THE INFLUENCE OF SOLVING HISTORICAL PROBLEMS ON MATHEMATICAL KNOWLEDGE FOR TEACHING

Kathleen CLARK

Florida State University, 1114 West Call Street, Tallahassee, Florida, USA
kclark@fsu.edu

ABSTRACT

This research investigated the role of solving historical problems on prospective mathematics teachers' mathematical knowledge for teaching. The primary research question was: *In what ways does a prospective secondary mathematics teacher's work on historical problems contribute to their developing mathematical knowledge for teaching?* In an effort to capture ways in which prospective secondary mathematics teachers (PSMTs; those who will teach pupils aged 10 – 18) engaged in solving problems found in historical sources during a history of mathematics course, I implemented an historical problems and analysis assignment. For the assignment each PSMT selected ten problems that they previously solved in the course. They then presented their solution and provided a reflection on how work on each problem informed their understanding of the underlying mathematical concepts addressed in the problem. The presentation will include a summary of the most often selected historical problems and will highlight the common themes identified as a response to the study's primary research question.

1 Mathematical knowledge for teaching

The concept of mathematical knowledge for teaching (MKT) is the most recent focus of trying to understand the special knowledge teachers must possess to teach mathematics – and to teach the subject matter well. This focus of mathematics education research began with large-scale efforts primarily focused on assessing teachers. Furthermore, the field has endeavored to define exactly what the nature of the special knowledge for teaching mathematics is, in much the same way that mathematics education (among other disciplines) sought to establish its unique definition of pedagogical content knowledge (PCK) in the wake of Schulman coining the term in 1986.

Ball, Thames, and Phelps (2008) described knowledge for teaching beyond the "obvious" knowledge of "topics and procedures that [teachers] teach" (p. 395). To do this, they concentrated on "how teachers need to know that content" (p. 395) and they sought to "determine what else teachers need to know about mathematics and how and where teachers might use such mathematical knowledge in practice" (p. 395). Several scholars with an interest in what history of mathematics contributes to teaching and learning mathematics (Clark, in press; Jankvist et al., forthcoming) have begun to focus on the potentiality of the history of mathematics to be one dimension of the "what else" described by Ball et al. and how this specialized knowledge contributes to prospective secondary mathematics teachers' (PSMTs') future practice.

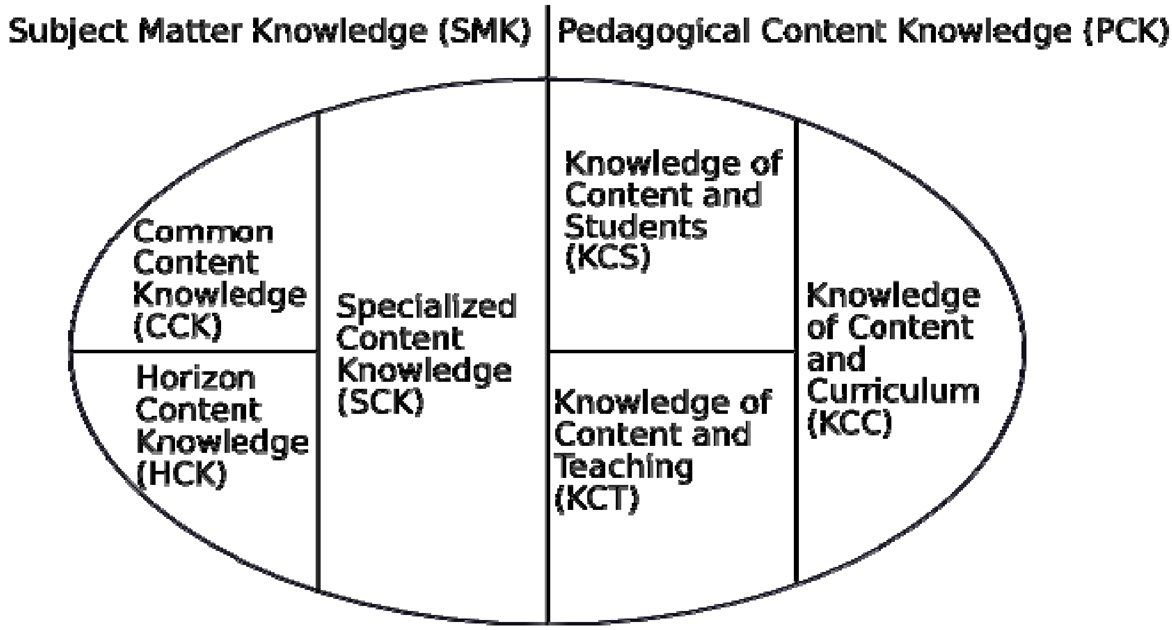

Figure 1. Division of Subject Matter Knowledge and Pedagogical Content Knowledge into further subdomains (Ball, Thames, and Phelps, 2008, p. 403).

Ball, et al. (2008) deconstructed Shulman's concepts of Subject Matter Knowledge (SMK) and Pedagogical Content Knowledge (PCK) into further subdomains (or subcategories), as shown in Figure 1. Although the "egg model" for mathematical knowledge for teaching may continue to evolve, the six subdomains provide a framework from which to gain insight into the ways that history of mathematics may contribute to PSMTs' MKT. SCK and KCT may be the obvious types of knowledge for which PSMTs' historical problem solving has the most influence. HCK, however, may also have a role to play, especially since many scholars have remarked on the tentative nature (e.g., Ruthven, 2011) of this subcategory – and that it certainly has interplay with the other five subcategories. One interpretation of HCK is that it relates to knowledge influenced by awareness, dispositions, and orientations towards particular instructional practice. Thus, history of mathematics may have strong ties to influencing PSMTs' MKT along this domain; however, stronger definitions of the HCK subcategory and research aimed at testing such definitions is needed. Rowland and his colleagues (2010) introduced the analytical framework of The Knowledge Quartet, or the different forms of knowledge trainee teachers possess. Of the four types of knowledge, Rowland's notion of foundation knowledge "concerns trainees' knowledge, understanding and ready recourse to their learning in the academy, in preparation (intentionally or otherwise) for their role in the classroom. It differs from the other three units [of the Knowledge Quartet] in the sense that it is about knowledge possessed, irrespective of whether it is being put to purposeful use" (p. 1842). This framework is also promising for the ways in which teacher educators can examine and characterize such foundational knowledge of mathematics teacher candidates. In particular, if, when, and how history of mathematics contributes to the development of foundation knowledge, serious implications exist regarding the policies and practices governing the role of history of mathematics (e.g., course content or courses in general) of mathematics teacher preparation programs.

2 Participants and study details

Twenty-four PSMTs, enrolled in a *Using History in Teaching Mathematics* course, participated in this study. The course was required for all PSMTs within two certification tracks, either middle grades mathematics certification (for teaching pupils aged 10-14) or secondary mathematics certification (for teaching pupils aged 11 – 18). Student work from one of the course assignments, "Historical Problems and Analysis", was used as the primary data source in the study. The assignment was described to students as follows:

> During most class sessions we will work with historical problems – either in groups during class or individually (or with a partner) outside of class. You will not be handing in each assignment. Instead, you will keep track of the work that you do and for the final task you will select ten problems, tasks, or activities that you feel represent your best effort toward achieving the course objectives. For each problem selection you will: (1) state each problem, task, or activity; (2) present your solution, work, or explanation (as appropriate); (3) describe which of the objectives you feel you addressed when completing and reflecting upon the problem, task, or activity and why; and (4) provide a reflection of how your work on the historical problem contributed to your understanding of the underlying mathematical concepts within the problem, task, or activity.

Students in the course – PSMTs – could elect any format to present their work on this assignment. In most cases, PSMTs used a format that included the stated problem, presentation of accompanying work, and a written narrative in response to the third and fourth items outlined in the assignment. Of the 24 students enrolled in the course, 23 completed the course with a passing grade, and of the 23 completed "Historical Problems and Analysis" assignments 13[1] were analyzed for the purpose of this paper.

The primary research question for this investigation was: *In what ways does a prospective secondary mathematics teacher's work on historical problems contribute to their developing mathematical knowledge for teaching?* Analysis of the collection of PSMTs' responses to the problems they selected included three tasks: (1) reviewing the accuracy and completeness of the solutions, (2) coding each reflection for revelations of how working on the problems improved PSMTs' understanding of the underlying mathematics, and (3) coding each reflection for expressions of beliefs about mathematics prompted by PSMTs' study of history of mathematics.

2.1 Data analysis

Although the problem solution (presentation, accuracy, completeness) was important to assess each PSMT's work for a course grade, for the purpose of this investigation, I was more interested in what each PSMT articulated in their analysis with regard to how the particular problems influenced either their mathematical knowledge or they way in which they now thought about particular mathematics concepts. I note that this investigation represents a preliminary effort to assess the potential for future research and as a result caution is offered regarding the use of such self-reported data. That is, without triangulation (e.g., using additional data sources) of each PSMT's claim, my interpretation of how

[1]Twelve of the 13 PSMTs were pursuing secondary mathematics certification and one was pursuing middle grades mathematics certification.

solving historical problems might influence their developing knowledge for teaching is just that: an interpretation. In future research, pre- and post-assessments of content and micro-teaching tasks and subsequent reflections will provide substantiation of claims and enable possible framework building to use for identifying influences on PSMTs' mathematical knowledge for teaching.

Table 1 presents a brief description of the problems selected by the PSMTs and the frequency of each. For this discussion I highlight two problems (or, types of problems in the case where multiple examples were available) that were selected and discussed by a majority of the PSMTs: problems on the method of false position (solving linear equations) and problems on the method of completing the square (for solving quadratic equations).[2]

Table 1. Historical problem selections.

Historical problems selected	**Number of students selecting**
1. Babylonian area calculation (Ancient Babylonian problems)	9
2. Egyptian unit fractions	8
3.The power of zero: 0^0	5
4. Method of false position	**10**
5. Acceptance of negative numbers	2
6. Use of different values of π	7
7. Euclid's Elements (various problems)	6
8. "A square and things" (method of completing the square)	**13**
9. Trigonometric identities	6
10. Imaginary numbers	6
11. Stifel's symbols	11
12. Proofs of the theorem of Pythagoras	9
13. Italian abacists	1
14. Figurate number tasks	1
15. Solving cubic equations	4
16. Decimal fractions	1

3 Problem reflections

3.1 Method of false position

The first type of problems, those based on the method of false position (found in surviving papyri from Ancient Egypt), were selected, solved, and discussed by ten students. Two problem sets were assigned to students, one set from Sketch 9 of *Math Through the Ages* (Berlinghoff & Gouvêa, 2004) and one from a task I created that required students to solve problems such as: A quantity three times; the quantity's three-fifths and one-fifth is added to it. It becomes 19. What is the quantity? The goal of including this topic in the *Using History in Teaching of Mathematics* course was to encourage PSMTs to consider the origins of solving linear equations and to connect the method of false position

[2]The problems based upon the algebraic symbols introduced and used by Michael Stifel were not considered for this investigation (even though 11 students discussed these problems in their historical problems analysis task) because these problems were not considered to contain sufficient rigor.

to important underlying ideas when solving linear equations, including, but not limited to rate of change and "undoing" arithmetic operations.

However, many PSMTs did not discuss the potential mathematical content of the false position problems, nor did they discuss how their knowledge developed as a result of their problem solving. Instead, many revealed their beliefs about solving historical problems using the method of false position and these were often superficial in that the PSMTs focused on issues of ease or familiarity. For example, many PSMTs shared some aspect about how the method was difficult to understand, how a modern method of solving linear equeation was "easier" to perform, or they were simply unable to analyze their mathematical understanding:

> "⋯it is frustrating to use 'guess and check' for solving linear equations when we know a way that comes much easier to us" (Clara[3]).
>
> "I understood it but did not understand why that method would be used⋯
> The false position is about guessing and proving [the answer] is right" (Julie).
>
> "It was quite difficult to solve their way, but solving it in the modern way was simple⋯.The false and double false position is not the most efficient way to solve for "x" (Katrina).
>
> "I found it more difficult to solve because it is a more involved method" (David).
>
> "The tricky part came when you had to decide how to go about solving the remaining problem based on whether your estimation gave an error above or under the desired number" (Chantal).

One PSMT, however, described her understanding of the method of false position by relating it to modern solution methods. Janine stated that, "I honestly feel more comfortable with the historical method⋯. The main difference with the two problems is the usage of a variable for the unknown [which is not used in the historical method]. For the most part, the method of solving for the "x" in the modern method is very similar to finding the solution in the historical version." Thus, instead of stating a key difference in the methods of solution, Janine tried to articulate that the actual solution process was the same – and that only symbolic representation set them apart.

In light of this sample of PSMTs' reflections, it is difficult to identify a subcategory of MKT (Ball et al., 2008) or Rowland's notion of foundation knowledge that was influenced by engaging in and reflecting about the false position problems. As a flawed application, aspects of each of the excerpts hint at a basic form of what Rowland described as the knowledge possessed – though in almost every case this may be interpreted as flawed knowledge possessed, or incomplete knowledge possessed.

3.2 Method of completing the square

As with the historical problems on the method of false position, PSMTs were assigned several problems to choose from for inclusion in their "Historical Problems and Analysis" assignment that focused on the method of completing the square. Problems from three different time periods were part of the course – from Ancient Babylonian mathematical texts, Euclid's *Elements*, and from the famous text of al-Khwarizmi. However, all 13 PSMTs who selected problems on the method of completing the square to solve quadratic equations selected those from al-Khwarizmi's text.

[3]All names are pseudonyms.

PSMTs' reflections in response to the prompt, "provide a reflection of how your work on the historical problem contributed to your understanding of the underlying mathematical concepts within the problem, task, or activity", about their work on the method of completing the square also lacked strong evidence of how their understanding was impacted. Furthermore, the strongest declaration expressed by these future teachers was that they had no previous experience with the historical roots of the method of completing the square, and applying that method to solve quadratic equations:

> "I had no clue that the quadratic formula and completing the square had actually anything to do with areas of square or rectangles" (Katrina).
>
> "Before these problems I never completely understood completing the square" (Julie).
>
> "Even though I was aware of completing the square method, I was completely unaware of where it originated" (Steven).

Other PSMTs discussed the connections between algebraic and geometric representations emphasized by the method of completing the square:

> "[Al-Khwarizmi's] use of the geometric representation, however, really helps you to see the completing of the square as you work through the problem" (Janine).
>
> "I have trouble considering a geometric representation to solve algebraic problems and [this method] really gave me ideas about having geometric representations for this and other sorts of problems" (Megan).
>
> "This task deepened my understanding through the use of geometrical methods and now I can finally say I understand completing the square" (Kevin).

Again, I anticipated that PSMTs would discuss what they now understood about solving quadratic equations after examination and successful application of the geometrical and numerical demonstrations provided by al-Khwarizmi. However, this proved difficult for most of the PSMTs. In addition to the two broad types of declarations (examples above), two students revealed incomplete understanding of the method of completing the square. Chantal, for example, began her reflection with "this was definitely something new for me". She added,

> I misunderstood at first, adding the missing portion to what the equation was equal to, just because I thought everything should equal that particular number (subtraction instead of addition of the two numbers given). I understand enough that I would feel confident if having to present this to students.

Chantal was able to identify how she incorrectly approached the problem; however, she still claimed that she only understood "enough" to teach the concept to students. Unfortunately, Chantal could not further articulate what her understanding was (or, was not).

Carrie's reflection on the method of completing the square revealed her incomplete mathematical understanding. She stated that

> ⋯the topic can show students the purpose of a topic that is constantly being drilled into them that they often do not understand. It uses the history of math to show what squares of quantities

were used for so that students can begin to make connections and understanding of exponential ideas and terminology.

Not only did Carrie's reflection shift to attention on students' understanding of completing the square (in fact, many PSMTs did this), she also incorrectly summarized the purpose of the method. Her conclusion that squares of quantities were used for understanding exponential ideas was mathematically incorrect compared to what other PSMTs discussed (e.g., geometrical representations of equations involving squares). Furthermore, this particular reflection raises the question of how history of mathematics – in particular, working on non-trivial historical problems – can actually highlight what mathematical knowledge prospective teachers do not possess (as opposed to the knowledge possessed that Rowland described) and that efforts to implement more historical problems in teacher training programs may prove beneficial.

4 Implications

This investigation intended to identify the ways in which history of mathematics influenced PSMTs' mathematical knowledge for teaching. I anticipated that PSMTs would choose problems from throughout the 15-week course and be able to discuss their understanding of the underlying mathematical concepts when viewed through the lens of their own work on the various historical problems. I was also hopeful that the prospective teachers would have sufficient practice at such written reflections given the numerous writing tasks assigned in the *Using History in Teaching Mathematics* course. Unfortunately, most of the 13 PSMTs' reflections did not contain explicit descriptions of what they really understood nor did they discuss how their work on historical problems assisted in that understanding.

Still, important lessons for future practice in mathematics teacher education can be learned from the initial analysis of the data. For example, prospective mathematics teachers must be provided ample opportunity to reflect on their own mathematical thinking and to articulate that thinking in order to prepare them for doing the same with their future pupils' mathematical thinking. Assigning such tasks to prospective mathematics teachers is not alone sufficient. Instead, as mathematics teacher educators interested in the role of history of mathematics in the preparation of future teachers, we must do a better job at modeling such reflective practices. Furthermore, we must create opportunities for which PSMTs can participate in public reflective practice. In this way PSMTs are called upon to listen and respond to their peers when undertaking these essential reflective tasks.

Finally, establishing the purpose of particular historical problems may yield important information regarding the question of whether historical problem solving contributes to mathematical knowledge for teaching. For example, explicitly presenting the method of false position as a way to analyze rate of change may have prevented many PSMTs' reflections focused on the perceived difficulty or inefficiency of the method. Instead, PSMTs could be prompted to focus on what mathematical ideas are present, why the mathematics "works", and the ways in which different mathematical ideas are related or represented. Such information may help reveal with more certainty the knowledge PSMTs possess (Rowland, 2010) and has the potential to tease out the influence of historical problem solving on the subdomains Ball, et al. (2008) described.

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