# BRIDGING THEORICAL AND EMPIRICAL ACCOUNT OF THE USE OF HISTORY IN MATHEMATICS EDUCATION? A CASE STUDY

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**ABSTRACT**

In this paper, I propose a study in which I take on this challenge and explore how *cultural understanding, replacement* and *reorientation* occur in the course of an experimentation in which students read and discussed a text by Pierre de Fermat as part of their calculus class. The data were collected within a study concerned with the metamathematical reflections (historicity of the concepts, rigor, intrinsic and extrinsic driving forces, etc.) students may develop when taking part in such activity. Two groups of students took part in the study (audio-recorded), and twenty-one interviews were conducted and transcribed. Using this set of data, I now confront them with Barbin and Janhke's three arguments regarding the use of history in the mathematics classroom and, in the same movement, confront the theoretical framework with the these data so to see how they could actually enrich one another.

**Keywords:** empirical studies, cultural understanding, replacement, reorientation, use of primary sources, learning and teaching calculus

## 1 Introduction

For several decades, many thinkers, researchers and teachers focused on the "how" and "why" of using the history of mathematics in mathematics education. Early in the 20th century, educators (Barwell, 1913), philosophers (Bachelard, 1938) and mathematicians (Poincaré, 1889; Klein, 1908; Toeplitz, 1927; Pólya, 1962) became interested. Until recently, it seemed like everyone, teachers and researchers, agreed that history was good and saw it as a motivating and effective tool for the learning of mathematics (Charbonneau, 2006). This enthusiasm has led to numerous studies concerning the use of the history of mathematics.

However, for 10 years now, the field of research around the use of history is being restructured. New and serious questions are born following the publication of the *History in mathematics education—The ICMI Study* (Fauvel & van Maanen, 2000). True health check of this area of research, the book brings together the beliefs, questions and concerns of researchers. This book invites researchers to take a step back from their beliefs. For example, the efficiency and relevance of many examples of application of history in class were questioned (Siu, 2000; Bakker, 2004). Some researchers stress the importance of being cautious when studying the historical aspects of mathematical concepts, they go as far as doubting the students and teachers ability to study these concepts from a historical point of view. (Fried, 2001; Charbonneau, 2002; Jankvist, 2009a). Others also question the "transferability" of positive experiences reported by practitioners from different academic levels (Tzanakis, 2000; Schubring, 2007). More broadly, we question the general way of conducting research in this field, in order to try to go beyond "the stories of practices". Thus, the lack of serious and systematic empirical studies questioning the possible contribution of history of mathematics in mathematics education is still around the table (Lederman, 2003; Siu & Tzanakis, 2004; Siu, 2007; Jankvist, 2009b).

## 2 Questions and research problem

Even today, if several studies inform us about positive experiences around specific activities involving the history of mathematics in the classroom (see Greenwald, 2005; Arcavi & Isoda, 2007; Hoyrup, 2007; Blomhøj & Kjeldsen, 2009), few of them, according to several (eg. Furinghetti, 2007; Charalambous, Panaoura & Philippou, 2008; Jankvist, 2009b; 2010; Guillemette, 2011), are making real analysis of how history is used, and how learners benefit from it.

### 2.1 Two types of studies

Looking through the scientific literature since the 1990s, one can classify the studies on the use of history in the mathematics classroom into two categories. There are those that usually take the form of stories of practices analyzed. These generally include initiatives of mathematics teachers of different academic levels who tried try to introduce history in various ways in their courses. However, most remain unsatisfactory from an experimental standpoint, because few of them present a framework for analyzing their data. They offer interesting reflections on the phenomenon, but they don't drive a precise analysis following a data collection methodologically established. In this sense, Gulikers and Blom (2001) observe that those cases are often isolated, creating a gap between "practical experiences" reported by some studies and theorical considerations reported by speculative reasearch. This brings me to the second type of studies, those who deal mainly with theoretical considerations. Their contribution is important because they provide ways of seeing and distinguishing that help us to look deeper on the arguments and methods regarding the use of history (Jahnke & al. 2000; Fried, 2007; 2008; Jankvist, 2009c). Having said this, despite the important contributions of these two types of studies, there is a need for empirical systematic studies on the mathematical learning process that unfold within the introduction of history of mathematics.

Moreover, these "practical experience" or theoretical considerations are rare. In his research, Jankvist (2007; 2009a) attempted to identify, from 1998 to 2009, all empirical studies published in English-language journals (Educational Studies in Mathematics, For the Learning of Mathematics, Mediterranean Journal of Research in Mathematics Education and Zentralblatt für Mathematik der Didaktik, as well as master's and doctoral theses and conference proceedings). In his work, he refers to empirical researches as "large scale quantitative studies to small scale qualitative studies, from experimental investigations to a teacher testing out a course using methods of questionnaires and interviews" (Jankvist, 2009a, p. 38). The studies he selected showed, in one way or another, empirical data on which the authors based their observations and conclusions. He found a total of 81 studies. On the other hand, he emphasized that throughout the 78 studies appearing in the HPM2004 & ESU4 (Furinghetti, Tzanakis & Kaijser, 2008) conference proceedings on the history of mathematics in mathematics education, only about 10% of them derive from empirical studies. That being said, he noticed an upsurge in the number of these studies since the middle of the decade, showing a change initiated in this field of study concerning this type of research.

Trying to bridge theorical framework and empirical account, my works is placing itself in this movement where researchers are looking to describe and explore what happens when using history for the learning of mathematics based on a conprehensive perspective.

The theoretical studies on the use of the history of mathematics in the class, discuss, among other things, the question of "why" history is used. From this central question emerge many arguments to justify the presence of history in the mathematics classroom. Two classifications stand out, on the one hand, Barbin (1994) and Janhke (2000) and, on the other hand, Jankvist (2009c). In the following, I present how each researcher dissected the question in their own way.

## 2.2 Jankvist and the "whys" of using history

Jankvist (2009c) divides the arguments for the use of history in two categories.

First, history can be seen as an effective and motivating cognitive tool that can assists and supports the teaching and learning of mathematics. Motivational factors, the humanization of mathematics, cognitive support for the student, the deepening of epistemological refexion for teaching, access to various problems and enriching the didactic reflexion around epistemological obstacles are clear arguments associated with this history perceived as a tool.

Second, a certain type of discourse that proclaims the teaching of the history of mathematics "as such" is a contribution to learning mathematics, in the sense that it teaches us what is mathematics. Jankvist does not hesitate to speak of the learning of "the sake" of mathematics through the history of mathematics (Jankvist, 2009c, p. 239). In this sense, the history of mathematics is seen as a goal in itself. It shows that mathematics evolves in time and space, and is "not something that has arisen out of thin air" (*ibid.*). Mathematics are a human activity wearing multiple facets trough cultures and societies. Its evolution is the result of intrinsic and extrinsic motivations animating the mathematicians in their day, are arguments that are part of a vision of history seen as a goal. These are arguments that are part of a vision of history seen as a goal.

Jankvist categorized the arguments that support the presence of history of mathematics in the classroom by observing the teachers' intention. If the intention relates more specifically to enrich the conceptual understanding of mathematical objects, the arguments are associated with the history seen as a tool. If the intention is mainly metamathematicals reflections (that is to say, the reflections that affect the historicity of the concepts presented, the historicity of the notation and the rigor associated, the mechanisms underlying the discovery of concepts, the intrinsic and extrinsic forces that drive mathematicians discoverers or the links between the development of these concepts and the development of societies and cultures) the arguments are then associated with the history seen as a goal.

## 2.3 Barbin and Jahnke and the "whys" of using history

For its part, Jahnke (2000) also questioned the arguments concerning the use of history, specifically concerning the use of primary sources. He highlights three main hypotheses. A first assumption is that history can provide a cultural understanding of mathematics. As he says: "The integration of the history of mathematics invite us to place the developement of mathematics in the scientific and technological context of a particular time and in the history of ideas and societies" (*id.*, p. 292). Thus, the history of mathematics is a way to place mathematicals objects studied in an historical continuum and an historical and social context, this enables us to see their progress and identify issues that have generated their development. Also, in this perspective, the links between the objects studied take a different shape from the simple sequence of concepts within a curriculum or ways in which the concepts of the discipline are typically organized.

The second assumption is that the integration of history leads to a replacement of mathematics, that is to say that "it allows mathematics to be seen as an intellectual activity rather than as just a corpus of knowledge or a set of techniques" (*ibid.*). These first two assumptions are related to the need to humanize mathematics, to emphasize its historicity and to emphasize the evolutionary aspect. It is a dimension frequently mentioned in literature (see Furinghetti, 2004; Tang, 2007; Blomhøj & Kjeldsen, 2009; Guillemette, 2009; Jankvist, 2010).

The final assumption is that history of mathematics brings a reorientation. In this sense, "the history of mathematics challenges one's perceptions through making the familiar unfamiliar. Getting to grips with a historical text can cause a reorientation of our view" (Jahnke & al., 2000, p. 292). From this perspective, history allows students to question their own assumptions and experiences related to mathematical objects by the encounter and comparison of another mathematical comprehension, one from another era. These concepts of reorientation, cultural understanding and replacement were first developed by Barbin (1994). Concerning reorientation, she stressed that "the history of mathematics, and this is perhaps the main attraction, has the virtue of allowing us to wonder what is obvious" (Barbin, 1997, p. 21, my translation).

With regard to research, if more fuel the discussion of these various arguments, they have seldom faced experimentation. The research, in terms of understanding how to operate this cultural understanding, this replacement and this reorientation in the learner and how it is articulated with the presence of historical elements in the mathematics classroom, is still in its infancy (Furinghetti, 2007; Siu, 2007; Charalambous, Panaoura and Philippou, 2008; Jankvist, 2009b). There is a real need to better understand these phenomena and what can potentially happen there. In a broader perspective, we must close this gap, raised by Guliker and Blom (2001) between the empirical studies and theoretical one.

Jankvist categorization between history seen as a tool and history seen as goal allows, firstly, avoiding a widespread confusion between arguments and methods and, secondly, facilitates the observation and the analysis of the relations between these two aspects of research. The categorization of Jankvist therefore aims to facilitate and guide the work of the researcher. On the other hand, the three arguments brought by Barbin and Janhke give a broad perspective of the potential benefits for the individual's learning and for the mathematics classroom.

Overall, this feed deep thought in the discussion about the "whys" of the use of history in the mathematics' classroom. From these theoretical considerations and assumptions, questions emerge: in wich particular manners the use the history in mathematics education can develops the components of cultural comprehension, replacement and reorientation for the learners? In particular, how are those components articulated with the use of primary sources in the mathematics classroom?

## 2.4 The "hows" of using history

Some researchers have considered the more specific "how" of the use of history. This is the case of Fried (2001; 2007; 2008), who highlights the difficulty of properly addressing the history of mathematics in class. He wants the history to be taken seriously and argues that its study should be very attentive. Otherwise, to Fried (2001), there is high risk of distortion of history, because history could be contaminated with a modern vision of mathematics that crushes the historicity of the concepts and sterilizes its exploration. The risk of anachronism and false readings of a progressive history are very high. Too often, historical aspects take the shape, as he said, of anecdotes and historical vignettes.

Concerning the "how", Jankvist (2009c) show three categories: the illumination approaches, the modules approaches and the history-based approaches. The first is the introduction of isolated facts of historical vignettes or anecdotes. A good exemple, described by Jankvist (*ibid.*), is the case of Lindstrøm (1995) who, at the end of each chapter of his book, added a small section on the development of the history of the concepts covered. The modules approaches offer problem-learning situations or sequences of lessons, varying in duration, based on the history of a specific mathematical topic. It is clear opportunities in the history that are supported mathematically and didactically and which may include the use of primary and secondary sources like reading historical texts or developing research projects by students and others. For the history-based approaches, it is based on the historical development of the mathematical object studied for the developement of a complete sequence of lessons. Directly or indirectly, the history is found in the mathematics classroom through the strategies adopted by the teacher, his attitude towards the presentation of the studied subjects, the issues raised from the historical context or the sequence of concepts discussed. Essentially, this third category includes practices based on genetic approach following the work of Toeplitz (1963) or, otherwise, Freudenthal (1991).

Thus, Jankvist (2009c) suggests that these different approaches, which include several specific methods of use, do not have all the same goals and that their scope is different from one another. In this sense, Fried (2007) states that reading historical texts appear as a preferred method when it comes to use history in a rigorous and serious ways. However, this difficult reading activity would imply a double perspective. To illustrate it, he stressed that the aim of the historian is to delve into the era of the mathematician, to perceive his idiosyncrasies and to situate his work within a continuum of mathematics development. The look of the mathematician, meanwhile, attempts to decode the obsolete symbols, returning them to the modern language and grasp the essential mathematical sens. He calls diachronic the reading of the historian and synchronic the reading of the mathematician, terms borrowed from the linguist de Saussure (1967/2005). He said that synchronic reading is too often reinforced by teachers. Also, the teacher's role is precisely to tip the student constantly between these two visions. This continuous back and forth work helps the learner to be aware of his own conceptions of mathematics, his personal insights and his ability to confront it constructively with those of others (self-knowledge). That is why the reading of historical texts appears as the preferred approach from the perspective of the learner to generate the three components of Barbin and Jahnke & al.

From these theoretical considerations and assumptions, one could ask: in which particular manners the use of primary sources can develop the components of cultural connotations, repositioning and reorientation for the learners? In particular, how are those components articulated with the reading of both synchronic and diachronic ancient texts in a mathematics class?

# 3 Collecting data, preliminary analyses and research perspectivs

Some preliminary data from my master degres' work (Guillemette, 2009) can give some clues. As mentioned above, the data were collected within a study concerned with the metamathematical reflections (historicity of the concepts, rigor, intrinsic and extrinsic driving forces, etc.) students may develop when taking part in historical text reading activity. In the next section, I will describe the context of this study and how the data were collected. Using this set of data, I will thereafter confront them with Barbin and Janhke's three arguments.

## 3.1 The use of an historical text

In this study, an activity based on the reading of a text by Pierre de Fermat was built and lived in a classroom. The text concerned the method of maxima and minima of Fermat. It is a well-known text through which Fermat provides an elegant method of solving optimization problems using similar principles of calculus that emerged at the time. The following text was used in class: *Method for finding the maximum and minimum (on the method of adegalation, 1629/1637)* (IREM de Basse-Normandie, 1999). This text present high potential for getting into anecdotes and stories surrounding the character. Moreover, the mathematical elements and the approach used by Fermat can be easily articulated with the elements of the calculus course that was taking place.

Fermat offers in this text a general method for finding the minimum and maximum of polynomial expressions. First, he proposes to put the problem in equations: "We first express the minimum or maximum using terms that may be of any degree". Then, he substitutes $(a + e)$ to the "primitive unknown $a$". Here, Fermat employs infinitesimal objects intuitively and without justification. He then "adegalise" the two expressions, the one using $a$ and the other using $(a+e)$. For Fermat, they are nearly equal since $e$ will be treated as an infinitesimal value.

In this equation, we find terms that are in $e$ or powers of $e$ : "affected of $e$", as he puts it. He can then divide each member as many times as he wants by $e$ to obtain at least one term without $e$. He then considers the value of $e$ as $0$ and thus eliminates the terms in $e$ or $e$ powers. Solving the remaining equation provides the maximum or minimum which sought.

In the piece chosen, Fermat gives an example of application of his method: "Lets divide the line AC with E, so that the rectangle AEC is maximum". He puts $m\overline{AC} = b$ and $a$ the length of a segment generated by the point E ($\overline{AE}$ or $\overline{EC}$). The other segment will be $b - a$. He then seeks to maximise $ba - a^2$.

By following his method explained earlier, he replaces $a$ by $a + e$ as the first segment length. The second segment becomes $b - a - e$ and $ba - a^2 + be - 2ae - e^2$ the new product. Thus, we have: $ba - a^2 \approx ba - a^2 + be - 2ae - e^2$ (Adegalation), $be \approx 2ae + e^2$, $b \approx 2a + e$ (Dividing each member by $e$), $b = 2a$ (Eliminating the terms in $e$). Thus, the area of the rectangle will be the maximum if $b = 2a$. This conclusion is not followed by any justification.

In summary, Fermat was looking to maximise the function $f(x) = x(b - x) = bx - x^2$, where $f(x)$ is the area of the rectangle posing $x$, the length of a side of the rectangle, and where $b$ is half the perimeter of that rectangle. Today we would have $f'(x) = b-2x$ and $f'(x) = 0 \Leftrightarrow b-2x = 0 \Leftrightarrow x = \frac{b}{2}$. Of course, because $f''\left(\frac{b}{2}\right) < 0$, $\left(\frac{b}{2}, f\left(\frac{b}{2}\right)\right)$ is a maximum of $f$.

More precisely, by moving closer to Fermat's method, we have: $f'(x) = \lim_{e \to 0} \left( \frac{f(x+e) - f(x)}{e} \right)$

$$0 = \lim_{e \to 0} \left( \frac{f(x+e) - f(x)}{e} \right) \quad \text{(with } f'(x) = 0\text{)}$$

$$\Leftrightarrow \quad 0 = \lim_{e \to 0} \left( \frac{(bx - x^2 + be - 2ex - e^2) - (bx - x^2)}{e} \right) \quad \Leftrightarrow \quad 0 = \lim_{e \to 0} \left( \frac{-2ex + be - e^2}{e} \right)$$

$$\Leftrightarrow \quad 0 = \lim_{e \to 0} (-2x - e + b) \quad \Leftrightarrow \quad 0 = -2x + b \quad \text{and} \quad \Leftrightarrow \quad x = \frac{b}{2}$$

### 3.2 The experiment

Based on Fermat's text, I built an activity that was conducted with preuniversity students in a cours of calculus. The activity consisted of three parts: a brief overview of socio-historical context and the mathematics at the time of Fermat, an individual reading of the text and a return in large group around that reading.

The socio-historical and mathematical context was presented from a PowerPoint document. It contained many evocative images of the socio-historical and scientific climate of the time of Fermat. This document presented several pictures of Fermat, various mathematicians of the time, the city of Paris and Toulouse and old documents. I have included various biographical elements of Fermat, concerning his correspondence with various scientifics of the time and references to his last theorem and the entire movement around it. The emergence of the Academy of Sciences and the tendency of scholars of the time for seeking global methods for solving a set of problems were mentioned. Subsequently, students were asked to read the extract individually. I circulated in the classroom to address different questions and guide students in reading. The final phase of the activity was a large pleanary. Students could then share and respond after reading the excerpt. I resumed the process of Fermat and tryed to conciliate it with modern methods, to highlight some idiosynccrasies of the mathematicien and to situate his work within a continuum of mathematics development.

The activity was experienced twice, first in a class of 30 students in the sector of natural sciences, and secondly in a class of 11 students in the sector of social sciences and humanities. These two phases of experimentation allowed me to confirm the feasibility and effectiveness of the reading activity and to be more comfortable conducting the activity and the individual interviews that followed.

### 3.3 Collecting data

Short interviews were conducted individually immediately after the workshop. Thus, among the 30 students in the first experiment, nine volunteered for interviews. For the second experiment, the 11 students were interviewed.

These were semistructured interwiews conducted by myself for about 10 minutes individualy. The discussion was around different questions: Overall, what struke you the most during this training workshop? Which elements from the presentation in the introduction hit you? What elements struck you during reading the text? And during the plenary phase? What did you learn about mathematics in general? What did you learn about calculus? What do you think this kind of reading can bring to a mathematics course? Do you think such an activity belongs in a mathematics class? All interviews, and the wokshops in class were audio recorded. All this was transcribed and constitued the data of the study.

### 3.4 Preleminary analysis and research perpectives

This research allowed me to observe metamathematical reflexions that emerged from pre-university students whom take part of such activity. Those metamathematical reflexions in question were those who, through a mathematical activity, concerned the historicity of the concepts presented, the historicity of the notation and the rigor associated, the mechanisms underlying the discovery of the concepts explored, the intrinsic and extrinsic forces that drive mathematicians and the links between the development of these concepts and the development of societies and cultures.

The transcripts of the interviews as well as the transcript of the activity helped to establish valuable data regarding the emergence of metamathematical reflections in connection with a reading of ancient texts. However, as I said above, it seems interesting, *a posteriori*, to try to highlight, through excerpts from the transcript, the three arguments concerning the use of history in the mathematics classroom: cultural understanding, replacement and reorientation. For example, the following two extracts which are the reactions of students who have experienced the reading activity:

> ”You say to yourself, ah! Math is going to be bad! However, you have inform us of the entourage of his discoveries, it seems that we know more from the inside. You know, you get the character and taste how his business is found. You know, math, just numbers, at least here you have something back [⋯] It is less abstract ”(*id.*, p. 67).

> ”Then that happened to him ⋯in the air as well and we finally we can do it with our limites and optimization to reach the same answer as him ⋯I do not understand, he still did it and now we can justify it ”(*id.*, p. 50).

It is possible to consider these reactions in the three components introduced by Barbin and Jahnke. Indeed, it is conceivable that the learner behind the first quote demonstrates a cultural understanding of mathematics by saying “we know more from inside”. He seems, somehow, to have anchored the concepts discussed in a socio-historical and cultural context. His look changed as he seemed to perceive the “entourage of discovery”, which allowed him to take a fresh look on mathematics. It is also conceivable that the activity of reading, for this same student, would have led to a replacement of the mathematical objects in question. He mentioned that with this kind of activity mathematics is not “just numbers”. It appears less frozen in time, unchanging or reified. In this sense, mathematical activity appears to be a true human activity. Objects and concepts discussed don’t come from heaven, but are developed by men in particular intrinsic and extrinsic motivations and they are the fruit of long and sometimes tortuous reflections. Finally, it is possible that a shift has taken place in the learner, considering the second quotation. Indeed, he was surprised by the intuitive approach of Fermat to such an extent that he felt the need to reclaim the concepts in question to better understand it. He asked and attemped to highlight the links between the two forms of understanding, his and Fermat.

This preliminary analysis of data from a particular research project, that are perceived by simple outlines, can provide clues about what each of the assumptions may mean. The fact remains that the contours of these theoretical considerations remain unclear and a refinement is needed. Many questions remain, for example: Does a cultural understanding implies necessarily a replacement of mathematics? Is there a form of gradation between each component? Is cultural understanding necessary for a shift to occur? More broadly, are these assumptions sufficient to interpret benefits for learners?

## 4 Conclusion

This first level of interpretation, on the one hand, provides a partial understanding of the phenomena in question and, secondly, clearly underlines the need to enter more deeply and more systematically in the analysis. In this sense, it seems necessary to build new experiments to investigate more precisely the impact of the introduction of history in the mathematics classroom to better illuminate the arguments in favor of this introduction. Thus, we must find effective ways to make the learners "talk": different kinds of interviews, written reflections, questionnaires, mathematical productions, etc. Through the systematic analysis of the experience of the class, it will be possible to fully grasp and understand the issues surrounding the introduction of history in the mathematics classroom. This understanding will provide tools to deal effectively with objects of study in this field of research.

**REFERENCES**


- Arcavi, A., Isoda, M., 2007, "Learning to listen: from historical sources to classroom practice", Educational Studies in Mathematics **66**(2), 111–129.
- Bachelard, G., 1938, *La formation de l'esprit scientifique*, Paris: Vrin.
- Bakker, A., 2004, *Design research in statistics education: on symbolizing and computer tools*, PhD Thesis, Utrech: The Freudenthal Institute.
- Barbin, E., 1994, "Préface", in *Quatrième université d'été d'histoire des mathématiques*, Commission Inter-IREM Épistémologie et histoire (ed.), Lille: IREM de Lille, pp. ii–iii.
- Barbin, E., 1997, "Histoire et enseignement des mathématiques: Pourquoi? Comment? ", *Bulletin de l'Association Mathématique du Québec* **37** (1), 20–25.
- Barwell, M., 1913, "The advisability of including some instruction in the school course on the history of mathematics", *The mathematical gazette* **7**, 72–79.
- Charalambous, C. Y., Panaoura, A. & Philippou, G., 2008, "Using the history of mathematics to induce changes in preservice teachers' beliefs and attitudes: insights from evaluating a teacher education program", *Educational Studies in Mathematics* **71**(2), 161–180.
- Charbonneau, L., 2002, "L'histoire des mathématiques peut-elle changer l'attitude des élèves face aux mathématiques? ", in *Comhishma: HPM*, Marrakech: École Normale Supérieure, pp. 99–120.
- Charbonneau, L., 2006, "Histoire des mathématiques et les nouveaux programmes au Québec: un défi de taille", in *L'enseignement des mathématiques face aux défis de l'école et des communautés: Actes du colloque EMF*, CRP (ed.), Sherbrooke: Faculté d'éducation, Université de Sherbrooke, p.11.
- Fauvel, J., Maanen, J. van. (eds.), 2000, *History in mathematics education: The ICMI study*, Dordrecht: Kluwer Academic Publishers.
- Freudenthal, H., 1991, *Revisiting mathematics education: China lectures*. Springer.
- Fried, M. N., 2001, "Can mathematics education and history of mathematics coexist? ", *Science & Education* **10**, 391–408.
- Fried, M. N., 2007, "Didactics and History of Mathematics: Knowledge and Self-Knowledge", *Educational Studies in Mathematics* **66**(2), 203–223.

- Fried, M. N., 2008, “History of mathematics in mathematics education: a saussurean perspective”, *The Montana Mathematics Enthusiast* **5**(2), 185–198.
- Furinghetti, F., 2004, “History and mathematics education: a look around the world with particular reference to Italy”, *Mediterranean Journal for Research in Mathematics Education* **3**(1–2), 125–146.
- Furinghetti, F., 2007, “Teacher education through the history of mathematics”, *Educational Studies in Mathematics* **66**(2), 131–143.
- Furinghetti, F., Kaijser, S. & Tzanakis, C. (eds.), 2008, *Proceedings of HPM2004 & ESU4 (revised edition)*, Uppsala: Uppsala Universitet.
- Greenwald, S., 2005, “Incorporating the mathematical achievements of women and minority mathematicians into classrooms”, in *From calculus to computers: Using the last 200 years of mathematics history in the classroom, MAA Notes (No. 68)*, A. Shell-Gellasch, D. Jardine (eds.), Washington DC: The Mathematical Association of America, pp. 183–200.
- Guillemette, D, 2009, *Utilisation de textes anciens dans l’enseignement du calcul différentiel*, Master degree thesis, Montréal: Université du Québec à Montréal.
- Guillemette, D., 2011, “Les études empiriques concernant l’utilisation de l’histoire dans l’enseignement des mathématiques: un regard sur la méthodologie”, *Petit x* **86**, 5–26.
- Gulikers, I., Blom, K., 2001, “A historical angle: a survey of recent literature on the use and value of history in geometrical education”, *Educational Studies in Mathematics* **47**, 223–258.
- Høyrup, J., 2007, “The roles of Mesopotamian bronze age mathematics tool for state formation and administration: carrier of teachers’ professional intellectual autonomy”, *Educational Studies in Mathematics* **66**(2), 257–271.
- IREM (Institut de recherche sur l’enseignement des mathématiques) de Basse-Normandie., 1999, *Aux origines du calcul infinitésimal*, Paris: Ellipse.
- Jahnke, H. N., Arcavi, A., & al., 2000, “The use of original sources in the mathematics classroom”, in *History in mathematics education: The ICMI Study*, J. Fauvel, J. van Maanen (eds.), Dordrecht: Kluwer Academic Publishers, pp. 291–328.
- Jankvist, U. T., 2007, “Empirical research in the field of using history in mathematics education: review of empirical studies in HPM2004 & ESU4”, *Nomad* **12**(3), 82–105.
- Jankvist, U. T., 2009a, *Using History as a "Goal" in Mathematics Education*, PhD Thesis, Roskilde: Roskilde University.
- Jankvist, U. T., 2009b, “On empirical research in the field of using history in mathematics education”, *ReLIME* **12**(1), 67–101.
- Jankvist, U. T., 2009c, “A categorization of the ”whys” and ”hows” of using history in mathematics education”, *Educational Studies in Mathematics* **71**(3), 235–261.
- Jankvist, U. T., 2010, “An empirical study of using history as a ”goal””, *Educational Studies in Mathematics* **74**(1), 53–74.
- Kjeldsen, T. H., Blomhøj, M., 2009, “Integrating history and philosophy in mathematics education at university level through problem-oriented project work”, *ZDM Mathematics Education* **41**(1–2), 87–103.
- Klein, F., 1908, *Elementarmathematik vom höheren Standpunkte aus*, Stuttgart: Leipzig.
- Lederman, N., 2003, “Is history of science stuck in the past?”, *Canadian Journal of Science, Mathematics and Technology Education* **3**(4), 521–523.

- Lindstrøm, T., 1995, *Kalkulus*, Oslo: Universitetsforlaget.
- Poincaré, H., 1889, "La logique et l'intuition dans la science mathématique et dans l'enseignement", *L'Enseignement Mathématique* **1**, 157–162.
- Pólya, G., 1962, *Mathematical discovery (combined edition)*, New York: Wiley.
- Saussure, F. de., 1967/2005, *Cours de linguistique générale*, Paris: Payot & Rivages.
- Schubring, G., 2007, "Ontogeny and phylogeny: categories for cognitive development", in *Proceedings of HPM2004 & ESU4 (revised edition)*, F. Furinghetti, S. Kaijser & C. Tzanakis (eds.), Uppsala: Uppsala Universitet, pp. 329–339.
- Siu, M.-K., 2000, "The ABCD of using history of mathematics in the (undergraduate) classroom", in *Using history to teach mathematics: an international perspective, MAA notes (Vol. 51)*, V. Katz (ed.), Washington DC: The Mathematical Association of America, pp. 3–9.
- Siu, M.-K., 2007, "No, I don't use history of mathematics in my class. Why?", in *Proceedings of HPM2004 &ESU4 (revised edition)*, F. Furinghetti, S. Kaijser, & C. Tzanakis (eds.), Uppsala: Uppsala Universitet, pp. 368–382.
- Siu, M.-K., Tzanakis, C., 2004, "History of Mathematics in Classroom Teaching: Appetizer? Main Course? Or Dessert? ", *Mediterranean Journal for Research in Mathematics Education* **3**(1), 5–10.
- Tang, K.-C., 2007, "History of mathematics for the young educated minds: A Hong Kong reflection", in *Proceedings of HPM2004 & ESU4 (revised edition)*, F. Furinghetti, S. Kaijser & C. Tzanakis (eds.), Uppsala: Uppsala Universitet, pp. 630–638.
- Toeplitz, O., 1927, "Das Problem der Universitätsvorlesungen über Infinitesimalrechnung und ihrer Abgrenzung gegenüber der Infinitesimalrechnung an den höheren Schulen", *Jahresbericht der Deutschen Mathematiker - Vereinigung* **36**, 88–100.
- Toeplitz, O., 1963, *The calculus: A genetic approach*, Chicago: University of Chicago.
- Tzanakis, C., 2000, "Presenting the relation between mathematics and physics on the basis of their history: A genetic approach", in *Using history to teach mathematics: an international perspective, MAA notes (Vol. 51)*, V. Katz (ed.), Washington DC: The Mathematical Association of America, pp. 111-–120.