of powers of 3.

For instance, if you want to express the number twenty-three in ordinary ternary, you will write it as $212_{(3)}$, because $23_{(10)} = 2\times 9 + 3 + 2 = 2\times 3^2 + 1\times 3^1 + 2\times 3^0$, whereas in balanced ternary, as you don't use the digit 2, you write twenty-three as $+\odot --$, because $23_{(10)} = 27 - 3 - 1 = 1\times 3^3 + 0\times 3^2 + (-1)\times 3^1 + (-1)\times 3^0$. When the students are familiar with the system, I can show them an excerpt from Fowler's *Tables*:

I may now conclude, with an Example of Multiplication, by the Ternary Scale, which scarcely requires any mental exertion whatever; no Multiplication, nor even Addition, is required, as ordinarily practised.

```
        Multiply + ⊙ − − + − +    =   628
              By + − + ⊙ ⊙ − ⊙    =   564
                 ---------------      ---
             − ⊙ + + − + − ⊙          2512
         + ⊙ − − + − + ⊙ ⊙           3768
     − ⊙ + + − + −                  3140
 + ⊙ − − + − +
 ---------------------------------   ------
 + − ⊙ ⊙ ⊙ ⊙ ⊙ ⊙ − − + − ⊙    =  354192
```

Did the reader try to verify the result in Balanced Ternary? That is what the students have to do, and then construct the addition and multiplication tables in Balanced Ternary; eventually, they have to perform several computations and conversions in Balanced Ternary: great fun with strange objects! Who would have time and motivation to create such a system nowadays?

5 The final touch: History of mathematics in the exams

The euromaths exam takes place every year in June, a couple of weeks before the other session of the Baccalaureate; it is an oral examination and, more precisely, an oral examination on foreign language. The students are given (short) texts on a variety of subject in everyday mathematics, history of mathematics, etc., and they have to comment the text they've got to study, answering several questions too; the total time is 20 minutes for each student; the jury is composed of a teacher of mathematics and a teacher of English

The original texts are usually adapted because of their difficulty, and they deal with rather easy matters, because the English teacher might be alone (official communication!) In this purpose, simple historical texts are highly valued, because usually make it easy for the English teacher to engage discussion on non-mathematical matters$\cdots$

5.1 Record's multiplication

It is a subject from the 2009 Baccalaureate session: *In 1542, the Welshman Robert Record published The Ground of Arts, in which he showed how to multiply two numbers between 5 and 10. Here is the multiplication* *of 8 by 7:*

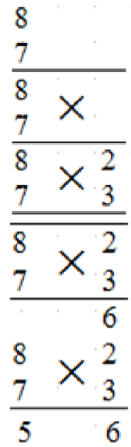
First set your digits one over the other:

Then from the uppermost downwards, and from the nethermost upwards, draw straight lines, so that they make a St. Andrew's cross:

Then look how many each of them lacks of 10, and write that against each of them at the end of the line, and that is called the difference.

Multiply the two differences, saying, "two times three make six", that you must ever set down under the differences.

Subtract one difference from the other digit (not from his own),as the lines of the cross warn you, and write it under the digits. You can take one or another, for all is like: if you subtract 3 from 8 or 2 from 7, it remains 5. So 7 multiplied by 8 is 56.



The text was adapted from Richard J. Gillings' *Mathematics in the times of the Pharaohs*; the follow-up questions were these:

1. Using Record's method, multiply 6 by 9.

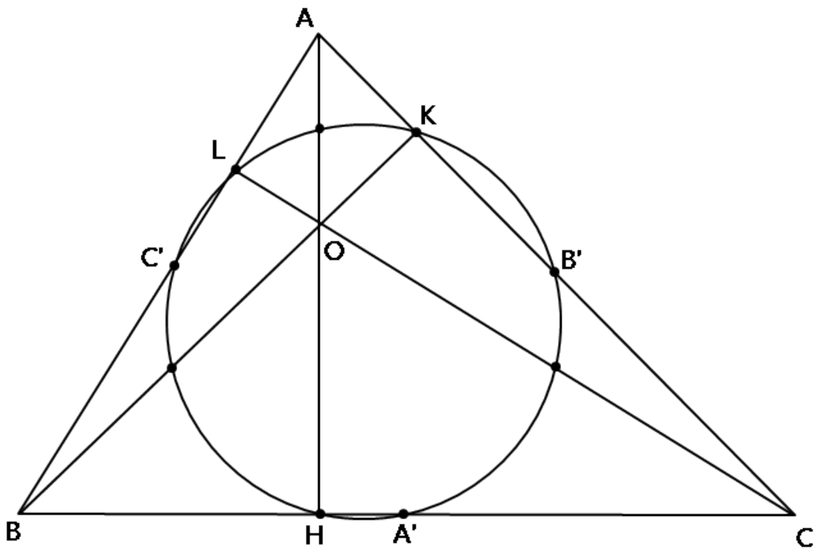
2. In Record's last explanation, we can read: You can take one or another, for all is like. Why is it true?

3. Prove that Record's method is true, by choosing two digits a and b and showing that the final line contains the product ab.

You can see that the examination is based on the comprehension of the text.

5.2 The Feuerbach circle

Another example of subject, from the 2010 Baccalaureate session: *In every triangle, the three midpoints of the sides, the three base points of the altitudes, and the midpoints of the three altitude sections touching the vertices lie on a circle.*



The proof consists of two steps: in the first we demonstrate that the circle circumscribing the triangle of the three midpoints of the sides passes through the base points of the altitudes; and in the second we show that the circle circumscribing the triangle of the altitude base points passes through the midpoints of the altitude sections.

Step 1*: Let A', B' and C' represent the midpoints, respectively, of sides BC, AC and AB. Let H be the base point of the altitude AH. Then the trapezoid $HA'B'C'$ is isosceles and it is therefore a quadrilateral inscribed in* *a circle, that we will name C. In the same manner we would demonstrate the other altitudes base points, namely K and L, lie on circle C, circumscribing triangle $A'B'C'$.*

Step 2: Let the altitudes of the triangle ABC be AH, BK, CL, and O their point of intersection. We will now show that the centre of each altitude section touching a vertex, let us say section OC, also lies on circle C. [End of this proof to be completed in q3]

The text was adapted from Heinrich Dörrie's books on the great problems of Mathematics, published in 1965. That is already history, isn't it? (Mind your answer, I was already born in 1965···) The questions were these ones:

1. What are the "midpoints of the three altitude sections touching the vertices"?

2. In step 1, explain why $HA'B'C'$ is a trapezoid and why it is isosceles [Hint: HC' is the radius of the circle which has AB as its diameter.]

3. Show that an isosceles trapezoid is always inscribed in a circle (you can sketch the situation; it may be useful to consider some perpendicular bisectors.)

4. Completing step 2: a) Consider triangle OBC: what are its altitude bases? What is their circumscribed circle? b) Apply the result of step I to triangle OBC: what is the circumcircle of these 3 points?

But this subject wouldn't be given anymore, because the present day students are not trained in geometry as much as they used to be some years ago. In fact, the euromaths programme is a space of freedom for the teachers and the students that are eager to learn deeper mathematics. Our current reform of the curriculum tends to skip the difficulties for all the students to succeed in their high school exams, but the counterpart of this is that the pleasure of meaningfulness little by little disappears. Teaching maths in English as a foreign language allows teachers to be creative and lead our students to "old fashioned" mathematics, including historical texts. Old fashioned maybe, but so tasty!

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