THE USE OF ORIGINAL SOURCES IN AN UNDERGRADUATE HISTORY OF MATHEMATICS CLASS

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ABSTRACT

When designing a course on the History of Mathematics, the instructor is faced with several questions. Certainly, the extent of the content covered is a primary problem. Once that had been decided, another primary concern becomes the approach, which is taken to the content. After years of telling second hand stories in similar courses, I have found it more interesting to let the material present itself. Why not let the student hear the original author speak for themselves?

Well, there are several problems with this approach. Much of what we need is neither accessible nor is in a language the student can understand. Much of what we have is unreliable, having been recopied many times over the years. Yet it is the basis for much of what we teach in such a course. If the instructor has done some research on the pitfalls of using original sources, the student will benefit from having been guided through them.

I would like to discuss resources, which are available in print form and on the web. I would also like to consider various approaches to the material including guided reading and assignments outside the classroom. The material might include works which are not strictly mathematical in content but demonstrate the place of mathematics in society.

It has been my great privilege for the past five semesters at Brooklyn College to teach MATH 41W: History of Mathematics. This is a class for undergraduates who are either mathematics majors or mathematics education majors. It is an elective course which is particularly recommended for prospective high school teachers. A typical group from Spring 2010 consisted of 20 students of whom 10 were mathematics majors, 9 were mathematics education majors, and one was a finance major, who proved to be one of the best. One catalog description of the course goes like this:

> "History of Mathematics. Development of mathematics from antiquity to recent times; emphasis on the inter-relationship of subject matter and on the rise of modern concepts. Prerequisite : Calculus II. 3 hours, 3 credits"

This particular description is from a 1962 catalogue when the instructor for the course was none other than the eminent mathematical historian Carl Boyer. He had taught the course from the beginnings of the college back in the 1930s until his demise in 1976. Since that time two changes have been made to the course description. The first is that the hyphen has been removed from the word "inter-relationship". The second is that a W has been appended to the course number. The latter change has been more formidable.

In an effort to equip the mathematics department with a writing-intensive course it was decided that MATH 41 would be the natural candidate for such a course. A prerequisite of English 2 was added to the course description. This is particularly important due to the number of non-native speakers in the class. Written work consisting of a minimum of ten pages would now be required. Some of this could be in the form of shorter exercises but the final result would naturally be some form of research paper. Students normally choose a research topic based on personal interest. I encourage them to write on topic not covered in class. As an additional incentive the more promising work can be expanded and submitted for the annual Boyer Essay competition.

One of the consequences of this writing intensive format is that students will have to do a significant amount of outside reading. The readings that they do for the class itself influence their choice of what they read for their papers. After years of telling stories like Archimedes at the battle of Syracuse, or the conflict over methods of solving the cubic, it became clear that these episodes are best told by the players themselves. It is instructive to let the students ruminate on the accuracy of these stories. In the words of Wardhaugh from his current work *How to Read Historical Mathematics*: “there’s always more to find out about the mathematical past” (Wardhaugh 2010, p.101). Students’ lack of confidence in their own abilities is diminished when they “see mathematicians struggling with problems they *can’t* solve” (*ibid* p.85). The evolution of the “problem of points” has always been a good example for me of how mathematicians grappled with a misunderstood concept. The ICMI study on *History in Mathematics Education* (Fauvel & van Maanen 2000) suggested reading the original sources will allow students to “appreciate the role representations play in the evolution of ideas” (*ibid* p.294). Freudenthal is quoted in that same study as suggesting that seeing mathematics in its original setting will give students “an opportunity to rediscover properties taken for granted” (*ibid* p.295). What students thought they understood will become truly clear.

So what exactly do we mean by an original source? First we need to distinguish between two types of sources: external and internal. External sources are those that deal with the backstory of the mathematics to be discussed. These can consist of biographies and general historical descriptions. They are not difficult to read but are important for clearing up historical misunderstandings. These readings can often be assigned as homework before they are discussed in class. Many teachers of the history of mathematics would not even consider including them in such a course. I think they are important for at least two reasons. First, the general historical preparation of the average mathematics student is poor and they will greatly benefit from such reading. Secondly, the readings are accessible and give the student motivation to try the more difficult readings. The mathematical content of the internal readings can frequently be challenging. For example, Apollonius cannot be read without the necessary prerequisites from Euclid. The ICMI Study has suggested it is always wise for the instructor to consider the level of difficulty presented by each original work and the background of the students (*ibid* p.317). In general, students will not be able to read these works in the original language. Since the translations are difficult enough, we use a students’ primary common language. In addition, they will be “required to integrate (both) the mathematical language of their lessons (and) their own way of expressing mathematics” (*ibid* p.299). Frequently, there is need to modify available translations but we need insure they don’t lose their meaning (*ibid* p.315). We always need to avoid “anachronistic thinking and evaluating mathematics out of context” (*ibid* p.296). It is therefore worthwhile for the instructor to be aware of Grattan-Guinness’ discussion on the difference between *history* and *heritage* (Grattan-Guinness 2009). While it is tempting to evaluate any mathematical result in terms of our present interpretation, we should not infer that that is what the original author intended. It is important that the student avoid coming to conclusions about the author's intent which are not contained in the work. This requires constant guidance from the instructor. Students should be made aware of the transmission process and modernization that occurs during translation.

Several approaches can be taken to the material. A direct approach will require no previous preparation. It has the benefit of "shock value". Students will work on a reading with a set of prepared questions. An alternative would use student generated questions. A more indirect approach could begin by introducing a "non-routine" problem to motivate the reading (Fauvel & van Maanen 2000, p.314). Michael Glaubitz recently has performed an empirical study comparing genetic and hermeneutic approaches. In the genetic approach a topic is first introduced in its original historical order while the hermeneutic approach assumes pre-knowledge of the topic. While the group using the genetic approach did worse than a similar group using a conventional approach, the group using the hermeneutic approach did better than either. Details can be found in this volume, ch.3.1.

I have divided MATH 41W chronologically into five distinct periods: Egyptian - Mesopotamian, Early Greek, Alexandrian Greek, Muslim-Indian-Chinese, and European (1000-1750). From the first period we examine the Rhind Papyrus, the Berlin Papyrus, and several cuneiform tablets from Mesopotamia. The Ishango Bone is presented as a possible topic for student research. Links are suggested where students may view some of these ancient artifacts. Some background history on Egypt can be found in Herodotus' *Histories* and Aristotle's *Metaphysics*. Several exercises from the Rhind Papyrus including unit fractions, ratio division, false position, and area of plane figures are discussed in class. Students discover the nature of the Mesopotamian sexagesimal system by deciphering a reproduction of a nine times table. The nature of Mesopotamian mathematics is further elucidated by examining a table of reciprocals, area problems (YBC6967), and the measure of the diagonal of a square (YBC7289). The Butler Library at Columbia University in Manhattan is the repository of an important artifact known as Plimpton 322. Students will read several explanations (Buck 1980, Robson 2002, etc.) of the nature of this tablet and be asked to compare them.

From the Early Greek period the class will read several fragments from writers spanning a period of a thousand years. Students will read further selections from Herodotus and Aristotle. Selections from Proclus, notably the Eudemian Summary, and fragments from Simplicius on Hippocrates of Chios are useful to understanding the beginnings of Greek mathematics. There are numerous writers who describe the Pythagoreans and their philosophy. These include Plato, Aristotle, Nichomachus, Plutarch, and the later Neo-Pythagoreans such as Proclus, Iamblichus, and Porphyry. The writings of Diogenes Laertius in the early 3rd Century are a large source of information. Unfortunately, much of it is untrustworthy. It is useful to have students compare several conflicting accounts.

From the Alexandrian Greek period we have the first notable mathematical text, Euclid's *Elements* (Heath 2007). By this time of the course students are equipped to understand that this is not an original work but the amalgamation of many earlier authors. Each instructor will need to determine which sections are most important for their class. Certainly, sections of

Books I, II, the number theory books (VII-IX) and the method of exhaustion (XII) are recommended. A few propositions from III and VI are vital for a later reading Apollonius (particularly I.11 on the parabola). Students can read up on the life and death of Archimedes in Plutarch's biography of the Roman general Marcellus. In class we discuss sections of Archimedes' *Measurement of the Circle*, *On Sphere and Cylinder*, and *Quadrature of the Parabola* (Heath 2002). Heron's discussion on the area of a triangle and square root methods from the *Metrica* along with Ptolemy's method for creating a table of chords from the *Almagest* are worthwhile readings. Problems from Diophantus' *Arithmetica* and Metrodorus' *Greek Anthology* (notably problem 126) are within the reach of most students.

From Asian mathematics (Muslim–Indian-Chinese) the availability of sources had been a major problem. While not completely solving this problem the recent sourcebook edited by Victor Katz has been a welcome addition (Katz 2007). It contains many sources from the Muslim world and yet I still find that I rely on the works of Al Khowarizmi and Omar Khayyam as they give a good overview of that culture's contributions. Indian mathematics has been more obscure. As for primary sources Bhaskara I's explanation of the work of Aryabatta (Keller, 2006) seems to be the most accessible for my students. The Colebrooke translations of the work of Brahmagupta and Bhaskara II are available online through Google Books (Colebrooke 1817). They are much more detailed but older. There is a recent translation in English of the Chinese *Nine Chapters of the Mathematical Arts* by Kangshen, Crossley & Lun (1999) but if your library cannot obtain it you may be out of luck. The price listed on Amazon was over $400. Several sections can be found in the Katz 2007 sourcebook. Libbrecht's translation of the work of Qin Jiu Shao is valuable (Libbrecht 2006). The diagram of the binomial triangle of Zhu Shiejie which can be found in most texts provides a lively discussion.

The last section of the course deals with mathematics in Europe. I like to begin with the nice round date of 1000 when a mathematician sat on the papal throne. Most of Gerbert of Aurillac's (Sylvester II) letters deal with church matters but a few have mathematical content. Leonardo of Pisa's *Liber Abaci* contains several kinds of problems worth discussing. The recent translation by Sigler (2002) has been slighted by some but is worth having. The works of Jordanus de Nemore and Nicole Oresme are available but can be challenging for most. Swetz' edition of the Treviso Arithmetic is very accessible. I am still waiting for a translation of Pacioli's *Summa*, the Euclid of his day. The first great mathematical work of the scientific revolution is Cardano's *Ars Magna* in 1545 (Cardano 2007). In this work he discusses solution of cubic and quartic equations, gives the background for the priority dispute, and also introduces imaginary quantities. Other worthwhile and accessible works from this period include Recorde's introduction of the equal sign, Stevin's La Disme (the tenth), and Viète's discussion on the nature of roots (Viète 2006). This last work can be found on the Gallica website in Latin but in a form which is readable by most students.

The Seventeenth and early Eighteenth Centuries offer a wide array of choices for original materials. Time constraints will limit the extent of what can be discussed in class. Perhaps, students can be encouraged to research mathematicians not talked about in class. My list of essentials includes Descartes, Fermat, Pascal, Newton, Leibniz, the Bernoullis, and Euler. Many of the works of the last author can be found at the Euler Archive: www.math.dartmouth.edu/~euler/ (see also Descartes 1954, Leibniz 2005). I believe it was

Laplace who recommended: “Lisez Euler, c'est notre maître à tous”. I have attached De Moivre to the above list. I’m sure that we all have our personal favorites. As background reading for this period I highly recommend Voltaire’s *Letters on England*. He gives a lively comparison of Descartes and Newton along with some insights on the mathematical community in general.

I have given some hints as to where to find these sources but I would now like to proceed in a more methodical fashion. I find that the sources fall into roughly four categories: individual translations, sourcebooks, instructor selected readers, and internet sources. Several of the individual translations can be found as inexpensive paperbacks published by Dover. These include Euclid, Archimedes, Cardano, Viete, Descartes, and Leibniz. There exists an out of print edition of Diophantus. Carl Boyer was nice enough to stock the Brooklyn College library with at least two copies of that work. My background in the Great Books Program has alerted me to the wealth of material in the *Great Books of the Western World* series. Titles include Herodotus (Vol.6), Plutarch (Vol.14), Plato (Vol.7), Aristotle (Vol.8), Euclid - Archimedes - Apollonius - Nichomachus (Vol.11), Ptolemy (Vol.16), Descartes (Vol.31), Pascal(Vol.33), and Newton (Vol.34). Although the translations of many of these works are significantly out of date, they do provide some possibilities.

Most courses of this nature that stress reading original sources include a sourcebook as a required text. I have never found one that completely suits my needs but have recently returned to using the one edited by John Fauvel and Jeremy Gray. It seems to work fine except for Asian mathematics. I have used Calinger’s *Classics of Mathematics* (Calinger 1995) in the past but found that it covered too much material and was cost prohibitive. A new choice has been offered by Jacqueline Stedall (2008) but mathematics before 1540 is given short shrift. Sourcebooks by D. E. Smith (1959) and Stephen Hawking (2005) just don’t contain enough of the material I have described. Two out of print sourcebooks which are excellent have been edited by D.J. Struik (1986) and Henrietta Midonick (1965). I wish they were available as texts. The latter contains selections on Indian and Chinese mathematics from Colebrooke (1817) and Wylie, respectively. The Katz sourcebook (2007) has been previously mentioned. A comprehensive sourcebook entitled *Early Greek Philosophy* has been edited by Jonathan Barnes (1987).

In an attempt to create a reader that would satisfy my need for external sources I contacted the good people at Pearson – Penguin. I found I could create my own custom reader online using their system. While I was able to locate many of my choices of authors, I found that the specific selections I wanted were not being offered. I could include some of these as “outside selections” (even though they were from the regular Penguin catalog) but I would have to pay a much higher price for them. In the end I decided to try their choices which included all of the following: Hammurabi’s Code, Herodotus, Aristotle, Plato, Plutarch, The Koran, Confucius, Hugh of St. Victor, Roger Bacon, Alberti, Vasari, Galileo, Descartes, Pascal, and Voltaire. In general, I would have to say that this reader did not satisfy our needs.

Lastly, many original sources (particularly external ones) can be found on the internet. One major problem I encountered with assigning sources from the internet is that the sources frequently do not have page numbers. This can lead to some confusion. Many Greek writers can be found at <classics.mit.edu>. Euclid’s *Elements* is available in a dynamic version at

<aleph0.clarku.edu/~djoyce/java/elements/elements.html>. I was duly impressed with Cambridge University's interactive page turner at <www.lib.cam.ac.uk/cgi-bin/PascalTriangle/ browse>. The Gallica website at Bibliothèque National contains a wealth of material. Google books can help locate many out of print works. As previously mentioned museum websites are a great source for mathematical artifacts.

The task of presenting original works of mathematics to a class of undergraduates may seem like a daunting task. It requires much preparation. How does one become confident that one can do this? Well, you must remember that you are not alone in this task. Attending any conference with themes on the history of mathematics will remind you that there are many others who have the same interest as you. Lately, I have found that regular attendance at reading groups such as the Euler Society or ARITHMOS have increased both my ability to deal with more difficult readings and my desire to read further. If you don't have such a group in your area, why not find some like-minded individuals and start one? You have nothing to lose but your pre-conceptions.

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