

A DISCUSSION ON THE SIGNIFICANCE OF THE DISCOVERY OF MATHEMATICS BAMBOO SLIPS FROM THE WARRING STATES PERIOD, QIN AND HAN DYNASTY

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ABSTRACT

The recent acquisitions of archaeological findings of bamboo slips in many institutions in China provide us a good significance for the study of development of mathematics in Ancient China. These includes the Tsinghua University (arithmetic table, dated from the Warring States period), the Yuelu Academy (bamboo slips of the book “mathematics”, Qin Dynasty), the Beijing University (bamboo slips of mathematics, dated in the Qin Dynasty), and the Hubei Museum (Han bamboo slips of mathematics at the found on “land of sleeping tiger”, Han Dynasty). Firstly, it gives us rare first-hand information on the knowledge of mathematics in Ancient China. Secondly, it helps to clarify that the nihilistic attitude holds by some scholars on the development of early Chinese mathematics was unfounded. Thirdly, it provided strong evidence in the study of Chinese mathematics, that the major methods and mathematics problems in *Jiuzhang Suanshu* (Nine Chapters) was completed in early Qin period. Most importantly, it provides reliable literature to support that the first peak of mathematics achievement in China was in the Spring and Autumn and Warring States period. In the past, the statement was only supported by the evidence of the work of *Jiuzhang Suanshu* and its explanations and remarks made by Liu Hui.

1 The recent discovery of mathematics bamboo slips from the Warring States Period, Qin and Han Dynasty

From 1983 to early 1984, about 200 mathematical bamboo slips “*Suanshu shu*” were discovered in Jingzhou Zhangjiashan Han Tomb No. 247 in Hubei. There was less significant harvest for 20 years in unearthed mathematical bamboo slips since then. Then there were much news on discovery of mathematical bamboo slips from the Warring States Period, Qin and Han Dynasty in the past 10 years. The news of discovery excited the circle of the scholars in history of Chinese mathematics. Some of the discovered bamboo slips have been treated and studied, some are under treatment. We introduce as follow these discoveries according to the date of history of these mathematical bamboo slips:

Calculation table (*suanbiao*), Warring States Period, collection of Tsinghua University The Centre of research and protection for relics of the Tsinghua University has a collection of calculation table (*suanbiao*) dating from the Warring States Period. The book has 21 bamboo slips and is dated 2300 years from now. The whole table consists of 441 unit cells, in the form of 19 rows time 19 columns.

The right column and the upper column start from bottom to above and from left to right. It could be reads in order, from $\frac{1}{2}$ to the cardinal number 1 to 9 and then all two-digit number. Their products were recorded at the intersection of the cells. Therefore, the table is a multiplication in the form of 9 times 9; the rest is the extension of the table.¹

Wooden Book of 9 times 9 table, Qin Dynasty, unearthed in Liye Town, Hunan In the year 2002, a multiplication table made from wood² dating from the Qin Dynasty in the form of 9 times 9 was unearthed from an old well in Liye Town of Hunan (湘西). There are in total 113 characters. Most of the bamboo slips of 9 times 9 multiplication table unearthed in 20th century were incomplete, and this table from the Qin Dynasty is a complete 9 times 9 table.

Bamboo slip «shu», Qin Dynasty, collection of Yuelu Academy Hunan University In December 2007, The Yuelu Academy in Hunan University had purchased a batch of bamboo slips from the antiques market in Hong Kong. Those bamboo slips were examined by a group of experts and it was confirmed that these were dated from Qin Dynasty. We are most interested in that collection of slips under the name “Shu”(mathematics). At present, 236 bamboo slips in Shu has been arranged and numbered, with 18 pieces of them not in good conditions (without numbering). The better-preserved slips are about 27.5 cm in length and 0.5 to 0.6 cm in width. There are three braided rope at the upper, middle and lower part of each bamboo slip. “Shu” was printed at the back of bamboo slip numbered 0956, which make the name of the book.³

In the format of “Shu”, a complete problem (suan ti) generally consisted of four parts, conditions, questions, answers and method. Only a few of the problems was given a title. There are 19 exemplars of methods (shu) in the book. For example, “he fen shu” (xx) and pithy for multiplication. There are 34 bamboo slips with the record of ratio of volume and weight of grains and their rate of exchange. Another 3 bamboo slips recorded measurement and units. The content of «shu» from Qin Dynasty included arithmetic operation of fractions, areas of fields, crop yield, ratios of grains and its rate of exchange, shuaifen, shaoguang, volume, ying bu zu, gougou, yingjun and zu wu quan etc.⁴ There was only one Gougou problem, and is in consistence with the gougou geyuan in GouGu Chapter in Jiuzhang Suanshu, with simple and unsophisticated wordings.⁵

Mathematical bamboo slip, Qin Dynasty, collection of the Beijing University In early 2010, a friend from Hong Kong donated a batch of bamboo slips to the Beijing University. Among these slips, there are in total 400 slips written with mathematics, a big proportion in this batch of slips. According to preliminary study, these mathematical bamboo slips is close to those bamboo slips Shu from Qin Dynasty by Yuelu Academy, Suanshu shu from Han Dynasty and the Jiuzhang Suanshu.

¹Tsinghua's new development of bamboo slip. “The recent development of the jian”, from the website of the Fudan University's Research Centre of Unearthed Literature and Ancient Text:
<http://www.gwz.fudan.edu.cn/ShowPost.asp?ThreadID=3522>, 2010-8-10

²Hunan Province Archaeological Research Office: Brief report of Hunan Long Shan Li Ye Warring States Period – ancient city of Qin Dynasty, No.1 well excavation. “Wen Wu” vol. 1, 2003.

³Chen Songchang : Content summarization of the bamboo slips from Qin Dynasty in Yuelu Academy. “Wen Wu” vol. 3, 2009.

⁴Xiao Can, Zhu Hanmin: Major contents and historical value of the bamboo slips “shu” from Qin Dynasty in Yuelu Academy. “Research of Chinese History” vol. 3, 2009.

⁵Xiao Can, Zhu Hanmin: New evidence of gougou – related research of the bamboo slips “shu” from Qin Dynasty in Yuelu Academy. “Research of the history of natural science” Paper 29, vol 3, 2010.

The problems of the slips are classified into different groups, which mean similar problems are put under the same group. There are two formats used for describing the mathematical methods and the structure of problems. One started by giving a description for algorithm, with the wordings “it is stated that. . .” and then listed the exemplar of problems, with some variation in the numerical values in the problem. The second one is by listing the problems, and then provides with the general form of algorithm with the wording “A says. . .”. As we know, these two formats is the main body of style Jiuzhang Suanshu. Another point worth mentioning is that there is a passage around 800 words, which starts with “Lu jib inquire mathematics from chin quid”, discussing the beginnings off mathematics, its role and significance.^{6,7}

Mathematical bamboo slips from Shui-Hu-Di (Land of the Sleeping Tiger), Han Dynasty, collection of Hubei Museum There were 216 bamboo slips unearthed in a Han Tomb in Shui-Hu-Di (Land of the Sleeping Tiger) in Hubei. The book is named “Suan shu”. These bamboo slips were slightly damaged, but the characters and the texts are clear.

And from the calendar unearthed at the same finding dated the seventh year of Hou-Yuan, the year of the emperor Wen-Di of the Han Dynasty (157 BC). There are 10 pictures of these bamboo slips on the “Jianghan kaogu”. These bamboo slips are being compiled, and the preliminary information revealed that some of the questions were similar with Shu (Qin Dynasty), Suanshu-Shu (Han Dynasty) and Jiuzhang-Suanshu, but there are also content which has not been appeared in any of the Chinese traditional mathematical works.⁸ In addition, pieces of mathematical bamboo slips were also unearthed from Fuyang of Anhui and Linyi Yinqueshan (mountain of silver bird) from Shandong .

2 The significance of mathematics bamboo slips from the Warring States Period, Qin and Han Dynasty

These mathematical bamboo slips unearthed are rich in mathematical content; and have great significance in the course of study in the history of Chinese mathematics. These have been discussed in Zou Taihai’s papers.⁹ This paper would like to share the following points.

2.1 It provides the firsthand literature from pre-Qin dynasty and Qin Dynasty

Over the past century, scholars in the field of history of Chinese Mathematics were much regret with the fact that very few mathematical literature were passed down from pre-Qin and Qin Dynasty. From the past relic exploration, only a few fragments of Chousuan and 9 times 9 multiplication table were unearthed. We could only deduce the achievement and development of mathematics in pre-Qin Dynasty and Qin Dynasty according to the shape, characterization and a few second hand information unearthed.

⁶Brief report of the work of Beijing University Unearthed Literature Research Office, vol 3, October 2010

⁷Discussion on China’s earliest mathematical theories found from Beijing University’s compilation of bamboo slips from Qin Dynasty (Guang Ming Daily), China News Web: <http://www.chinanews.com.cn/cul/2010/10-25/2609393.shtml>, 2010-10-25.

⁸Cai Dan: A report at the explanation meeting of bamboo slips from Qin Dynasty, September 2010.

⁹Zou Dahai: unearthed bamboo slips and the early Chinese mathematics history, “Humanity and Society” Paper 2, vol 3, 2008.

The discovery of mathematical bamboo slips from the Warring States Period, Qin and Han Dynasty provides researchers with the first hand information about mathematics research from pre-Qin Dynasty and Qin Dynasty, so that we can truly understand some of the real development of mathematics at that particular time. From these bamboo slips, we found that the development of mathematics was well-progressed in pre-Qin Dynasty and Qin Dynasty. There was a complete multiplication table in the form of 9 times 9, also complete methods and solution of arithmetic operation of fraction, fang tian, sumi, shuai fen, shaoguang, shanggong, ying bu zu, gougu. It can be said that in addition to technique of solving equation(fangcheng shu), which means the general solution of linear equations and the standard problem of junshu, other seven methods and problems in "jiushu" were discovered, also some difficult problems in "Jiushu" session in *«Jiuzhang Suanshu»* were found, these greatly enriched the content and range for study in the history of Chinese mathematics.

2.2 It breaks the nihilism towards the early development of Chinese mathematics

Certain people in Chinese and overseas academic sectors doubt about the existence of China's achievements in mathematics before Song and Yuan Dynasty. This does not refer to those Eurocentric academics and their followers in China who do not understand and not even try to understand the mathematics of ancient China. For example, there is one claiming to be a senior researcher at Chinese Academy of Sciences Shanghai Branch who made such a big mistake as identifying the well-known scientist Rene du Perron Descartes and Gottfried Wilhelm Leibniz as ancient Greeks. He also overblew the dark European Middle Age Era of mathematics, but ridiculed that in ancient China people only knew about gougu and achieved nothing in mathematics. In addition he blamed that the reason why China could not get the Nobel Prize was mainly because of its poor performance in ancient China.

Unfortunately, this kind of belief was constantly found in the publications, even including "China Science News"(now "Science Review"), "Dialectics of Nature" and other prestigious academic publications. This does not refer to ignorant people who are not afraid to give his or her opinion("Wu Zhi Zhe Wu Wei"), but to scholars who have in-depth study of mathematics in ancient China. Apart from the fragmentary information of Dunhuang Suanshu - the contents of these materials were relatively not rich enough, and all of them were the 5th to 10th century works -, there is no mathematical texts before the Song Dynasty handed down, therefore they suspected that the authenticity of achievements in mathematics in two Han Dynasty as well as Wei Dynasty of Southern and Northern Dynasties. Moreover, they also doubted about the existence of a mathematics book at that time, and saying that "Jiuzhang Suanshu" was a book claimed to have formed in Han Dynasty, but in fact, its earliest text was appeared in Southern Song Dynasty only. This implied that the mathematics information in ancient China is only reliable after formation of Song Ben Suanjing by Bao Huanzhi in Southern Song Dynasty Jiading period, all the previous information are unreliable.

By the end of 1983 to early 1984, "Suanshu shu" was unearthed; this to some extent refuted the above erroneous view. However, the latest period shown in li ri, which was unearthed on the same date as "Suanshu shu", was l hou er nian (186 BC). Saying that the vast majority of problems arose from pre-Qin and Qin Dynasty and existed in pre-Qin Dynasty¹⁰, and there existed more than one

¹⁰Peng Hao: Explanation of Zhang Jiashan bamboo slips "Suanshu shu" from Han Dynasty, Beijing: Science Publishing Company, 2001.

mathematics book,¹¹ in pre-Qin Dynasty was a conclusion come up with scholars after verification and it was not obvious. The mathematical bamboo slips unearthed from Warring States Period, Qin Dynasty and Han Dynasty provide the world with the mathematics in pre-Qin Dynasty and Qin Dynasty without any change of the original text made by descendants. This not only refuted the wrong saying of no existence of any mathematics work in two Han Dynasty, but also broke the nihilistic attitude adopted by some scholars towards the early development of Chinese mathematics.

2.3 It provides a strong evidence in solving problems about formation of Jiuzhang Suanshu

Over the past 1700 years, views on the development of Jiuzhang Suanshu varied a lot. As Jiuzhang Suanshu is the most important classic mathematics book in ancient China, ranking the top of all canons, and therefore the development of Jiuzhang Suanshu is an important issue academically since the 20th Century. There are three interrelated issues with different focuses. The first one is the date of completion of the major methods and mathematics problems in Jiuzhang Suanshu. The second issue is whether there was a certain kind of form that Jiuzhang Suanshu in existence before the Qin Dynasty. The third issue is the completion date of the book Jiuzhang Suanshu which Liu Hui had referred.

2.3.1 When the main mathematics methods and problems in《Jiuzhang Suanshu》were completed

On this issue, one of the founders of the history of Chinese mathematics branch Qian Baocong (1892–1974), said, “Without a doubt that the techniques in solving problems in fang tian, sumi, shuai fen, shaoguang, shanggong are mainly formed before Qin Dynasty.” I believe that: “apart from data of equation(fangcheng) which was not found in books from pre-Qin Dynasty—it is very difficult to find information on mathematical methods from literature and history books—the mathematical methods of the remaining eight chapters and even some of the questions, evidence can be found from the texts and historical relics from pre-Qin Dynasty.”¹² This saying is made before looking at the translated texts of 《Suanshu shu》 from Han Dynasty and 《Jiuzhang Suanshu》 from Qin Dynasty.¹³ I actually think, “the main body of 《Jiuzhang Suanshu》 which means using texts with technical methods(shu wen) to group examples together as well as most of the examples in books were completed in Warring States Period and Qin Dynasty.”¹⁴ This is essential for understanding the compilation of 《Jiuzhang Suanshu》.

Although Suanshu shu was not the predecessor of Jiuzhang Suanshu¹⁵, Suanshu shu had a lot of commonalities in the methods and problems with the bamboo slips of Shu from Qin Dynasty, the

¹¹Suanshu shu” is not a systematically compiled publication, but rather one that is selectively compiled with many publications. See Guo Shuchun: Preliminary analysis of “Suanshu shu”, “Research of Chinese Literature” Page 307–349, vol 11, June 2003.

¹²Qian Baocong: Abstract of Jiuzhang Suanshu. See Qian Baocong-edited “suan jing shi shu” volume 1. Beijing: Zhong Hua Book Store, 2003. “Li Yan, Qian Baocong The whole collection of science history” vol 4, Shen Yang: Liao Ning Education Publishing Company, 1998.

¹³Guo Shuchun: “Liu Hui, the world-class mathematics leader in ancient time” Ji Nan: Shan Dong Science Technology Publishing Company, 1992. revised edition in original complex, Tai Bei: Ming Wen Book Store, 1995.

¹⁴Guo Shuchun: “Preface, explanation of Jiuzhang Suanshu” Shanghai Ancient Text Publishing Company, October 2009, April 2010.

collection of bamboo slips from Beijing University and Jiuzhang Suanshu. These commonalities obviously are the consensus of the academic circle in the early-Qin Dynasty. And this reinforces the saying that the main methods and questions of Jiuzhang Suanshu were completed in early-Qin Dynasty.

2.3.2 The existence of a particular form of *«Jiuzhang Suanshu»* in early-Qin Dynasty

It is commonly recognised in the academic circle that *«Jiuzhang Suanshu»* was the result of long-term accumulation, developed from “Jiushu,”¹⁶ and was completed in Han Dynasty. However, mathematicians before Ming Dynasty have different arguments. Starting from the mid-Qing Dynasty, many scholars expressed their own views. In the existing data, the first one who talked about the compilation of *«Jiuzhang Suanshu»* was Liu Hui. He said: “Zhou Gong introduced the social norms and therefore “Jiushu” existed. The details of “Jiushu” are actually “Jiuzhang”. As the Qin Emperor burnt so many books and the remaining ones were broken and incomplete. After Shi Jue, both Zha Chang, bei ping hou and Geng Shou Chang, da si nong zhong cheng of Han Dynasty were skillful in mathematics. They prepared the new edition of the books as the old ones were incomplete. When compared the content lists, they might be different from the old ones, but the actual contents were more or less the same.”¹⁷

That is, Liu Hui believes that Jiuzhang Suanshu was developed from Jiushu and certain format of the text was formed in early-Qin Dynasty. This format of Jiuzhang Suanshu was damaged in Qin-fire. (I believe that the damage was come from the commotion in late Qin Dynasty, especially the burning and plunder by Xiang Yu and his men.)

It should be pointed out that we cannot put the views and comments by Liu Hui and the views of people after him to have the same weight on a balance. In other words, only by successfully refuting Liu Hui’s argument, could we consider whether the argument held by people after Liu Hui being reasonable. If we negate the argument of Liu Hui without evidence of any loophole, and make speculations, is far from correct. We will argue later on, the negation opinion held by Dai Zhen on the Liu’s historical material was wrong. From the analysis on the format of the Jiuzhang Suanshu and analysis on the information of the price of material of that time, Liu’s argument is perfectly correct.

«Jiuzhang Suanshu» can be divided into two forms, one is using texts with technical methods (shu wen) to group problems together, another one is the applied question bank. The style of using texts with technical methods (shu wen) to group problems together can be divided into three styles. The differences in styles illustrates that *«Jiuzhang Suanshu»* cannot be compiled in a single epoch. It was the efforts of many mathematicians from different generations. The three styles in using texts with technical methods (shu wen) to group problems together have a total of 82 methods (shu) and 196 problems (wen), covers all part of the six chapters included fang tian, sumi, shaoguang, shanggong, ying bu zu, fangcheng, also shuai fen, shuai fen in jun shu zhang, problem of jun shu and gougu shu in gougu zhang, gougu rong fang, rong yuan, ce yi zhu shu. Applied question bank is used in

¹⁵Guo Shuchun: “About the relationship between Suanshu shu” and “Jiuzhang Suanshu”, “Zi, Qu Fu University of Education Journal”, paper 34, vol 3, 2008.

¹⁶Zheng Xuan of Eastern Han Dynasty (127–200) referred to Zheng Zhong’s(?–83) “Zhouli zho” to explain “Jiushu” and said, “Jiushu: fang tian, sumi, cha fen, shao guang, shang gong, jiu shu, fang cheng, ing bu zu, pang yao. Now it has zhong cha, xi jie, gougu.” Lu Deming reckoned xi jie stream detailed article. See “Zhouli”, “Explanation of 13 books”. Beijing, Zhong Hua Book Store, 1982.

¹⁷Guo Shuchun edited: Edited “Jiuzhang Suanshu” supplementary edition. Shen Yang: Liao Ning Education Publishing Company, Tai Bei: Jiuzhang Publishing Company, 2004.

the remaining non-shuai fen problem in shuai fen zhang, non standard jun shu problem in jun shu zhang, questions in solving gougu shape in gougu zhang and three questions¹⁸ of yin mu wang shan. If non-shuai fen problem and non-jun shu problems from shuai fen and jun shu zhang are deleted respectively, the content of *«Jiuzhang Suanshu»* using texts with technical methods (shu wen) to group problems together match with the titles of different chapters and is surprisingly consistent with *«Jiushu»* mentioned by two Chongs. This proves that what Liu Hui said “jiu shu zhi liu equals to *«jiu zhang shi yi»*” and “compared its content list might differ from the past” are supported by evidence. *«Jiushu»* is truly a source of *«Jiuzhang Suanshu»*.

Hori in Japan studied *«Jiuzhang Suanshu»* and the price of goods reflected from *«Shiji»*, *«Hanshu»* and *«Juyan Hanjian»*. He concludes that it is rather weak to say that the price of goods found in *«Jiuzhang Suanshu»* equals to the price of goods in Han Dynasty. Basically the price of goods found in *«Jiuzhang Suanshu»* reflects to price of goods in Warring States Period and Qin Dynasty.¹⁹ This conclusion is consistent with that of Liu Hui. To analyse after combining the difference between eras reflected by the price of goods shown in *«Jiuzhang Suanshu»* as well as its variation of problems, Liu Hui's view will be strengthened. There are 31 problems involved when compared and analyzed *«Jiuzhang Suanshu»* and the price of goods in Han Dynasty, 20 of them showed a large price difference when compared with that of Han Dynasty but rather close to the price in Warring States Period and Qin Dynasty. 18 out of these 20 problems are in the form of using texts with technical methods (shu wen) to group problems together. And 11 problems showed very close to the price in Han Dynasty but having a larger price difference when compared with that of Warring States Period and Qin Dynasty. 7 of them belong to the applied question bank; and 4 of them are in the form of using texts with technical methods (shu wen) to group problems together.

All in all, the present historical information not only did not contradict with Liu Hui's discussion on the compilation of *«Jiuzhang Suanshu»*, but also prove that Liu Hui's discussion on the compilation of *«Jiuzhang Suanshu»* was totally correct. In addition, Liu Hui has a realistic and rigorous learning attitude as well as a high level of morality. We should believe the words of Liu Hui. He designed the mouhe fanggai and pointed out the right way to solve the volume of a sphere. Although the result fell short of his prediction and he could not calculate the volume of mouhe fanggai, he did not hide his failure, but spoke frankly, “When making the judgment and conclusion, there is the confusion of square and circle, and also the mixture of thickness and thinness, it is unable to set the equality. If a rough idea is simply made, it is afraid that the truth might be missed. Therefore the problem is not solved, it should wait for the capable one to get it done.” “li shou zuo shu” was the traditional view at that time. But he said, “I haven't heard of its details.” After the description of the shape of qiandu, he said, “I haven't heard why it is named qiandu”. The entire explanation of Liu Hui showed his supreme spirit of expressing his opinions with evidence and never tells unproved idea. Therefore Liu Hui's words do have the full reliability. In short, regarding the compilation of *«Jiuzhang Suanshu»*, we should believe the words of Liu Hui. Freely denied the words of Liu Hui and even invented another argument was not a scientific attitude.

¹⁸Guo Shuchun: “About Chinese traditional mathematics ‘Shu’”. Li Wenlin & others edited: “Mathematics and mechanization of mathematics”. Ji Nan: Shan Dong Education Publishing Company, 2001. This article made certain amendments on the related discussion basis of the author's “Liu Hui, the world-class mathematics leader in ancient time”.

¹⁹(Japan)Hori: “Study on price of goods in Qin and Han Dynasty”. “Study and discussion on the history of legal system in Qin and Han Dynasty”. Beijing: Law Publishing Company, 1988.

2.3.3 When the *«Jiuzhang Suanshu»* which Liu Hui read was formed

Liu Hui believed that *«Jiuzhang Suanshu»* which he read was compiled by Zhang Cang (?–152 BC) and Geng Shou Chang (1st century BC) in Western Han Dynasty. As noted above, the words of Liu Hui are worth to believe. As compilation of *«Jiuzhang Suanshu»* was such a serious issue, if he did not have reliable information and had not read the concrete piece of the Zhang Cang and Geng Shou Chang compiled *«Jiuzhang Suanshu»*, it was absolutely impossible for him to talk about this. It was unreasonable to reject the words of Liu Hui only based on the reason that it was a single case and no other circumstantial evidence was available. Because of years of delay and natural disasters, only very limited data which Liu Hui had seen were kept till mid-Qing Dynasty to today. Among those limited data handed down to the era of Dai Zhen, what Dai Zhen and other people could read and memorize was a very small proportion only. Therefore one can imagine how prejudiced it was when Dai Zhen and other people based on their own knowledge to reject Liu Hui's discussion. In fact, Dai Zhen and other people rejected the Zhang Cang compiled *«Jiuzhang Suanshu»* was mainly due to two reasons. Firstly, the existence of the name of a place called "Shang Lin".²⁰ Secondly, they said that problems of junshu existed only from the era of Han Wu Di, thus Zhang Cang was unable to involve in the compilation of *«Jiuzhang Suanshu»*. In fact, as early as the era of Qin Emperor, there was a Shang Lin Parkland²¹, and junshu 1 was appeared in bamboo slip which was unearthed at the same time with *«Suanshu shu»*.²² As a result, the two main reasons for rejecting Liu Hui's discussion were no longer existed.

Conditions in *«Jiuzhang Suanshu»* also proved that Liu Hui's words are correct. The examples and styles used in the part that adopting applied question bank were totally different from the part that using texts with technical methods (shu wen) to group problems together. Moreover, differences were also found between the nature of questions and the nature of titles of chapters which they were classified, a clear patch nature could obviously be seen. Also, a great variation existed in ideologies for compilation.²³ By comparing with bamboo slips *«Shu»* from Qin Dynasty, *«Suanshu shu»* from Han Dynasty and problem of shaoguang from *«Jiuzhang Suanshu»*, one would find that the former two slips were written by simple ancient words, while the later was written by the languages of Han Dynasty. This proved that what Liu Hui said "the actual contents were more or less the same" was supported by evidence.

In addition, from the guiding ideology for compilation of *«Jiuzhang Suanshu»*, Qian Baocong²⁴ thought that the characteristics of the calculation techniques (suanfa) in *«Jiuzhang Suanshu»* as to solve practical problems as its fundamental purpose showed its realistic style. This reflected its acceptance of Xunzi's materialist ideology (wei wu zhu yi). On the other hand, *«Jiuzhang Suanshu»* did not define any mathematical concepts, and also no mathematical formula and explanation were derived and proved. This also reflected Xunzi's ideologies of "conventionalism (yue ding su cheng)"

²⁰(Qing) Dai Zhen: Abstract of "Jiuzhang Suanshu". In "Jiuzhang Suanshu" of "Wu Ying Hall collection of selected books". See Guo Shuchun edited: "Mathematics Paper - Collection of Chinese Science Technology Books" vol 1. Zheng Zhou: He Nan Education Publishing Company, 1993.

²¹(Han) Si Maqian: "Shi Ji—Qin Emperor original record". Beijing, Zhong Hua Book Store, 1959.

²²Li Xueqin: Significant findings in Chinese mathematics history". "Wen Wu Tian Di", vol 1, 1985.

²³Guo Shuchun edited: "Mathematics Paper—Chinese Science Technology History", Science Publishing Company, October 2010.

²⁴Qian Baocong: "Relationship between Jiuzhang Suanshu" and its explanation by Liu Hui and philosophical ideas". "Li Yan, Qian Baocong The whole collection of science history" vol 9, Liao Ning Education Publishing Company, 1998.

and “there was an end in learning (xue you suo zhi)”.²⁵ That is, *«Jiuzhang Suanshu»* is compiled under the guidance of Confucianism of the Xun School. There were very few historical records about the thinking of Zhang Cang. However, Xunzi (313BC–238 BC) taught *«chun qiu zuo shi zhuan»* to Zhang Cang. Zhang Cang taught *«Zuozhuan»* to Jia Yi.²⁶ It could be concluded that Xunzi, Zhang Cang and Jia Yi did have the lineal teacher-student relationship. As Jia Yi was the main representative of Confucianism of Xun School in the early Western Han Dynasty, therefore Zhang Cang did believe in Confucianism of Xun School.²⁷ This was consistent with the ideology for compilation of *«Jiuzhang Suanshu»*.

In short, the fact that *«Jiuzhang Suanshu»* was compiled by Zhang Cang, Geng Shou Chang, should not be rejected.

2.4 It provides reliable literature to support the first climax of traditional Chinese mathematics occurred in Spring & Autumn and Warring States Period

In 1990s, I concluded that the first climax of traditional Chinese mathematics was occurred in Spring & Autumn and Warring States Period and Western Han Dynasty compiled *«Jiuzhang Suanshu»* was only a conclusion²⁸ of this climax after studying *«Jiuzhang Suanshu»* and its Liu Hui's zhu (commentary). Although I firmly believed that this view was correct, lack of evidence was the problem at that time. In 2000, interpretation of *«Suanshu shu»* was announced, in which the rich mathematical contents and Mr. Peng Hao's conclusion about *«Suanshu shu»* saying that the vast majority of problems arose from Qin Dynasty and pre-Qin Dynasty made me settled. Now, several batches of bamboo slips from Warring States Period and Qin and Han Dynasty were found, which provide more reliable literature for the study of mathematics in Qin Dynasty and pre-Qin Dynasty. This ultimately ends the situation of mainly relied on *«Jiuzhang Suanshu»* and its Liu Hui's zhu (commentary) to derive that the first climax of traditional Chinese mathematics was occurred in Spring & Autumn and Warring States Period.

3 Expectations and Recommendations

Currently, study in bamboo slips *«Shu»* from Yuelu Academy in Qin Dynasty is still very popular. While the mathematical bamboo slips from Qin Dynasty in Beijing University and the mathematical bamboo slips from Han Dynasty in Hubei Museum are still being compiled, so we are unable to glimpse the whole picture. We propose two expectations:

First is to accelerate the compilation of the mathematical bamboo slips from Qin Dynasty in Beijing University and the mathematical bamboo slips from Han Dynasty in Hubei Museum, so that interpretation of the text will be available as soon as possible.

²⁵(Warring States Period): Xun Qing: “Xun Zi” “Simplified explanation of Xun Zi”, Shanghai People Publishing Company, 1975.

²⁶(Western Han) Liu : “Preface of Spring and Autumn”. “Explanation of Spring and Autumn Zuo Chuan “Kong Yingda's explanation referred to Liu Xiang ‘Bie Lu’, See “Explanation of 13 books”. Beijing, Zhong Hua Book Store, 1980.

²⁷Guo Shuchun: “Zhang Cang and ‘Jiuzhang Suanshu’”. In “Ke Shi Xin Chuan”, Shen Yang: Liao Ning Education Publishing Company, 1997.

²⁸Zou Dahai: “The rising of Chinese mathematics and Early Qin mathematics – the late report”. Shi Jia Zhuang: He Bei Education Publishing Company, 2001.

Second is hoping more colleagues in history of mathematics sector can participate in the study in mathematical bamboo slips from Warring States Period, Qin Dynasty and Han Dynasty. Mathematics in pre-Qin Dynasty is the origin of source and the fundamental stone of traditional Chinese mathematics. In the past, we were in the state of blurred understanding. With the findings and studies of mathematical bamboo slips from Warring States Period, Qin Dynasty and Han Dynasty, the mystery of mathematics in pre-Qin Dynasty was gradually unveiled, so that our knowledge on mathematics in pre-Qin Dynasty can truly get close to the history.

In the basis of in-depth study in mathematical bamboo slips from Warring States Period, Qin Dynasty and Han Dynasty, two tasks should be conducted:

Firstly, to co-ordinate experts of history of mathematics, archaeology and ancient text to conduct a comprehensive research and interpretation of mathematical bamboo slips from Warring States Period, Qin Dynasty and Han Dynasty.

Secondly, as proposed by Mr. Dao Benzhou, to launch an international academic symposium about mathematical bamboo slips from Warring States Period, Qin Dynasty and Han Dynasty at an appropriate time, in order to conclude and promote the study in mathematical bamboo slips from Warring States Period, Qin Dynasty & Han Dynasty, and the study in mathematics in pre-Qin, Qin and Han Dynasty and even the entire Chinese history.



湖南里耶出土的秦九九表
Multiplication table in the form of 9 times
9 in Hunan Liye Town



秦简《数》的部分简(岳麓书院供图)
Some Bamboo slips