NEWTON'S PRINCIPIA MATHEMATICA IN A TWENTY FIRST CENTURY CLASSROOM

Alejandro ROSAS, Leticia del Rocío PARDO
CICATA-IPN, Legaria #694 Col. Irrigación, México D.F., México
alerosas@ipn.mx, rociopardo2000@yahoo.com.mx

ABSTRACT

In this paper we present a didactic activity developed under the Project Based Learning approach. We wanted that our students experienced the way the seventeen-century scientists did calculations without using computers or electronic calculators. The didactic activity involved the use of Lemma V included in Newton's Principia Mathematica. We discuss our experience applying this activity to Engineering students and a group of in service teachers of high school and university. We also show some results of our students concluding that the main goals stated when we designed the activity were fulfilled.

1 Introduction

In our classrooms students live with electronic calculators and computers without any idea of how scientist made their discoveries in the past. Most students do not imagine how Archimedes, Pythagoras, Pascal, Newton, Euler or every famous mathematician and physicist did the calculations they needed. To be honest neither we did as students. Many of us do not imagine how ancient scientist could work without calculator or computers, or even without the Calculus and a gravitational theory like Newton's.

We designed our activity with three main goals; the first is to discover the way scientist worked in the past centuries. The second goal is to justify the need of numerical methods and the third one is to rediscover one of the most famous books in the History in a twenty first century classroom. In addition we want students to solve a problem that is not included in pre-calculus classes.

Project Based Learning (PjBL) approach was used to design the activity, we gave a short context and a set of real data to be used.

2 Origin of the activity

By 2005 two colleagues of us were working with Taylor's Series and its original source when they were told about some interpolation techniques proposed by Newton in his Lemma V included in "Principia Mathematica" (part of Proposition XL Theorem XXI, Book 3). After some time, one of the authors of this paper was researching about numerical series and its origins as part of his PhD thesis (Rosas, 2007) and Lemma V was mentioned again.

In Instituto Tecnológico y de Estudios Superiores de Monterrey (ITESM for short) Engineering students must learn mathematics based on "real world applications". This way teachers need to apply some didactic techniques like Problem Solving, Problem Based Learning, Cases Study, Project Based Learning and others. After some years of using these techniques students feel fine with this kind of learning (at least we think they feel fine). As a result of our technological era our students think that only computer solutions are good and sometimes they do not respect ancient methods.

In 2009 because of these reasons we decided to design a didactic activity that included Project Based Learning, an ancient technique and a "real world" situation. Our first idea was to give to our students some invented data of a car to approximate its position. Then we remembered that Newton's Lemma V deals with asteroids and we thought that it would be a good idea to use real data of an asteroid. After a brief quest in diaries we found an asteroid (Ruiz, 2009) named 2009ST19, one kilometre in diameter and passing around 600,000 kilometres of the Earth, it was discovered by the Spaniard astronomer Josep Maria Bosh on September 16, 2009. We visited the Jet Laboratory Propulsion web page (Jet Laboratory Propulsion [JLP], n.d.) to find some data about it.

The Solar System Dynamics is a sub web page of JLP where we can find any data about celestial bodies in the section Small-Body Database Browser [SSD-SBDB] (Solar System Dynamics, n.d.). After writing the asteroid's code in the search field we can find every known data about the asteroid. With the Ephemerides option we can choose the period of time where we want the system calculates asteroid's data.

Some of the data we can get are number of observations, data-arc span, first observation used, last obs. used, elevation angle, perihelion, period, and Sun-Observer-Target (S-O-T). The called S-O-T is the angle between the Sun, object and observer; a short explanation of its different values is:

> Sun-Observer-Target angle; target's apparent solar elongation seen from observer location at print-time. If negative, the target center is behind the Sun. Angular units: DEGREES. (Solar System Dynamics, n.d.)

In our first design we choose the elevation angle of the asteroid, but we thought that its values were changing so much that our students could make mistakes. So we decided to use the S-O-T angle because its variations were more suitable to the example provided by Newton.

This way our design included an ancient method of interpolation used in a time of no computers and involved the use of a real object that could endanger the Earth's life; this last thing was interesting for our students. To increase the feeling of the historical context we decided to use the original Latin version of Newton's Lemma V.

3 Activity in detail

PjBL is a didactic technique built over real learning activities that catch students' interest and also encourage students to search new knowledge. Activities under this approach are designed to solve a problem and generally show the many ways people learn and work in the real world.

Some characteristics of PjBL are: Environment focused on learning, Collaborative work, Real world tasks, Many ways of solving a problem, Time management and Different ways of evaluate students' work. These characteristics let teachers to act like a guide instead of a "know-all" people. This guiding is important because in some projects students do not know how to solve the problem and neither the teacher.

The didactic activity was called "finding an asteroid". In SSD-SBDB we selected 13 observations from July 18 to November 15 in 2009, with ten days of separation. We show the full table of these data in Image 1.

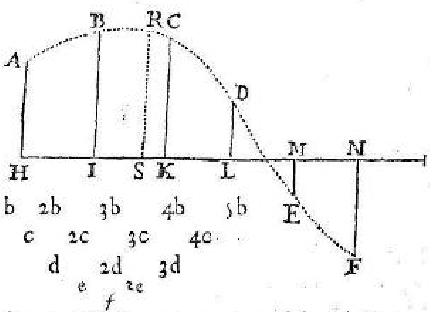
Date__(UT)__HR:MN		S-O-T
2009-Jul-18	00:00	110.6386
2009-Jul-28	00:00	98.2399
2009-Aug-07	00:00	85.0185
2009-Aug-17	00:00	68.5472
2009-Aug-27	00:00	48.2178
2009-Sep-06	00:00	57.0115
2009-Sep-16	00:00	80.3307
2009-Sep-26	00:00	97.0950
2009-Oct-06	00:00	111.1146
2009-Oct-16	00:00	124.4429
2009-Oct-26	00:00	137.5462
2009-Nov-05	00:00	149.7362
2009-Nov-15	00:00	159.0660

Image 1. Values of the elevation angle for asteroid 2009 ST19

In the activity we ask students to apply Lemma V from Newton's Principia Mathematica to the table in image 1 and to approximate the value of the S-O-T angle of the asteroid for two different dates. We choose September 10th because it is a date inside the given observations like Newton's example and Lemma V should give a good approximation of the real S-O-T angle, this was the first date. The second date was November 25th, a date outside the given observations. Lemma V then gives a wrong answer and we wanted to find what students would do when their calculated value were visible wrong. In image 2 we can see part of Lemma V schema and formulas.

[482]

2 c, 3 c, 4 c, &c. tertias d, 2 d, 3 d, &c. id est, ita ut sit $HA - BI = b$, $BI - CK = 2b$, $CK - DL = 3b$, $DL + EM = 4b$, $-EM + FN = 5b$, &c. dein $b - 2b = c$ &c. Deinde erecta quacunque perpendiculari RS, quæ fuerit ordinatim applicata ad curvam quæsitam: ut inveniatur hujus longitudo, pone intervalla HI, IK, KL, LM, &c. unitates esse, & dic $AH = a$, $-HS = p$, $\frac{1}{2}p$ in $-IS = q$, $\frac{1}{3}q$ in $+SK = r$, $\frac{1}{4}r$ in $+SL = s$, $\frac{1}{5}s$ in $+SM = t$; pergendo videlicet ad usque penultimum perpendiculum ME, & præponendo signa negativa terminis HS, IS, &c. qui jacent ad partes puncti S versus A, & signa affirmativa terminis SK, SL, &c. qui jacent ad alteras partes puncti S. Et signis probe observatis erit $RS = a + bp, + cq + dr + es + ft$ &c.



Cas. 2. Quod si punctorum *H, I, K, L,* &c. inæqualia sint in

Image 2. Image of the schema and formula of Newton's Lemma V (Newton, 1667, p. 482).

The complete activity was available in a webpage format in the course's web page. Students had to open the activity and solve it in a one-week period. There was not a specific format for the final report, but it should include calculations, diagrams, graph or anything else they used to solve the activity. In the appendix we include the translated version of the activity.

4 Experiences applying the activity

So far we have used four times this activity. Three times were used with students in first year of Engineering and the fourth time the activity was solved by students in first semester of Master Degree in Mathematics Education.

Our first experience was with a group of approximately 25 first-year Engineering students. This time the activity included the original text written in Latin by Newton. Many students complained about the activity and asked for a translation in order to solve the problem. Because students could not apply the algorithm to the data we gave them, their teacher had to solve a short example in the classroom. After that students were able to repeat the process with the data set of the 2009ST19 asteroid. After so many complaints we had students' answers just like the example of the teacher. We decided not to include the answers as part of the research because all of them were almost the same and we did not observe any personal details. We think that the example drove the students too much.

As we wanted the students to discover how the seventeenth-century scientists calculated a planet's orbit without computers (and electronic calculators), we decided to include a short explanation of the algorithm in Spanish for the second time we applied this activity. After this modification, we gave the activity to the students and they focused on the algorithm and the involved mathematics. Two groups of Engineering with 30 and 32 students solved the activity.

Most students did the calculations by hand but they tested their results using calculators or a spreadsheet. This time they showed interest in the usefulness of basic arithmetic to solve something complicated as the orbit of an asteroid. A couple of students delivered solutions including remarks that we did not expect.

Last time we used this activity in a class was in an introductory course of first semester of an Online Master Degree in Mathematics Education. The students of this MD program need to be in-service teachers of high school or university (they usually are over 28 year old and we will refer to them as teacher-students in contrast to university students). Because of this characteristic we decided to use the Latin version of the activity. Teacher-students did not complain, they just asked if there were Spanish, or English versions of Newton's Principia Mathematica. But they did not wait for our answers; they searched Latin to Spanish dictionaries or automatic translators. Some teacher-students translated the Latin text; others found Spanish versions of the text. After a couple of days every teacher-student could read and follow Newton's text. We thought that the translations would have some mistakes but teacher-students shared their texts and fixed the mistakes of their translations.

The common experiences of this group were the spontaneous and cooperative work among teacher-students; they also liked the use of arithmetic to found the position of an asteroid. Some remarks of them were about use a Spanish version of the activity in their own classrooms.

5 Some students' answers

Now we analyse some of the answers that our students gave us. There were no restrictions on the final report, but many answers had common elements in technical details and format.

One of those common elements was a graph where students plotted the asteroid's data set as points. Some students included in their graph a point representing the unknown value but it was not a general characteristic. Almost half of these graphs were drawn with a continuous curve joining the points.

Most students included images of the paper sheets where they wrote their calculations. Divisions, products, sums and subtractions were included in the reports as an evidence of no using computers. The S-O-T angle of the asteroid calculated with the algorithm was quite near the real value.

On the left side of image 3 we can see the calculations of letters a, o, p, q, r, s, t, u, v, w, x, y and z following Newton's calculations. On the top middle of the image we can see a text inside a box titled "altura de A" ("the height of A") and below it the substitutions of the values found on the left side of the sheet. This is the case of the S-O-T angle for July 18. The last calculations are for the case of November 15, and as it can be seen in the last line the angle is negative (but it should be positive).

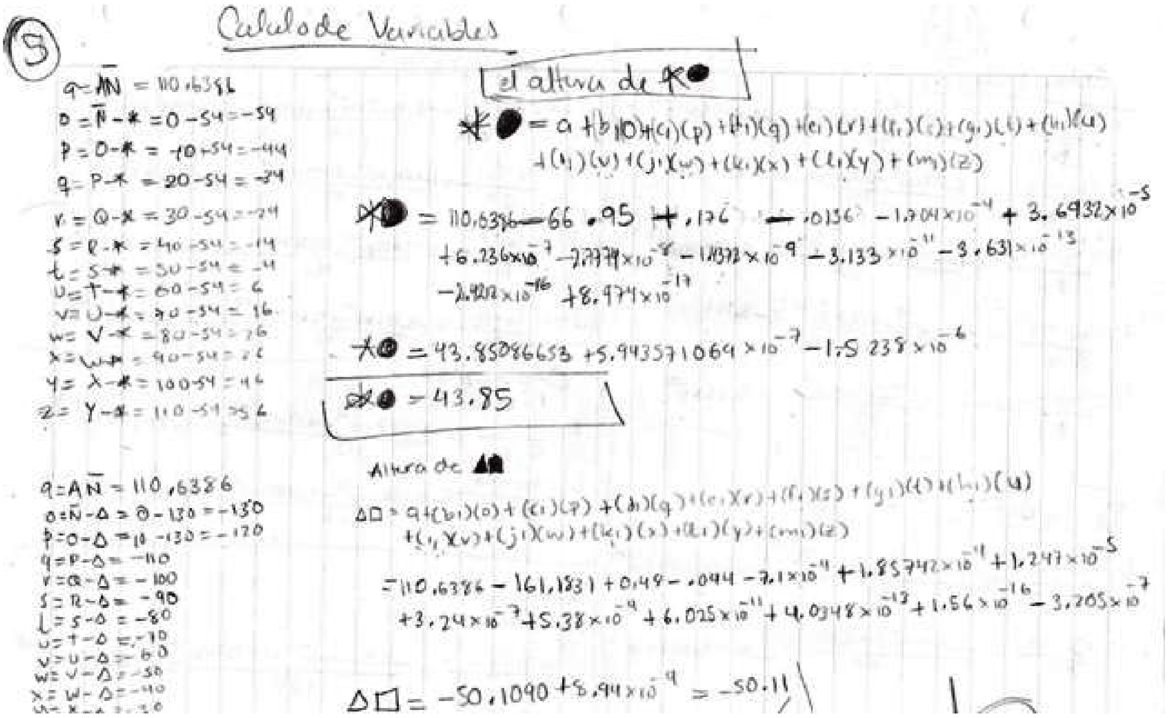


Image 3. Students' hand made calculations

One student's answer includes some remarks we think are interesting. He made his calculations by hand but he thought that he could be wrong. As a way of verification he used Stellarium (a free open source planetarium for the computer with 3D visualizations of the sky) (Stellarium, n.d.) and he found that his value of the elevation data calculated with Lemma V was a very good approximation. He said that with his telescope he took a photo of the asteroid but he gave us not details about the class of telescope and camera he used so we were not sure if he really took the photo. With the same kind of calculations another student commented that she used the software Mathematica to interpolate the value of the S-O-T angle and she found that her results were good.

Teacher-students' solutions were like students' solutions, the only difference were that teacher-students did not complain about the Latin version of Newton's Lemma. Most teacher-students delivered their calculations made by hand and a spreadsheet as a kind of validation of their results.

6 Conclusions

In every one of the three experiences we considered applying this activity we found the same situation: Students gave respect to ancient scientist and their work done without computers. We also found some students really involved in the activity; they felt very interested in Lemma V as a good way to approximate unknown values based in a few observations. We found students, who did not like the activity but they were no more than five and we think it is a good result. Every time we used "finding an asteroid" students were interested and we think it is because the problem is not a common problem like "find the equation of…"

We did not mention to our students that this technique involved an interpolation; our idea was to generate in our students' minds the need for this kind of methods. One student tested her results using interpolating software and this was unexpected. But it was only this case.

As a final remark we can say that we fulfilled our three goals in last three times we used this activity. Some teachers had given us suggestions to modify or strengthen the activity and we are working on that. We hope to get better results.

REFERENCES

- Jet Propulsion Laboratory (n.d.). Retrieved from the JPL web page http:// ssd.jpl.nasa.gov/sbdb.cgi/ on February 10, 2011.
- Newton, I. (1687). *PhilosophiaNaturalis Principia Mathematica*. Burndy Library.
- Rosas, A. (2007). *Transposición Didáctica de las series numéricas infinitas. Una caracterización del Discurso Matemático Escolar vigente* [Didactic transposition of the nummerical infinite series. A characterization of the actual mathematical speech in the school]. Mexico. CICATA-IPN. Unpublished PhD thesis.
- Ruiz, M. (2009, September 25th). El gran asteroide que más se ha acercado a la tierra [The great asteroid that has ever got closer to earth]. El país. Madrid, Spain.
- Solar System Dynamics (n.d.). Small-Body Data Browser. Retrieved from http://www.ssd.jpl.nasa.gov/sbdb.cgi on February 25, 2011.
- Stellarium. (n.d.). Retrieved from http://www.stellarium.org/es/ on February 25, 2011.

APPENDIX

Didactic Activity: Finding an asteroid
Date of application: December 01, 2009.
Remarks: This activity was available in a web page and the images were of high quality, but in this appendix we include those images with low quality.

This activity is named "*Finding an asteroid*" because you will work with real data of the position of an asteroid, and you will use a method created by Newton in 1667.

The next two images show two pages of the book Philosophiae Naturalis Principia Mathematica written by Isaac Newton in 1687, as you can see it is written in Latin because at that time scientists wrote in Latin (actually we mostly write in English).

The first part of your job is to read Lemma V, yes in Latin.

xima ex parte ſupra Planetas verſantes, & eo nomine orbes axibus majoribus deſcribentes, tardius revolventur. Ut ſi axis orbis Cometæ ſit quadruplo major axe orbis Saturni, tempus revolutionis Cometæ erit ad tempus revolutionis Saturni, id eſt ad annos 30, ut $4\sqrt{4}$ (ſeu 8) ad 1, ideoque erit annorum 240.

Corol. 2. Orbes autem erunt Parabolis adeo finitimi, ut eorum vice Parabolæ abſque erroribus ſenſibilibus adhiberi poſſint.

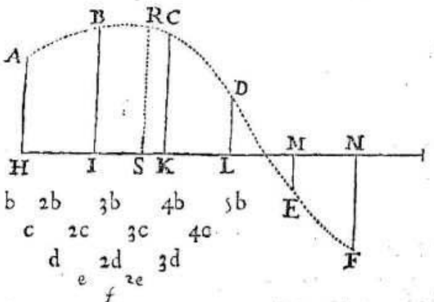
Corol. 3. Et propterea, per Corol. 7. Prop. XVI. Lib. I. velocitas Cometæ omnis erit ſemper ad velocitatem Planetæ cujuſvis circa Solem in circulo revolventis, in dimidiata ratione duplicatæ diſtantiæ Planetæ ~~Cometæ~~ à centro Solis ad diſtantiam Cometæ ~~Planetæ~~ à centro Solis quamproximè. Ponamus radium orbis magni, ſeu Ellipſeos in qua Terra revolvitur ſemidiametrum tranſverſam, eſſe partium 100000000, & Terra motu ſuo diurno mediocri deſcribet partes 1720212, & motu horario partes $71675\frac{1}{2}$. Ideoque Cometa in eadem Telluris à Sole diſtantia mediocri, ea cum velocitate quæ ſit ad velocitatem Telluris ut $\sqrt{2}$ ad 1, deſcribet motu ſuo diurno partes 2432747, & motu horario partes $101364\frac{1}{2}$. In majoribus autem vel minoribus diſtantiis, motus tum diurnus tum horarius erit ad hunc motum diurnum & horarium in dimidiata ratione diſtantiarum circiter ~~reſpective~~, ideoque datur.

Lemma V.

Invenire lineam curvam generis Parabolici, quæ per data quotcunque puncta tranſibit.

Sunto puncta illa A, B, C, D, E, F, &c. & ab iiſdem ad rectam quamvis poſitione datam HN demitte perpendicula quotcunque AH, BI, CK, DL, EM, FN.

Caſ. 1. Si punctorum H, I, K, L, M, N æqualia ſunt intervalla HI, IK, KL, &c. collige perpendiculorum AH, BI, CK &c. differentias primas b, $2b$, $3b$, $4b$, $5b$, &c. ſecundas c, 2 c, 3 c, 4 c, &c. tertias d, 2 d, 3 d, &c. id eſt, ita ut ſit $HA - BI = b$, $BI - CK = 2\,b$, $CK - DL = 3\,b$, $DL + EM = 4\,b$, $-EM + FN = 5\,b$, &c. dein $b - 2\,b = c$ &c. Deinde erecta quacunque perpendiculari RS, quæ fuerit ordinatim applicata ad curvam quæſitam: ut inveniatur hujus longitudo, pone intervalla HI, IK, KL, LM, &c. unitates eſſe, & dic $AH = a$, $-HS = p$, $\frac{1}{2}p$ in $-IS = q$, $\frac{1}{3}q$ in $+SK = r$, $\frac{1}{4}r$ in $+SL = s$, $\frac{1}{5}s$ in $+SM = t$; pergendo videlicet ad uſque penultimum perpendiculum ME, & præponendo ſigna negativa terminis HS, IS, &c. qui jacent ad partes puncti S verſus A, & ſigna affirmativa terminis SK, SL, &c. qui jacent ad alteras partes puncti S. Et ſignis probe obſervatis erit $RS = a + bp, + cq + dr + es + ft$ &c.

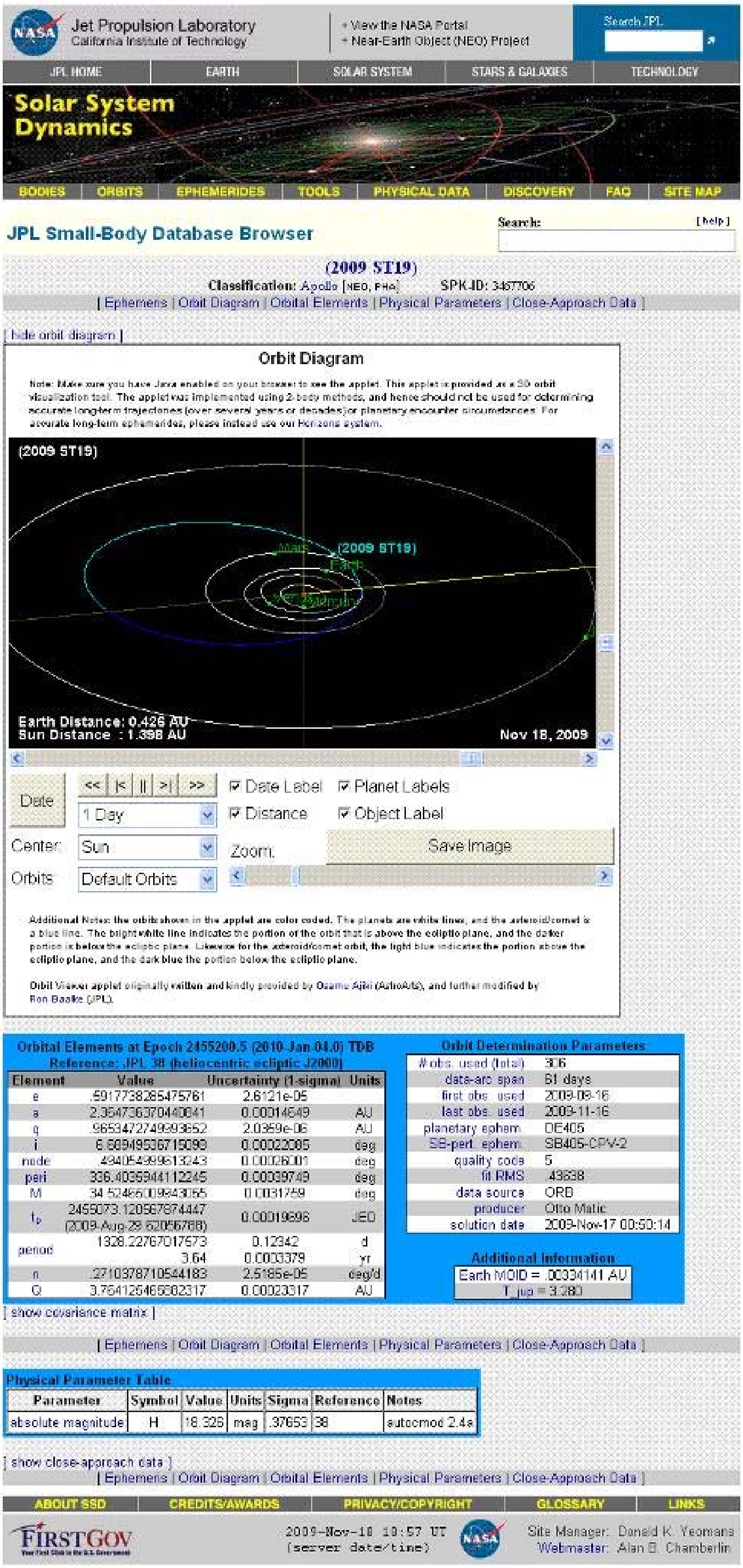


Caſ. 2. Quod ſi punctorum H, I, K, L, &c. inæqualia ſint intervalla HI, IK, &c. collige perpendiculorum AH, BI, CK, &c. differentias primas per intervalla perpendiculorum diviſas b, 2 b, 3 b, 4 b, 5 b; ſecundas per intervalla bina diviſas c, 2 c, 3 c, 4 c, &c. tertias per intervalla terna diviſas d, 2 d, 3 d, &c. quartas per intervalla quaterna diviſas e, 2 e, &c. & ſic deinceps; id eſt ita ut ſit $b = \frac{AH - BI}{HI}$, $2\,b = \frac{BI - CK}{IK}$, $3\,b = \frac{CK - DL}{KL}$ &c. dein $c = \frac{b - 2b}{HK}$, $2c = \frac{2b - 3b}{IL}$, $3\,c = \frac{3b - 4b}{KM}$ &c. Poſtea $d = \frac{c - 2c}{HL}$, $2\,d = \frac{2c - 3c}{IM}$ &c. Inventis differentiis, dic $AH = a$, $-HS = p$, p in $-IS = q$, q in $+SK = r$, r in $+SL = s$, s in $+SM = t$; pergendo ſcilicet ad uſque perpendiculum penultimum ME, & erit ordinatim applicata $RS = a + bp + cq + dr + es + ft$, &c.

Corol. Hinc areæ curvarum omnium inveniri poſſunt quamproximè. Nam ſi curvæ cujuſvis quadrandæ inveniantur puncta aliquot,

The second part of your job will be to apply the method described by Newton to a set of values of the angle called S-O-T. The red line shows the formula you can use in this activity.

The set of data belongs to the asteroid 2009ST19 that we got from the Jet Propulsion Laboratory web page:



Jet Propulsion Laboratory
California Institute of Technology

+ View the NASA Portal
+ Near-Earth Object (NEO) Project

Search JPL

JPL HOME | EARTH | SOLAR SYSTEM | STARS & GALAXIES | TECHNOLOGY

Solar System Dynamics

BODIES | ORBITS | EPHEMERIDES | TOOLS | PHYSICAL DATA | DISCOVERY | FAQ | SITE MAP

JPL Small-Body Database Browser

Search: [help]

(2009 ST19)

Classification: Apollo [NEO, PHA] SPK-ID: 3467706

[Ephemeris | Orbit Diagram | Orbital Elements | Physical Parameters | Close-Approach Data]

[hide orbit diagram]

Orbit Diagram

Note: Make sure you have Java enabled on your browser to see the applet. This applet is provided as a 3D orbit visualization tool. The applet was implemented using 2-body methods, and hence should not be used for determining accurate long-term trajectories (over several years or decades) or planetary encounter circumstances. For accurate long-term ephemerides, please instead use our Horizons system.

Date | << | |< | || | >| | >> | Date Label | Planet Labels
1 Day | Distance | Object Label
Center: Sun | Zoom: | Save Image
Orbits: Default Orbits

Additional Notes: the orbits shown in the applet are color coded. The planets are white lines, and the asteroid/comet is a blue line. The bright white line indicates the portion of the orbit that is above the ecliptic plane, and the darker portion is below the ecliptic plane. Likewise for the asteroid/comet orbit, the light blue indicates the portion above the ecliptic plane, and the dark blue the portion below the ecliptic plane.

Orbit Viewer applet originally written and kindly provided by Osamu Ajiki (AstroArts), and further modified by Ron Baalke (JPL).

Orbital Elements at Epoch 2455200.5 (2010-Jan-04.0) TDB
Reference: JPL 38 (heliocentric ecliptic J2000)

Element	Value	Uncertainty (1-sigma)	Units
e	.5917738285475761	2.6121e-05	
a	2.354736370440841	0.00014649	AU
q	.9653472749993652	2.0359e-06	AU
i	6.68949536715098	0.00022085	deg
node	494054999613243	0.00026001	deg
peri	336.4035944112245	0.00039749	deg
M	34.52466009943055	0.0031759	deg
t_p	2455073.120567874447 (2009-Aug-28.62056788)	0.00019696	JED
period	1328.22767017573	0.12342	d
	3.64	0.0003379	yr
n	.2710378710544183	2.5185e-05	deg/d
Q	3.764125465502317	0.00023317	AU

[show covariance matrix]

Orbit Determination Parameters

# obs. used (total)	306
data-arc span	61 days
first obs. used	2009-09-16
last obs. used	2009-11-16
planetary ephem.	DE405
SB-pert. ephem.	SB405-CPV-2
quality code	5
fit RMS	.43638
data source	ORB
producer	Otto Matic
solution date	2009-Nov-17 00:50:14

Additional Information

Earth MOID = .0034141 AU
T_jup = 3.280

[Ephemeris | Orbit Diagram | Orbital Elements | Physical Parameters | Close-Approach Data]

Physical Parameter Table

Parameter	Symbol	Value	Units	Sigma	Reference	Notes
absolute magnitude	H	18.326	mag	.37653	38	autocmod 2.4a

[show close-approach data]

[Ephemeris | Orbit Diagram | Orbital Elements | Physical Parameters | Close-Approach Data]

ABOUT SSD | CREDITS/AWARDS | PRIVACY/COPYRIGHT | GLOSSARY | LINKS

FIRSTGOV Your First Click to the U.S. Government

2009-Nov-18 10:57 UT (server date/time)

Site Manager: Donald K. Yeomans
Webmaster: Alan B. Chamberlin

Now we give you some values of the S-O-T angle from July 18, 2009 to November 15, 2009:

Date__(UT)__HR:MN		S-O-T
2009-Jul-18	00:00	110.6386
2009-Jul-28	00:00	98.2399
2009-Aug-07	00:00	85.0185
2009-Aug-17	00:00	68.5472
2009-Aug-27	00:00	48.2178
2009-Sep-06	00:00	57.0115
2009-Sep-16	00:00	80.3307
2009-Sep-26	00:00	97.0950
2009-Oct-06	00:00	111.1146
2009-Oct-16	00:00	124.4429
2009-Oct-26	00:00	137.5462
2009-Nov-05	00:00	149.7362
2009-Nov-15	00:00	159.0660

Now you have to calculate to positions of 2009ST19 using Newton's formula:
1_ The angle on September 10.
2_ The angle on November 25.
The conditions of this activity are:

a) You cannot use computers
b) You cannot use calculators

This is because Newton had no any of them.

You have to deliver your report in DigitalDrop Box of our Blackboard's course; you must include an image of the calculations made by hand. Feel free to include any other data or image you think is important for your report.

Thanks…