“CAUCHY IN GORIZIA” IN THE MEMORY OF GIORGIO BAGNI

A year after his passing away

Caterina VICENTINI
Istituto d’Arte “Max Fabiani”, Piazzale Medaglie d’Oro, 2, Gorizia, Italy
“Scienza under 18 isontina”, Fogliano Redipuglia, Italy
caterina.vicentini@alice.it

ABSTRACT

This publication is meant to be a tribute to Giorgio Bagni a year after his passing away and it deals with the work we did together in 2007 on the occasion of Augustin Louis Cauchy's 150th death anniversary. The starting point was giving to the local teachers and students information about the 2 years, from 1836 to 1838 that A.L. Cauchy spent in Gorizia. Moreover we aimed to afford the question of the inheritance this great mathematician left to nowadays students and teachers by means of an epistemological and hermeneutic approach to history of mathematics.

1 Introduction

Not many people know that Augustin-Louis Cauchy lived in Gorizia for two years, from 1836 to 1838. He arrived from Prague, helped by the Habsburgs, together with the Bourbon Court exiled from France after the revolution of July 1830. He was the mathematics and science tutor of Henry V , the grandson of Charles X, who was supposed to become the King of France if the Bourbons had managed to rule France again.

1.1 The project „Cauchy in Gorizia“

The research that Giorgio Bagni and I did in 2007 was financed by the project called “Progetto Lauree Scientifiche” promoted by the Italian Ministry of Education and Scientific Research together with the “Conferenza Nazionale dei Presidi delle Facoltà Scientifiche” and “Confindustria” (the national association of industrials), thorough a cooperation between the Mathematics Department of Udine University (thanks to prof. Fabio Zanolin), the Ufficio Scolastico Regionale del Friuli Venezia Giulia (thanks to dr. Luigi Torchio), the Istituto d’Arte “Max Fabiani” (thanks to the headmaster prof. Fabio Della Picca), the Comune di Gorizia (thanks to the Culture Counsellor dr. Claudio Cressati) and the Administration of the Grand Hotel Entourage (thanks to dr. Antonella Lovato).

On the occasion of Augustin Louis Cauchy's 150th death anniversary, Giorgio held two conferences at the Istituto d’Arte “Max Fabiani”, open both to the students and the teachers, to present the work of Cauchy from the epistemological and hermeneutic point of view, while I held a public conference in collaboration with the “Comune di Gorizia” at the “Grand Hotel Entourage” more focusing on the life of Augustin Louis with special regard on the years he spent in Gorizia. At the end of my conference the Town Culture Counsellor unveiled a memorial marble plate, offered by the Administration of the Grand Hotel Entourage, at the entrance of the palace where Cauchy used to give his lessons to Henry V.

Hereafter you can find the historical content conveyed and the thorough examination of the foundations of Calculus under a cognitive as well as a meta-cognitive point of view in order to examine which can be considered the most valuable issues of the inheritance of

Cauchy for nowadays students and teachers.

2 Hard life of a Royalist mathematician

Augustin - Louis Cauchy was born in Paris on the 21 August 1789 , slightly over one month after the historical date, 14 July 1789, the fall of the Bastille, which marked the beginning of the French Revolution.

His father, Louis Francois Cauchy worked in very close collaboration with the police liutenant general of Louis XVI of Bourbon, who fled to England to survive the Revolution. Despite the hard times Augustin's family managed to live in Paris for a while, but at a certain moment was forced to seek refuge in Arcueil (set half way between the centre of Paris and Orly airport).

Augustin Louis's life was nothing but easy at the beginning. The Cauchys had to face a period of hard economic difficulties and fear for their personal safety. This would mark forever their young son. He not only suffered physically and remained rather weak and not in good health all his life long; but also, in a very stubborn way, never considered in his life any other revolutionary movement or idea which might cause to part him from the Bourbon Family. He had the same stubborn attitude with respect to his strong catholic beliefs.

In 1794 Roberspierre was beheaded and the Cauchys went back to Paris. Augustin's father showed great skills in adapting himself to different circumstances and political situations which offered him new opportunities: during Napoleon's consulate he was nominated file clerk at the Senate. He was able to keep his position until 1830 despite all the various political and institutional changes of that period.

Since his young age Augustin showed great learning skills (Belhoste, 1990 & 1991; Mahwin, 1995; Valson, 1968). He was bestowed all the prices for ancient languages by the Central Pantheon School which he had attended since 1802; but despite his literary success, he started, against his family tradition, an engineering course at the Ecole Polytechnique. At the Polytechnique he joined the Congregation de la Sainte Vierge, an association founded in 1801 by the Jesuit father Bourdier-Delpuits in order to organize prayer meetings. Later on, this association would have fought prevailing anti religious feelings from inside the most important institutions. After the Ecole Polytechnique Augustin-Louis was admitted to the School of Civil Engineering (l'Ecole de Ponts et Chaussées): everywhere he obtained brilliant results. In 1809 he became Engineer and the year after he was given a position at the harbour of Cherbourg where fortification works ordered by Napoleon were under way.

Thanks to the ardour and the zeal he carried in every task, Cauchy was quickly appreciated for his technical qualities, but his great interest soon turned towards pure science, which he initially cultivated in his free time. In 1813 he left the job in Cherbourg in order to return to Paris. Some intense activities on the polyhedrons, on the theory of the substitutions, on the double integrals, and a memory on the theory of the waves, rewarded by the Académie des Sciences, caught the attention of the experts on the young brilliant mathematician. Nevertheless, despite several attempts, he did not succeed in obtaining neither a chair, nor the election to the Académie des Sciences. Only after the restoration, during the reign of Louis XVIII of Bourbon, brother of Louis XVI, Cauchy was offered a post as teacher directly by the governor of the École Politechnique and than was named member of the Académie des Sciences by Royal Decree. In 1817 he was appointed teacher

also by the Collége de France and later even at the Sorbonne.

His father arranged for Augustin-Louis the wedding with Louise de Bure which was celebrated in 1818, and signed by Louis XVIII in person. Louise was the descendant of a famous dynasty of publishers in Paris, and gave him two daughters, Alicia and Mathilde. In this period Cauchy applied his extraordinary energy to the mathematical research, to the Congregation and in a very committed way to the teaching at the École Polytechnique. In this period he took his time to rework the foundations of the mathematical Analysis and to base them systematically on the idea of limit of which he also specified the meaning. Unfortunately his efforts were little appreciated, so far very often the young university professor had to fight against the negative reactions of the Conseils de l'Instruction et de Perfectionnement of the École Polytechnique.

In 1821, at the end of a lesson which lasted far beyond the established timetable, Cauchy was hissed by five or six students. The Director made a report about the incident penalizing the university professor, but the Ministry considered the hisses mainly as a political manifestation against the ultrarealistic convictions of the teacher. The publication of the course of Analysis, several times required by the Conseil d' Instruction, was eventually assured by Augustin Louis. The *Cours d'Analyse algebrique* was issued in 1821, the *Résumés des leçons sur le calcul infinitésimal*, in 1823, the *Applications du Calcul Infinitesimal à la Géometrie*, between 1826 and 1828, and the *Leçons de Calcul Différentiel* in 1829, all published by the publisher De Bure, it has to be said. These works nowadays are considered the models of any modern treaty of analysis.

In 1824 Louis XVIII of Bourbon died and the throne passed to his brother Charles X, who would not be given the same high consideration and would be dethroned in July 1830 after the revolution. The Bourbon Family was forced to the exile and, before leaving to England, Charles X and his son, the Duke of Angouleme, abdicated in favour of their grandson, the Duke of Bordeaux, still under-age, orphan of father since his birth as he was the son of the assassinated second son of Charles X, and of Maria Carolina de Berry. After Charles X, Louis Philip of Orléans succeeded on the French throne and required from the public institutions an oath of fidelity to the crown: but Cauchy refused to swear and was consequently removed from the chair. He abandoned voluntarily France, seeking refugee in Friburg (Switzerland), where in 1831 he received the invitation from the king of Sardinia Charles Albert to move to Turin in order to hold the chair of Sublime Physics.

In 1833 Augustin Louis was called to Prague by Charles X of Bourbon in exile, with the task to provide for the scientific education of the Duke of Bordeaux, presumed heir to the throne of France that he would assume with the name of Henry V (but the thing never came true). The reason of this call must be connected to the disagreement between Charles X and his daughter in law Maria Carolina de Berry about the education of Henry V, disagreement that crossed all the party of the Bourbons' followers. Henry V's mother, from whom Charles X had withdrawn the regency in favour of himself, would wish for her son a kind of education more open to the libertarian ideas introduced by the French Revolution, because her dream was to be recognized one day as the mother of the effective king of France. On the other hand Charles X kept defending the principles of the Ancien Régime and trusting the Providence for the effective rescue of the throne by the Bourbon, so he claimed for his grandson an education more faithful to the old traditional principles. In this context it is clear why Cauchy, who did not have a good reputation as teacher and pedagogue, was offered the position of tutor for Henry V. The new tutor was asked not

only to teach sciences, but to do it in a frame of deep catholic principles and not as an illuminated scientist. Who could ever have done it better than Augustin Louis?

Strongly faithful to his feelings of Bourbons follower, Cauchy did not hesitate to leave the residence of Turin, where he could dedicate himself entirely to science, in order to assume the new delicate task that would absorb a good part of his time. For five long years Cauchy followed his student in the peregrinations of the exile and tried to teach to the Duke of Bordeaux, who in the meantime had assumed also the title of Count of Chambord, the elements of mathematics and of other sciences. The pedagogical skills of the tutor were debatable, but his patience was heroic. Apparently the prince used to play him dirty tricks that went far beyond the limits of the simple joke. Cauchy was joined by his family in Prague and then he followed the Bourbon Court in exile to Teplitz, Budweitz, Kirchberg and Gorizia, where he arrived in the October of 1836 and where he witnessed the death of Charles X (Bader, 1995; Juznic, 2005). After the death of his grandfather, Henry V moved to Palazzo Strassoldo (the current Hotel Entourage) by his uncle and aunt, the Dukes of Angouleme. Cauchy was lodged at the hotel "Tre Corone" and he used to go to the Palace in order to give his tuitions to the Prince.

During his stay in Gorizia, Cauchy carried on his research. It has be estimated that 4% of his total production dates back to this period (Juznic, 2005). This means more or less 500 pages of articles. They were about the theory and the propagation of light and the complex functions. Besides this, during the period spent in Gorizia, Cauchy not only was the tutor of Henry V but he also followed and encouraged the studies of prof. Mocnik. He promoted his doctorate thesis and therefore helped him to leave the Normal School of Gorizia for the University of Graz (Juznic, 2005).

In October 1838 Henry V was 18 so Cauchy's task ended. Augustin Louis, who was almost 50, moved back to Paris with his family. He had been conferred in appreciation the title of baron by the Bourbons, title to which the mathematician attributed great importance. When back in Paris, he was offered different chairs and assignments, included a post by the Bureau des Longitudes, but his reluctance in taking the required oath held him far from every public office until the advent of the Second Republic in 1848, when the oath was suppressed. He attended only the Académie des Sciences, where every week he passed some communication. In 1849 Cauchy was named professor of Mathematical Astronomy at the Faculty of Sciences of Paris. In 1852 the emperor Napoleon III restored the oath, but after a while he exempted the royalist Cauchy and the republican Arago from swearing, so Cauchy was given the post of teacher of Mathematical Physics at the Faculty of Sciences and he held it till his death, on 23 May 1857. His last worlds pronounced in front to the priest of Sceaux were *"Les hommes passent, les oeuvres restent"* and, by good, he was right (Mawhin, 1995).

3 The mathematic work of Cauchy and the issue of the rigour

"Cauchy is insane" but he's the only one "who knows how to do mathematics" and "the only one that nowadays does pure mathematics" (Bottazzini, 1990). The young and enthusiastic Niels Henrik Abel (1802-1829) introduces in such an irreverent but stimulating way the thought of one of the main characters of the history of pure and applied mathematics.

Augustin-Louis Cauchy's work is huge and covers all the ranges of mathematics. Everywhere he was, he kept on writing books and memoirs and was second in publishing

only to Euler.

His complete work started to be published by Gauthier-Villars in 1882 and was over only in 1974. It includes 27 volumes with more than 13,000 pages. Most important works about mathematics were *Cours d'Analyse algébrique* (Paris, 1821), *Leçons sur les applications du calcul infinitésimal à la géométrie* (Paris, 1826), *Exercices de mathématique* (Paris, 1826) and *Exercices d'Analyse et de Physique mathématique* (Paris, 1841-1844). He was involved in mechanics and optics too and was also keen on poetry, even though, as a poet, was not that great (Mawhin, 1995).

The importance of his analytical works is double: both substantial and formal. In them, we can find a lot of definitions and demonstrations which are considered, according to the XIX century sensibility, fully "rigorous".

Here follow some of Cauchy's words:

«*Quant aux méthodes, j'ai cherché à leur donner toute la rigueur qu'on exige en géométrie, de manière à ne jamais recourir aux raisons tirées de l'algèbre. Les raisons de cette espèce, quoique assez communément admises […], ne peuvent être considérées, ce me semble, que comme des inductions propres à faire quelquefois pressentir la vérité, mais que peu s'accordent avec l'exactitude vantée des sciences mathématiques...*»

that can be translated:

«*As for the methods, I have sought to give them all the rigour which one demands from geometry, so that one need never rely on arguments drawn from the generality of algebra. Arguments of this kind, although they are commonly accepted […], may be considered, it seems to me, only as examples serving to introduce the truth some of the time, but which are not in harmony with the exactness so vaunted in the mathematical sciences…*»

Cauchy's cultural project, therefore, appears to be very clear. He wanted to part the analysis from the procedures, currently used in his time, which had merely the aim to justify empirical results. These methods were almost never completely justified, and revealed in some cases to be clearly incorrect. They were deriving from an application of algebraic techniques to analytic situations without any real theoretical reason. Cauchy, instead, perceived the pressing need to base all the analytical concepts on precise foundations and was the first mathematician to do it.

In this perspective, what can be considered Cauchy's inheritance for mathematicians, and overall for nowadays teachers and students?

Let's consider the definitions of infinitesimal and limit given by Cauchy in the *Cours d'Analyse*:

"When the value successively attributed to a particular variable indefinitely approach a fixed value in such a way as to end up by differing from it as little as we wish, this fixed value is called the limit of all the other values. Thus, for example, an irrational number is the limit of the various fractions that give better and better approximations to it. In geometry, the area of a circle is the limit towards which the areas of the inscribed polygons converge when the number of their sides grows more and more, etc. When the successive numerical values of such a variable […] decrease indefinetely, in such a way as to fall below any given number, this variable becomes what we call an infinitesimal, or an infinetely small quantity. A variable of this kind as zero as its limit." (Bottazzini, Freguglia & Toti Rigatelli 1992)

Could we define them as “absolutely rigorous”? Probably not, if examined on the light of the mathematical sensibility of our days. This answer, however, appears too simple to be considered satisfactory. The comparison with the previous mathematical literature certainly shows a clear radical change of trend: **Cauchy placed the notion of limit instead of that of derivative as a fundamental concept** for the first time in a treatise on Analysis.

The importance of analytical work of Augustin Louis is also due to a clear theoretical organisation of some geometric notions which were previously introduced only by intuition. From the end of XVII century on, the notions of area limited by a curve, of length of a curve, of volume limited by a surface and of area of a surface had been simply accepted as understood by everyone, and the fact that such quantities could be measured through integrals was considered one of the main realizations of calculus (Kline, 1991). Cauchy was the first considering **those quantities as defined by the integrals normally used only to calculate them.** This fact clearly represents an important and theoretically delicate change of perspective.

The question therefore is: what's Cauchy's inheritance for mathematicians, and above all for nowadays teachers and students?

In order to try to give an answer, we should introduce some considerations about the use of the history of mathematics with reference to teaching.

4 The inheritance of Cauchy: an hermeneutic approach

Let's take again in account Cauchy's definitions of limit and infinitesimal to discuss about their being or not “rigorous”. If we consider them in the light of today's mathematical sensibility, surely they are not, at least not completely. Whereas such an answer, as already said, seems too simple to be satisfactory.

There are some observations which are worth discussing.

From the point of view of a historical reconstruction, the comparison with the preceding mathematical literature highlights a clear reversal. In his *Traité élémentaire du Calcul Différentiel et du Calcul Intégral* published between 1810 and 1819, S.F. Lacroix hadn’t put the notion of limit at the heart of differential calculus, but the notion of derivative and differential (Lacroix, 1837).

The new treatment, which we owe Cauchy, reflects however the general lines which are often adopted by modern mathematical analysis, in which the limit itself is considered as the fundamental concept. (Boyer, 1982).

Therefore neither a historical reconstruction nor a today’s point of view has to be the exclusive horizon from which we would consider Cauchy's position.

The very question is: with reference to what notion of rigour, should we nowadays interpret the works of the great French mathematician?

According to Umberto Bottazzini, rigour in Mathematics is itself a historical concept and so it is in progress. Appealing to the requirement of rigour when explaining the development of mathematics seems actually a circular process. In fact, we come to the formulation of new *standards* of rigour when the old criteria don't allow a suitable reply to the questions that come from mathematical practice (Bottazzini, 1981).

We find ourselves in a difficult situation which presents two components which are quite different and for some aspects, at least apparently, contrasting. On one side an interpretation based on historical reconstruction carried out from the point of view of 19th century mathematical analysis would lead us to emphasize Cauchy's process of

introduction of rigour, on the other side, a “modern” approach would draw us to consider critically the definitions and the concepts from our point of view and so with explicit reference to the epistemological statute of mathematics which is used, taught and learnt nowadays.

These two different interpretations are not anyway conflicting. A correct hermeneutic approach, referable to the ideas of Hans-Georg Gadamer (1900-2002) may help us considering the situation in all its complexity and richness, especially from the point of view of teaching (Bagni, 2006). We have to quote Wilhelm Dilthey (1833-1911) who underlined that the first condition of the possibility of a science of history consists of the awareness that everybody is a historical being even the person that studies and investigates history (Dilthey, 1962).

Gadamer (2000) moreover remarks that a historian often chooses the concepts with which he describes the characteristic historical physiognomy of its objects without explicitly caring for their origin and justification, omitting in this way to realize that the descriptive suitability he finds in the concepts he chooses might flatten what is historically distant on what is familiar.

To sum up, the risk of making something present-day without any awareness is concrete and is explicitly marked by different authors.

Should a more careful attitude be necessary for a historian?

Should it be advisable to model concepts on those which are characteristic of the period we are examining?

With reference to Cauchy's huge and important work of introduction of rigour, should we put ourselves in a mental attitude which is somehow close to the one of XIX century mathematicians?

Gadamer strongly claims that such a choice would be both illusory and groundless and that the naivety of an historian becomes an abyss when he starts realizing the problematic nature of his position and sets the principle that, in historic understanding, one has to put his own ideas aside, trying to think only according to the concepts of the period one wants to know (Gadamer, 2000).

Such naivety doesn't come simply from the inevitable failure to which this choice would be condemned, the main problem is different and much deeper: the historic conscience disregards itself if, in order to understand, leaves out just what makes understanding possible. To think historically actually means to bring to an end the transposition that the concepts of the past undergo when we try to think according to them. To think historically always involves mediation between those concepts and the actual way of thinking (Gadamer, 2000).

So, we are neither allowed to judge Cauchy's work as it had been done nowadays nor really able to evaluate it “from inside the XIX century”, so to speak. To interpret a historical event considering our point of view is somehow inevitable, but it mustn’t happen in direct and radical terms. *Mediation* is the key concept. It is not possible to appreciate the work of a great protagonist of the history of mathematics and more generally of the history of human thought, without considering its historicity; as well as at the same time, it is not possible give up completely our role of readers and interpreters.

The fact that we can't help duly considering a scholar's point of view while examining and interpreting the mathematical work of an author of the past, must not lead us either to consider any historical reconstruction useless or make us fear any form of arbitrariness

when we use historical elements in our teaching.

We just want to recall that also very “hard” sciences have progressively loosened their hold as regards a demand of absolute objectivity. The epistemological statute of physics, for example, has had Heisenberg's uncertainty principle to reckon with. So it seems understandable the right, becoming at the same time a duty, to “mediate” inside a discipline like the history of mathematics which, since it is history, undoubtedly presents clear “humanistic” features.

In the light of these reflections, we can declare that Augustin-Louis Cauchy's lesson for the students and the teachers of the XXI century is alive, intense and involving. It is the great, rich attempt, historically placed, to give rigour to concepts and procedures of mathematical analysis. His correct interpretation, by nowadays readers, will give a lot of precious ideas for a wide cultural growth, both form the point of view of the true understanding of contents, and from a methodological approach.

In conclusion, it can be asserted that the work done in Gorizia in 2007 is an example of a possible collaboration between School, University, local authorities and private citizens in order to integrate the History of the Mathematics with regard to this hermeneutic approach not only in the Didactics and the increase of the epistemological awareness of teachers and students, but also in the increase of the historical knowledge of the citizens.

In short: the History of the Mathematics as an important, inseparable and unavoidable part not only of the teaching of Mathematics, but also of History as such.

REFERENCES

- Bader, L., 1995, *I Borboni di Francia in esilio a Gorizia,* Casa Editrice Goriziana per la Cassa di Risparmio di Gorizia, Gorizia.
- Bagni, G.T., 2005, “The historical roots of the limit notion. Cognitive development and development of representation registers”, *Canadian Journal of Science, Mathematics and Technology Education* **5**, 4, 453-468
- Bagni, G.T., 2006, *Linguaggio, storia e didattica della matematica*, Pitagora, Bologna
- Bagni G. T., Vicentini C., 2007, *Augustin-Louis Cauchy (1789-1857) a centocinquant’anni dalla scomparsa*, «Progetto Alice», n.24, 391-401.
- Belhoste, B., 1985, *Cauchy. Un mathématicien legittimiste au XIXe siècle,* Belin, Paris
- Belhoste, B., 1991, *Augustin Louis Cauchy. A Biography*, Springer, New York
- Bottazzini, U., 1981, *Il calcolo sublime*, Boringhieri, Torino
- Bottazzini, U., 1990, *Il flauto di Hilbert. Storia della matematica moderna e contemporanea*, UTET, Torino
- Bottazzini, U., Freguglia, P., Toti Rigatelli, L., 1992, *Fonti per la storia della matematica*, Sansoni, Firenze
- Boyer, C., 1982, *Storia della matematica*, Mondadori, Milano
- Dilthey, W., 1962, *Gesammelte Schriften*. Teubner, Stuttgart e Vandenhoeck & Ruprecht, Göttingen
- Gadamer, H.G., 2000, *Verità e metodo*. Vattimo, G. (a cura di). Bompiani, Milano (*Wahrheit und Methode: Grundzüge einer philosophischen Hermeneutik*. Mohr, Tübingen 1960)
- Grabiner, J.V., 1981, *The origins of Cauchy's rigorous calculus*, MIT Press, Cambridge (MA)
- Juznic, S., 2005, Cauchyjeva in Mocnicova Gorica kot sredisce evropske matematike (ob 190.obletnici Mocnikovega rojstva), *Archivi* 28, 15-35, Ljubljana
- Kline, M., 1991, *Storia del pensiero matematico. II. Dal Settecento a oggi*, Einaudi, Torino
- Lacroix, S.F., 1837, *Traité élémentaire du Calcul Différentiel et du Calcul Intégral*, 5ème édition, Bachelier, Paris
- Mawhin, J., 1995, Augustin Louis Cauchy: l’itinéraire tourmenté d’un mathématicien légitimiste, *Mathématique et Pédagogie* 103, 5-20
- Valson, C.A., 1968, *La vie et les œuvres du baron Cauchy*, I-II, Gauthier-Villars, Paris