THE USE OF ORIGINAL SOURCES IN THE CLASSROOM

Empirical Research Findings

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ABSTRACT

This article describes the theoretical framework and empirical results of a threefold comparative study in which original sources were used in the ordinary mathematics classroom according to a genetic, hermeneutic and conventional approach.

1 Introduction

In the last thirty years the use of history of mathematics has become a widespread and ever-growing approach in the teaching of mathematics. Its conjectured educational potential has been described extensively in many publications and conferences during that time. Very often, the proponents of teaching with history are also very passionate about the history of the subject itself. Obviously, this is no coincidence, and it surely applies to me. I remember that *my* own interest in the history of science began when I saw an image of Johannes Kepler's famous *Mysterium Cosmographicum.* It was printed on the cover of my physics textbook when I was a schoolboy. In fact, this was my first encounter with an original source and I was fascinated right away. I quickly learnt that the *Mysterium Cosmographicum* was meant to be a model of the solar system. It was very different from what I had ever seen before until then, although Kepler had already conceived it in the 16th century. And *because* it was so different, it piqued my interest. The words *Mysterium cosmographicum* seemed to indicate nothing less than that Kepler thought he had revealed God's geometrical plan for the universe. In fact, much of Kepler's passion for science stemmed from his belief that there were a link between the physical and the spiritual world. Of course, I had no idea about this when I saw the image for the first time. But my interest for the history of science and the history of thinking was aroused by this encounter with an original source.

My old physics textbook is out of use nowadays and has been replaced by newer editions, that I – a teacher myself by now – would use with *my* students. The newer editions still have the *mysterium* image on their covers, but not quite as large and eye-catching, and maybe not quite as thought-provoking as it had been before.

While those images have been shrinked by some layout artists, *my* interest in the history of science and mathematics has endured and become even greater. As a teacher I have always been eager to use history in the classroom whenever I think it is possible and helpful. And yet, what I could not be sure about, was the question if – apart from my subjective impression and maybe selective perception – there were any evidence that the history of mathematics really is as exciting and helpful for the majority of my students as it is for me? To say it in more general terms: That the history of mathematics is indeed a valuable resource for the learning and teaching of mathematics?

When I began to review the literature, I quickly realized – as has often been noted these days – that rather little is known empirically about this question, especially when it comes to teaching ordinary students in ordinary classrooms. So this is where my own commitment began. In order to find some answers I designed and conducted a theoretical and empirical study in which teaching with historical elements – 'historical teaching' – was compared with non-historical, 'conventional' teaching. My specific research questions were:

- How does the historical teaching affect the students' mathematical skills compared to students who had conventional teaching?
- How does the historical teaching affect the students' views on mathematics, teaching and their relationship to it?
- In what way and to what extent does the historical teaching allow for (meta-)reflection and discussion in the classroom?

There are, of course, many possible approaches to teaching with history. Personally I think, the study of original sources is the most demanding among them, both for teachers and for students. It is demanding in many ways: Of course, on part of the teacher it involves quite some preparation, but this is always the case when you are going to try something new. What is more, the study of original sources requires teachers and students to be prepared to dive into some strange and unknown realm of thinking, to appreciate cultural and historical contexts and – last but not least – to deal competently with written text that is more extensive than the word problems they are used to in mathematics. On the other hand, reading original sources does offer some exceptional opportunities (for a comprehensive account see [Jahnke et al. 2000, 292 ff.]). They might very well be worth the effort.

Original sources, however, can be used within at least two different theoretical frameworks, namely the genetic approach and the hermeneutic approach. Both of them were considered in this altogether threefold comparative study then. Its parts were

- teaching a certain topic with historical sources according to a genetic approach,
- teaching the same topic with historical sources according to a hermeneutic approach,
- and finally teaching the same topic with no history at all, that is to say 'conventionally'.

In the next two chapters I am going to describe these different approaches and its effects respectively.

2 Teaching with historical sources according to a genetic approach

2.1 Theoretical considerations

Teaching with history according to a genetic approach is, indeed, an old idea. It was the great Felix Klein, who in his classic "Elementary Mathematics from an Advanced Standpoint", argues for a genetic method of teaching wherever possible. This idea later has been adopted by Otto Toeplitz who explicitly proposed to use history in service of a genetic approach to modern mathematics. Within it history is used as a means of introducing new subjects in class or reconstructing a whole development. Toeplitz suggested that by doing so students could be guided to participate in the intriguing process of discovery that once captivated the heroes of the past. On the other hand he explicitly dissociated himself from the idea of doing history in class and said:

> I want to pick from history only the basics of those things that have stood the test of time and make use of them. Nothing is further from my thoughts than giving a lecture on history

> … as a student I have run away from a similar lecture. It is not the history of problems, theorems and proofs that is important to me, it is their genesis. [Toeplitz 1927, 95]

Thus, according to this view, history of mathematics is a tool in order to get the "real" mathematics, the mathematics of today, across to the students, and that in a genetic way.
Of course, Toeplitz' idea has been challenged by noted researchers. Walther Lietzmann for example objected that "if you wish to develop young people's mathematical skills you have to focus on the young people and not on the history of mathematics." [Lietzmann 1919, 135]. Wolfgang Klafki put it very trenchantly and said: "If you want to educate for the present, the present must be your starting point." [Klafki 1963, 127]. Serious objections against Toeplitz' way of using history can further be raised from an epistemological point of view. Toeplitz' approach in which he picks from history everything that is suitable in order to present, what he calls, "a great ascending line" from elementary to sophisticated knowledge advocates a continustic misconception of the history of science and mathematics. As Herbert Mehrtens puts it "the mathematician of today tends to declare all history the prehistory of the mathematics he knows" [Mehrtens 1992, 25]. Yet, since Kuhn's "Structure of scientific revolutions" we know that this perception is disputable. Of course, we find analogies in ancient and modern mathematics but as Niels Jahnke quite rightly points out: the things that are analogous are found in very different realms of experience and of thinking. [Jahnke 1991, 8]. For example, the Greek geometric algebra – as found in Euclid II – is no simple translation of Babylonian traditions, just as our own algebra is no cumulative continuation of Greek mathematics. Niels Jahnke concludes that

> A history of algebra does not exist in a strict sense. Instead we find a variety of related scientific work that has developed historically without forming a consistent and continuous history. [Jahnke 1991, 11]

much less a "great ascending line" that Toeplitz wishes to expose. We have to bear this in mind when we try to teach according to the genetic approach. So if for principal reasons we cannot find a great ascending line in history, we should at least just try to identify important stages. With this modifications and limitations in mind, we are prepared to do the first step of a comparative study in which historico-genetic teaching with original sources were to be compared with conventional teaching.

2.2 The teaching topic

The first thing you have to do is, find an appropriate topic to teach. After some consideration I chose 'quadratic equations and the quadratic formula', because this is really a classic, both in teaching with history *and* in the conventional secondary syllabus. The conventional goal of a unit on this topic is to have students learn the properties of quadratic equations, let them master the methods to solve them and give them some real-life problems to apply what they have learnt. In the genetic unit the goal should be the same, of course. History is just a tool then to achieve it in a different way and hopefully with some extra profit.

While it is never a problem to conceive a modern teaching unit on quadratic equations, it is different when it comes to developing a historical teaching unit to present, if not a "great ascending line" then at least some important stages in the history.

The first thing you have to do is look through the history to identify them. Depending on your demands this might become quite an effort. What I can do here is giving you a quick overview in the trust that some keywords will be sufficient for this audience. This overview is essentially based on the classic literature by Smith, Cantor and Tropfke.

The first known solution of a quadratic equation is the one given in the Berlin papyrus from the Middle Kingdom in Egypt. We have to note, of course, that Egyptian mathematics did not know equations and numbers like we do nowadays; it was instead descriptive and rhetorical. The problem I refer to is very well known and in our terminology reduces to a system of equations: $x^2 + y^2 = 100$ and $x = (3/4)\, y$. The solution that is given in the papyrus is a case of false position method.

On clay tablets the ancient Babylonians left another evidence of problems that, in our terminology, give rise to quadratic equations. The solving method is essentially one of completing the square. We've had a presentation on that this week.

"The Greeks were able to solve quadratic equations by geometric methods." Now this is a statement by David Eugene Smith, which of course can lead to some misunderstanding or even dispute. Let us agree that the Greeks dealt with geometrical problems that usually amounted to finding a *length* in a figure, which in our notion can be regarded as the root of a quadratic equation.

Hindu mathematicians took the Babylonian methods further so that Brahmagupta gives an almost modern method which admits negative quantities. He also used abbreviations for the unknown. Usually the initial letter of a color was used.

The Arabs did not know about the advances of the Hindus or at least they chose to ignore them, so they had neither negative quantities nor abbreviations for their unknowns. However Al-Khwarizmi gave a well known classification of different types of quadratics (although only numerical examples of each). The different types arise since Al-Khwarizmi had no zero or negatives. In his famous book algebra he has six chapters, each devoted to a different type of equation.

Al-Khwarizmi gives the rule for solving each type of equation, which essentially amount to the familiar quadratic formula given for a numerical example in each case, and then a proof for each example which is a geometrical *completing the square.* This particular derivation of the quadratic formula later was brought to Europe, where Al-Khwarizmi was given the name "father of algebra".

During the Renaissance in Europe, several mathematicians compiled the works related to the quadratic equations – Cardano for example blended Al-Khwarizmi's solution with the Euclidean geometry. At the end of the 16th Century the mathematical notation and symbolism was invented by François Viète. Viète was among the first to replace geometric methods of solution with analytic ones, although he apparently did not grasp the idea of a general quadratic equation. In 1637, when René Descartes published *La Géométrie*, modern mathematics was born, and the quadratic formula has adopted the form we know today.

And there it is – not a great ascending line, not a consistent history but a collection of work from the past that is related to quadratic equations and highlights some important stages.

2.3 The teaching units

Certainly, you have to reduce the material a little before you can make it a teaching unit, because otherwise it would be too much for the students and for the teachers. So I deleted Euclid, the Hindu mathematicians and the European renaissance from the list. Then I took the rest of the material and composed a historical teaching unit from it. It comprised nine lessons. In six of them the students read the original sources, in the remaining three lessons they learnt about the modern solution method by way of building upon Al-Khwarizmi's methods. They proceeded then with conventional exercises, problems and applications with modern methods.

The genetic teaching unit	**The conventional teaching unit**
1. Egyptian papyrus (false position method)	1. Introductory problem
2. Babylonian tablets (completion method)	2. Completing the square
3. Arab mathematics (eq. type 1)	3. The quadratic formula
4. … continued (eq. type 2)	4. Easy exercises
5. … continued (eq. type 3)	5. Word problems
6. … continued (the author's preface)	6. … continued
7. The modern formula	7. Easy applications
8. Word problems and applications	8. More difficult applications
9. Modeling problem	9. Modeling problem

Table 1: The genetic and the conventional (non-historic) teaching unit (outlines).

The conventional unit also comprised nine lessons. Its material was composed from standard textbooks in such a way that the students – from a technical point of view – were doing the same problems, but without any historical reference. An important difference was: In the conventional unit the students got an introduction into quadratic equations, completing the square and the quadratic formula right at the beginning, of course, with definition, theorem, proof and everything. After that they did the usual exercises, word and modeling problems.

The study was carried out with 175 students in 6 classes with 6 teachers from 3 different schools. 3 experimental classes studied the historical material, while 3 control classes pursued the 'conventional' way. Both groups then took an identical achievement test and a second one six weeks later in order to find out if one group had learnt the topic significantly better than the other.

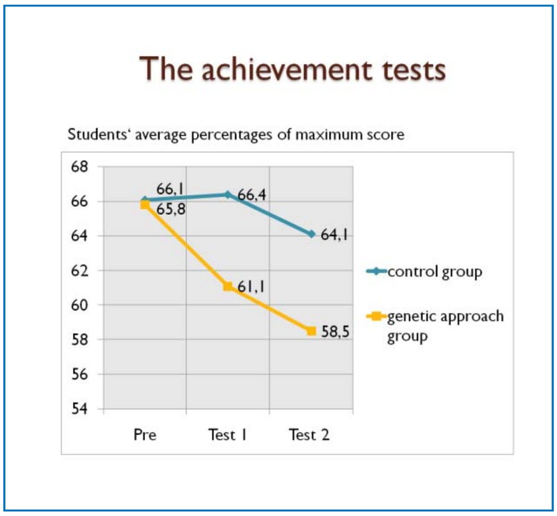
The participating teachers were offered special training. However, none of them made use of it, and evidently, I think, this is part of the reality in which historical teaching in schools would have to grow, at least in Germany. At any rate every teacher was supplied with a special accompanying booklet that explained the main ideas.

Additionally, the students were asked to fill in two questionnaires, one in advance of the experiment and the second one right after its conclusion. The questionnaires were to gather information about students' characteristics, their views on mathematics and, of course, on the teaching unit they had completed. By any means they were to account for the comparability of both groups in terms of similarity, which, in fact, they did.

All this amassed to huge amounts of data. Yet, the message was very clear: The historical teaching according to the genetic approach was a complete failure.

2.4 Some results of the teaching experiment

Let me begin with the achievement tests. They were identical in the experimental and in the control classes. Statistics show that the in-advance competence and performance levels were nearly the same in both groups. This was determined by an identical pre-test for all participants that was based on a standard competence test for German schools (figure 1; the numbers are percentages of the maximum score the students could achieve.)



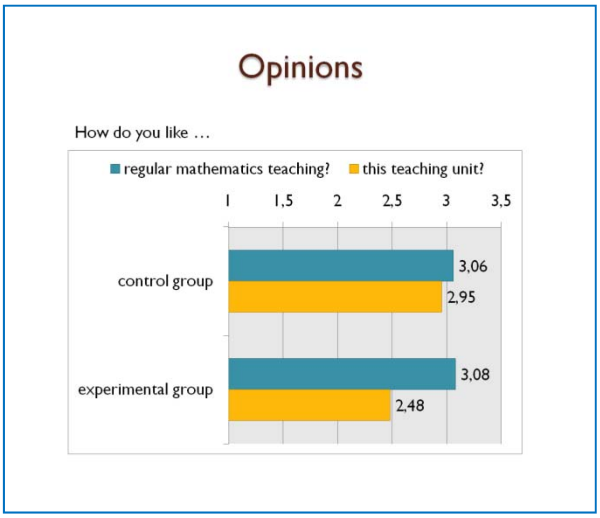


Figure 1: Results from the achievement tests   Figure 2: Opinions on the teaching units

Now look at the results from the first test right at the end of the teaching units. It consisted of problems of the type the students had worked with before. We observe nearly no change within the control group, but the students in the experimental group on average performed significantly worse. 66.4 versus 61.1; likewise in the second test, six weeks later; the performance was worse in both groups, but the difference has even increased and amounts to about half a grade. Thus we have to admit: the historical unit did not get the mathematics across in a satisfactory way.

When we have a look at the students' typical mistakes and problems, we find that they were particularly unobservant with plus- and minus-signs, that they often used a wrong formula which seemed to be a mixture from old and modern, that they in general quite often confused historical with modern approaches, that they had particular problems with applications and word problems (as opposed to the control group, which might be an indication of too little practice) and that some of them simply were not fast enough to finish the test in time.

Now let's have a look on some opinions (figure 2). How did the students like the teaching units? In both groups, Mathematics generally is a very popular subject. On a liking scale from 1 to 4 – with 4 meaning "I like it very much" –, regular mathematics teaching reached 3.06 in the control group, 3.08 in the experimental group. While this value dropped only insignificantly in the control group it has fallen sharply in the experimental group. 2.48 is a very mediocre value. A closer look at the experimental group reveals that even students who are rather interested in "normal" maths – this would be a three or a four on the liking scale – average only 2.86 when asked about the historical teaching. The others, who are rather not interested in mathematics only reach 1.83. If at all, the genetic approach is better suited for students who like mathematics very much.

What could be the reasons for these results? Let us have a look at some of the statements, the students have made.

> *"Although it was interesting, I think it was quite confusing at the same time. You never knew the important from the unimportant, as you did not know where it all should lead to."*

"It was too much about history and too little about the modern methods."
"Although it was impressive to have a look at the long and winding way, I think the matter is rather attractive for our maths experts."
"Too much material, I did not know what to focus on."
"Interesting in principle, but too long."

I think this tells us, that the students didn't get any solid ground underneath their feet, but – so to speak – rather were swept away by the river of time. Or to put it less metaphorically: I think, the historical methods they were studying – basically the well-known Al-Khwarizimi methods – did not find any anchor in their previous knowledge. It appears that they were all too new and not connected to anything, maybe not even to each other. We have to keep in mind, that those students had no knowledge of quadratic equations before. When reading the three chapters from the Al-Khwarizimi algebra they possibly did not fully understand that they had something to do with each other. They did not have this kind of modern view on it, like we do. And when the history part came to a close the students would not connect the old methods to the modern ones although the teachers did spend some time on this issue. I think the majority of the students needed their mental capacities *now* to understand *the modern methods* in the first place, before they could think about linking them to Al-Khawarizmi. Note for example, that the modern methods would produce negative solutions, whereas the old ones would not.

I could show you more charts and results and talk about them, but let it suffice to say – this teaching did not work. The students rather got confused and appeared very displeased in the end. And of course they achieved none of the aforementioned extra goals, as it was an all-in-all too frustrating experience.

3 Teaching with historical sources according to a hermeneutic approach

3.1 Theoretical considerations

As stated above, this was a threefold study and I haven't yet said anything about its third part. So far we have covered only the genetic approach. There is an alternative to it, that is called the historico-hermeneutic approach or just the hermeneutic approach. It was proposed by Niels Jahnke. What does this thorny word 'hermeneutics' mean? If you look it up in a dictionary you will find, it is "the art or principles of interpretation". Hermeneutics, in fact, is involved when it comes to the reading of original sources. Traditionally these sources were the Holy Scriptures, and later the law, but by now, since Heidegger and Gadamer, hermeneutics has broadened its range.

In our context the big difference to the genetic approach is, that the students are not expected to trace a history of thoughts that leads them from past roots to the standards of today. Essentially that is, because the students should be very familiar with a topic before they even touch a historical text that deals with it. In the hermeneutic approach, students are asked to examine a source in close detail and explore its various contexts of historical, religious, scientific etc. nature. In hermeneutics this is called: to move within hermeneutic circles – and these are circles of ever new understanding.

Of course, historically this is a rather local and perhaps even modest approach compared to what you have in mind when you imagine a historically guided reconstruction of mathematics or parts of it. The hermeneutic approach would not give you an overview. But in this case I think: less might be more. As a matter of fact, the hermeneutic approach allows for some kind of deep-drilling and aims at very fundamental preconditions of learning by addressing and stiring the prior beliefs or views in the learners' minds. According to Niels Jahnke the historical material – which is in fact the original source – should

> show something, that is accessible to the ordinary student and at the same time strange and different from what he has known hitherto. [Jahnke 1991, 11]

This is noteworthy. In teaching contexts strangeness very often is associated with obstacles to learning. We saw how strangeness in fact led to failure in the genetic teaching unit. In this approach, however, strangeness is nothing negative. Quite the opposite, it is a lever to learn and, in fact, to understand in a broader sense. So how can this be? Because the strange elements do have some anchor point. The student who deals with something that he already knows but that is presented in a radically different, unfamiliar way, in a strange context, in an unknown guise and so on, can make the connections to this anchor point. In hermeneutics you would say: His horizon merges with the horizon of the past. *Horizon merger* is a term that was coined by Hans Georg Gadamer. In the horizon merger the student may begin to wonder and to reflect upon what he possibly had never thought about before. In essence he begins to develop deeper awareness. This is in fact an instance of broadening one's horizon. The hermeneutic approach puts great emphasis on this possibility. And it does so by utilizing a strategy of dissonance. According to this strategy you let students have experiences of dissonance in a moderate amount, which – in turn – arouse the students' epistemic curiosity. This term that has been introduced by the British psychologist Daniel Berlyne. Epistemic curiosity is aroused when a person is confronted with information that is partially incompatible with his prior knowledge, his beliefs or his expectations. It is also confirmed that this kind of incompatible information ensures greater retention and ease of retrieval from memory. The hermeneutic approach makes use of these psychological findings. It puts emphasis on strangeness. But to do so, there must be a familiar reference frame. Unlike the historico-genetic approach it is therefore applied only to subject-matters that students are already familiar with. Usually the structure is as follows: First, the students have a quite conventional introduction to the topic. No history is involved until the second step, in which the students read a historical source. In this source, the same topic is covered, but in a way, that is historically distant, different in its representation, used in strange contexts and so forth. This is the step where the students' epistemic curiosity is – hopefully – aroused. In the third and final step students are required to explore the source in even greater detail, perform a horizon merger and reflect upon questions that occur to them.

3.2 The teaching unit

With regard to our hermeneutic teaching unit, this should mean that we be content with even less historical material. We merely select one instance in history and work with it in greater detail. In this case I chose Al-Khwarizmis algebra, which is a classic choice. Here's the outline of the unit:

After a plain conventional three-lessons-introduction to quadratic equations that is taken right from the conventional unit the students read the corresponding excerpts and the author's preface to his book. Of course, they discuss the material with each other, they identify its strangeness – for example the rhetorical form, the absence of abbreviations, mathematical symbols and of negative numbers, its graphical clearness – and compare Al-Khwarizmi's methods to our modern ones. They do some exercises, the same as in the genetic unit, by the way. But above all, – they get into the hermeneutic circle and merge the horizons by doing some individual research on Al-Khwarizmi and his background, his scientific, social, cultural, religious and historical background. Ten years ago this might have been impossible. But nowadays with the internet at hand students can learn quite a lot about Arab mathematics, the Golden Age of the Islam, the House of Wisdom and so on.

You might ask: why should students put so much an effort into the context instead of better doing more exercises or applications? The answer is: because it helps them to make broader sense of what they have read and learnt. This is the core of hermeneutics – to achieve a balance between text and context. The context helps to clarify the reasons for a particular piece of work, it sheds light on the author's motivations and intentions. And we know, that is what people care about. To see other people as acting, thinking and suffering. Why is this? Because, as the philosopher and theologian Emilio Betti puts it,

> "There is nothing that man is so concerned about, as to understand his fellow man; nothing that appeals as temptingly to his mind as the lost trace of human being, that shines and speaks to him." [Betti 1962, 7]

The hermeneutic approach therefore puts an emphasis on understanding the trace of human beings. But – when you are with school students you have to provide a setting that helps them. So, in this teaching unit the students did not just do some internet reading on Al-Khwarizmi. They also were given the assignment to make use of their findings in a creative and productive way

- by writing a fictitious interview with Al-Khwarizmi in which they let him give answers to who he was, why he had written the algebra, where his knowledge came from etc.;
- by writing a book review for Al-Khwarizmi's contemporaries, in which they appreciate his achievements, their reasons and their use for the society and for the Islam;
- by thinking up and performing a conversation between Al-Khwarizmi and the caliph who had asked him to write the algebra;
- even by writing some kind of science-fiction story in which Al-Khwarizmi meets a time traveler from today who tells him about his influence as "father of algebra" and who tries to persuade him to combine his three types of equations to get a unified approach.

The students worked in teams and could choose one of those assignments or suggest another. When they worked on it, they had to review again and again what they had read and learnt from the original source. The assignment, therefore, was also a means of knowledge consolidation. But it wasn't just consolidation, it was expansion as well. In hermeneutics you would say, they performed successive turns in the hermeneutic circle, and with each turn they were broadening their view and deepening their insight. – Insight into a world and a way of thinking that was very different from their own. And this is a crucial point. Hans Georg Gadamer, – I've mentioned him a couple of times by now, – puts it

> Every encounter with tradition that takes place within historical consciousness involves the experience of a tension between the text and the present. The hermeneutic task consists in not covering up this tension by attempting a naïve assimilation of the two but in consciously bringing it out. [Gadamer 1990, 311]

3.3 Some results of the teaching experiment

Where does this kind of hermeneutic movement in circles lead to? To answer this, I have done the second part of my comparative study, this time with 250 students, 10 classes in a 7-historical/3-conventional arrangement, 6 teachers and 6 schools. Please note, that the same achievement tests and pre-tests were used as before. Here are the results.

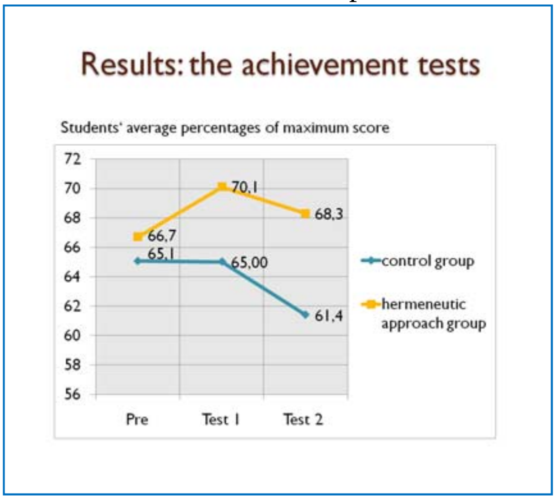


Figure 3: Results from the achievement tests.

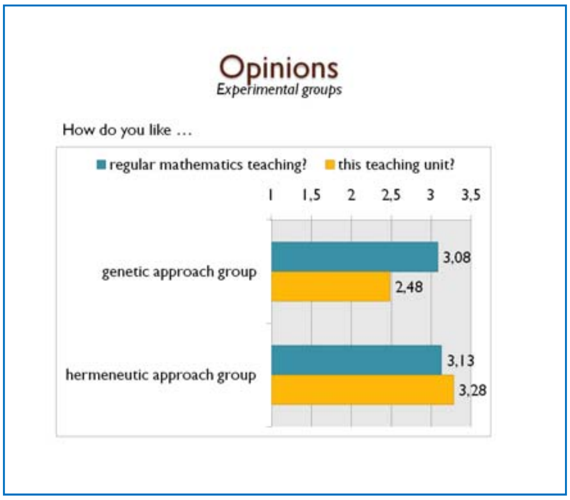


Figure 4: Opinions on the teaching units.

In the pretest the control group averaged on 65.1, in test 1 on 65.0 which is practically no change, and in test 2 on 61,4. The experimental group performed significantly better than the control group, especially with regards to the middle-term memory. I believe this has to do with the memory effects described by Daniel Berlyne in his strangeness-research.

Let us also have a look on the opinions. We compare the genetic and the hermeneutic approach groups right away. How did they like the teaching? Referring to the regular teaching, the genetic group reached 3.08 on the liking-scale, the hermeneutic group 3.13. This is no significant difference. But when we look at the values for the historical teaching, the genetic group only reached 2.48 whereas the hermeneutic group reaches 3.28.

On the one hand this difference can be explained by the students' impression of "too much and too confusing" in the genetic approach. On the other hand the hermeneutic approach led to a somewhat higher level of teaching. Students were given some freedom and responsibility when they were asked to do some research on Al-Khwarizmi and to make use of it in a creative way. I am sure that this fact played an important role because many students explicitly mentioned it. Also, it is my experience that the appreciation for the teaching unit drops, when a teacher does not make any use of these creative opportunities. Another observation is important. As we saw before, the genetic approach is better suited for students who like mathematics very much. With the hermeneutic approach this is not the case. The study shows that nearly half the students who like regular teaching very much did not like the hermeneutical teaching just the same. Rather their liking values dropped from 4 to 3 or even to 2. Nonetheless the overall average increased from 3.13 to 3.28. This is mainly due to the changes in the rest of the population. Quite a lot of students liked the hermeneutical teaching unit more than the regular teaching or, at any rate, they did not like it less. Thus we can say: the hermeneutic approach is suited for the great majority of students.

What about our intention to let the students reflect upon what they encounter? Did they seize the opportunity? Did they achieve some more of the extra goals of teaching with history? We can best answer this by looking at some of the statements they have made.

> *"I did not know that the Arabs have done so much for mathematics and science. Today, you would think that they were a little behind. Therefore I find it important to learn, what the people there have achieved so early."*
>
> *"Now I think in a different way about the Arabian countries and their culture, against which I have been prejudiced before."*
>
> *"Until now I thought that science and religion contradict each other and that scientists were not religious. But it may be different in their culture."*
>
> *"From visits to Spain with my family I knew that the Arabs had been superior in the Middle Ages. What I did not know was that this was true for mathematics as well."*
>
> *"I ask myself: Why have they lost their lead? Why did the Americans land on the moon and not the Arabs?"*

Statements like these are impressive, they are even touching. They show students on the trace of human beings instead of being on the trace of mere mathematical development.

4 Final remark

Let me finish this lecture with another personal remark. In the beginning, I told you about my first encounter with Kepler's *Mysterium Cosmographicum* and the impact it had on me. When I look at the results of this study, I begin to understand what it was that did captivate me so much then. Actually, it wasn't the model itself – of course, even then I knew it was wrong. Rather, it was the man's perplexingly original and unheard-of way of thinking, that was very strange and at the same time very fascinating to me. Betti calls it the lost trace of human being that shines and speaks to us. Elsewhere he writes that it should therefore be our finest duty to educate our youth in a way that – above all – they ask and learn about other people's thinking [Betti 1967, 203]. The students in our hermeneutic approach group eventually did learn at least as much about solving quadratic equations as they learned about a hitherto unknown, strange world, its people, their culture and their different ways of thinking and acting. Many of them even developed new appreciation for a culture they unconsciously had underestimated so far. Given this perspective, the hermeneutic approach in our teaching of mathematics could make a notable contribution to humanist, democratic and peace-promoting traditions in education.

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