EVOLUTION OF COLLEGE STUDENTS' EPISTEMOLOGICAL VIEWS OF MATHEMATICS IN A HISTORY-BASED CLASS

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ABSTRACT

Among all potential approaches for developing students' understanding of the nature of mathematics, history is seen as one of significant means for achieving the goal. The present study aimed to investigate in what way and to what extent Taiwanese college students' epistemological views of mathematics had evolved in a history-based general education courses. A course, titled "When Liu Meets Archimedes", was designed to help college students comprehend the cultural features of the development of mathematics through highlighting the similarities and differences of the process formation of mathematics between Eastern and Western worlds. Through comparing and contrasting collected data, students demonstrated different epistemological views of mathematics in several aspects: (1) they were able to recognize diverse features of mathematics; (2) thy were more likely to realize how mathematics had interacted with cultural aspects; (3) they tended to comprehend the different mathematical cultures between the East and West. However, it was also found that they were unable to realize the intuition-based inductive approaches employed by ancient Chinese mathematicians may play an indispensable and complimentary role with deduction in mathematical thinking and less likely to acknowledge how creativity and imagination were involved in the process of mathematical thinking.

1 Introduction

Schoenfeld (1985, 1992) claimed that belief systems are one's mathematical worldview shaping the way one does mathematics. Its influence could be realized by that mathematics-related beliefs are the subjective conceptions that students implicitly or explicitly hold to be true, that influence their mathematical learning and problem solving. An individual's epistemology is the conception regarding the nature of knowledge and the processes of knowing, which has been identified as a significant factor in students' academic performance (Hofer, 1999; Schoenfeld, 1992; Whitemire, 2004). For instance, Whitemire (2004) found that college students at higher stages of epistemological development usually demonstrated a better performance at handling conflicting situations and showed more sophisticated information-seeking behavior. Therefore, the research on students' epistemology in mathematics (their beliefs about the nature of knowing and knowledge of mathematics) has increasingly received greater attention.

Among all potential approaches for developing students' understanding of the nature of mathematics, history is seen as one of significant means for achieving the goal. The essence of mathematics lies in its intellectual adventure, beauty of abstract form, and application in physical world. Kitcher (1984) stressed that any account of the growth of mathematical knowledge should be referred to its historical development. Therefore, to understand the epistemological order of mathematics, one must understand its historical order. Furthermore, mathematics is conventionally understood as a fixed and absolute knowledge in which social and cultural aspects play minor role. In this manner, knowledge base of mathematics is mistakenly viewed as unquestionably solid, and all human struggles and intellectual adventures in the development of mathematics are hidden (Lakatos, 1976). Actually, despite their different fashions, both Eastern and Western mathematics experienced a long period of uncertainty. As Kline's (1980) claim, in the early 1800s, no branch of mathematics was logically secure.

For revealing humanistic aspects of mathematical knowledge, relevant scholars have called for integrating history into school mathematics curriculum to demonstrate the dynamic, potentially fallible, and socio-cultural nature of mathematical knowledge and thinking (Barbin, 1996; Furinghetti, 1997; Horng, 2000; Radford, 1997 ; Siu, 1995). By referring to relevant studies, I listed five major reasons for using history in school curriculum (Liu, 2003):

1. History can help increase motivation and helps develop a positive attitude toward learning.
2. Past obstacles in the development of mathematics can help explain what today's students find difficult.
3. Historical problems can help develop students' mathematical thinking.
4. History reveals humanistic facets of mathematical knowledge.
5. History gives teachers a guide for teaching.

Following above claims, several studies were conducted to explore the effect of integrating history into mathematics curriculum. Results showed that history not only helped college students improve their attitudes toward mathematics, but also developed their own understanding of the nature of mathematical knowledge and thinking (Liu, 2006; 2007; 2009). These experimental studies were all carried out in calculus classes and content were tied to conventional topics. It was found that students' understanding of cultural and societal aspects of mathematics remained meager. By taking this concern into account, the present study aimed to investigate in what way and to what extent Taiwanese college students' epistemological views of mathematics had evolved in a history-based general education courses.

2 The Content and Context of the Course

The course, titled "When Liu Meets Archimedes--Development of Eastern and Western Mathematics", is a history-based general education course focusing on how the development of mathematics was related to Eastern and Western cultures. The course objective was to help students comprehend the cultural features of the development of mathematics through highlighting the similarities and differences of the process formation of mathematics between Eastern and Western worlds. A brief introduction of main topics is as follows:

2.1 Mesopotamia civilization and Babylonian mathematics

The course began with Mesopotamia civilization and Babylonian mathematic including the sexagesimal (base-60) numeral system and secret codes on the recovered clay tablets. For instance, students were shown a picture of a surviving clay tablet and asked to decode the numerical codes (Figure 1). They were reminded to reveal the code by transferring base-60 numeral system to base-10 numeral system. Then they got,

“1; 24, 51, 10 = 1+24/60+51/3600+10/216000 = 1.414212963 $\cong\sqrt{2}$ =1.414213562” and

“30×1.414212963 = 42.42638889 = 42+25/60+35/3600 = 42; 25, 35”.

Above equalities imply that, in modern terms, the length of diagonal of a unit square is $\sqrt{2}$ and the length of diagonal of a square with side length of 30 is 42.42638889, an application of Pythagorean Theorem. Furthermore, higher Babylonian algebra such as solving system of equations was also introduced. Students were encouraged to compare the Babylonian style and modern fashion.

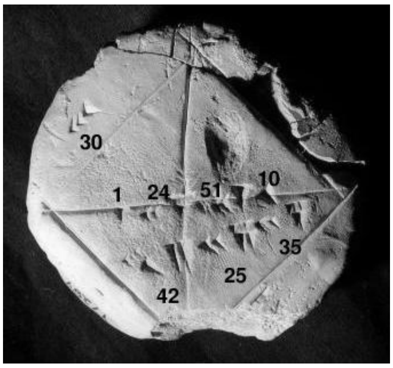

Figure 1 A Babylonian Clay Tablet

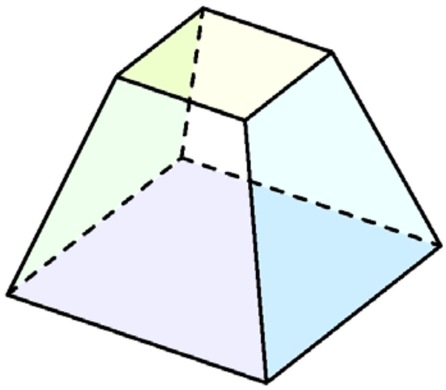
Figure 2 A truncated square pyramid

2.2 Egyptian mathematics

Hieroglyphic numeral system and unit fractions are two most particular features of Egyptian mathematics and a potential connection of Egyptian hieroglyph may be made to Sumerian script. Because Chinese characters are also hieroglyphic, students were attracted by this similar writing system but, on the other hand, were confused by the process of unit fractions. It is easy to factor 3/4 into 1/2+1/4 and 4/5 into 1/2+1/4+1/20. However, the factorization may not be unique. For instance, another representation of 4/5 is 1/3+1/4+1/5+1/60. The multiplicity of unit fraction representation brings an issue to the fore: How ancient Egyptian determined which is the simplest (in terms of mathematics) or best (in terms of practical needs) representation? Students were thus situated in a mathematical as well as cultural puzzle.

In addition, Moscow Mathematical Papyrus and Rhind Mathematical Papyrus not only reflect the achievement of ancient Egypt ca. 1800 B.C., but also reveal how mathematics was related to society at that time. A problem on Moscow Mathematical Papyrus receiving much attention is the volume of truncated square pyramid. For a truncated square pyramid whose top area is a square of length *a*, the bottom a square of length *b*, and the height *h*, the problem 14 actually asserted its volume V is:

$$V = \frac{1}{3}h(a^2 + ab + b^2)$$

Vetter (1933) had earlier indicated that how ancient Egyptian might reach the formula is a puzzle. The puzzle was then left to students as an assignment to work with, but yielded little response as a result.

2.3 Ancient Greece mathematics

Two particular features of ancient Greece mathematics were addressed in the class: the belief that mathematics is the key for understanding the universe and the relationship between philosophy and mathematics in general, geometry in particular. Both trends could be traced backed to Thales of Miletus for his rational thinking in explaining natural phenomena and deductive reasoning in establishing geometrical propositions, followed by Pythagorean thought of “*All are numbers*” and Platonic motto of “*Let no one ignorant of geometry enter here*”. Students were encouraged to think about why mathematics was viewed by ancient Greek thinkers as an essential discipline for developing philosophical literacy and how it might be related to culture at the time.

Euclid and Archimedes’ mathematical thoughts stand for two major trends of Western mathematics. The former established the logical foundation of mathematics in a deductive way, and the latter was complimentary with creative thinking and associated mathematics with physics. In addition to realizing how Euclid created a paradigm of modern mathematics on the basis of axiomatic systems, students were impressed by Archimedes’ sophisticated use of his imagination in making connections between different mathematical objects. For instance, in what ways he had an insight of the area of a circle is equal to that of a right-angled triangle where the sides including the right angle are respectively equal to the radius and circumference of the circle. Further, under what circumstance he was able to think of employing the principle of leverage to derive the volume of a sphere. Students were learned that Archimedes’ achievement not only signifies a landmark of ancient Greece mathematics, but also denotes a temporary rest of Western mathematics.

2.4 Scientific revolution and modern mathematics

Focus of the development of mathematics during 16 and 17 centuries was placed on the mathematical model of planetary motion and the quantification of physical motions. Using geometrical models to describe the pattern of planetary motion can be traced back to ancient Greek astronomer Hipparchus and Ptolemy of 2nd century creating deferent-epicycle model for explaining the regression of planets. This well-designed but incorrect model was on the basis of geocentric theory and a mathematical belief of harmony, dominating Western scientific thought over a millenarian, later replaced by Copernicus' simpler heliocentric model and Kepler's three laws of planetary motion. On the other hand, students were aware of how Galileo designed experiments to measure the movement of falling bodies, not only inspiring Newton's discovery of gravity but establishing a paradigm for later scientific investigations.

Furthermore, Euler's strategy for solving Seven-Bridge Problem of Königsberg (Figure 3) was treated as an early typical instance of the beginning of western abstract mathematics, followed by the birth of group theory and invention of non-Euclidean geometry. Students were also reminded to keep an eye on the issue that, regardless of its impractical nature, why abstract mathematics eventually shows its unreasonable effectiveness of mathematics in the natural science, as Wigner (1960) claimed.

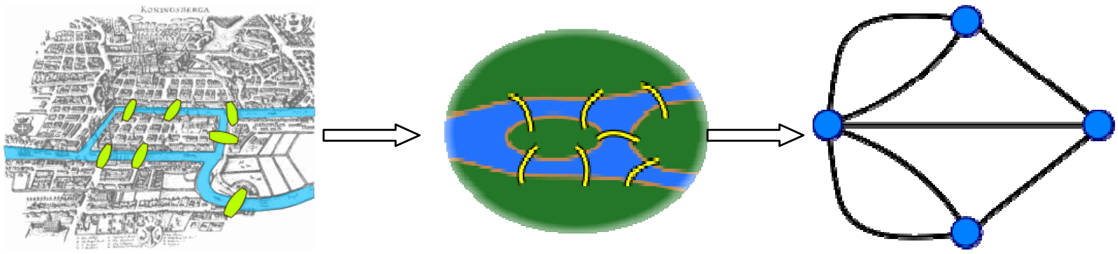

Figure 3 Euler's strategy for Seven-Bridge Problem of Königsberg

2.5 Ancient Chinese mathematics

Contrary to the deductive and abstract fashion of Western mathematics, the inductive and intuition-based style of ancient Chinese mathematics represents another paradigm. Students were told the mythical origin of Ho Tu (河圖), a diagram marked on the back of a dragon-horse rising from the Yellow River 5000 years ago, and Lo Shu (洛書), a figure scribed on the shell of a tortoise surfacing from Lo River 4000 years ago (Figure 4). Both Ho Tu and Lo Shu contain mathematical patterns (Lo Shu actually is a 3×3 magic square), but were then combined with Ba Gua (八卦) and Yin Yang (陰陽) to serve as fundamental elements of Feng Shui (風水) theory.

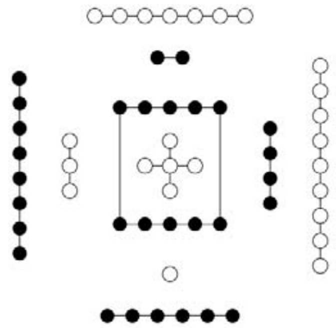

Figure 4 (a) Ho Tu

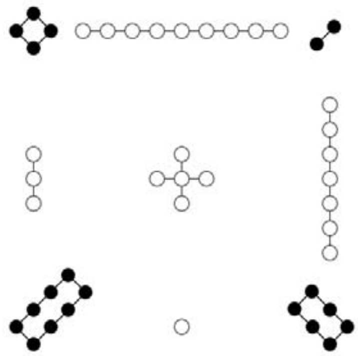

Figure 4(b) Lo Shu

Figure 4(c) Ba Gua & Yin Yang

Jiuzhang Suanshu (Nine Chapters on Mathematical Art , 九章算術) was the most important and representative mathematics book of ancient China. It distinguished itself from ancient Greece mathematical texts for its fashion of "solving without proof". A typical problem solved by the circular technique in *Jiuzhang Suanshu* can be seen as follows:

Question: Given a circle with area 300, what is the circumference?
Answer: 60
Solution: Multiply the given area by 12 and take the square root, it is the circumference.

It was not ready to know why the given area 300 should be multiplied by 12 followed by taking the square root. The puzzle was given as a problem-solving activity for students to figure it out. After a while of testing, they were soon realized that the answer was incorrect because the technique was based upon the experimental but wrong assumption that $\pi = 3$, which was widely accepted at that time.

The typical empirical nature of ancient Chinese mathematics was also manifested by Liu Hui's (劉徽) interpretation of *Jiuzhang Suanshu*. For example, he employed "Out-In Mutual Patching Technique" (出入相補) to "prove" the Pythagorean Theorem, which was called Gou-Gu (勾股) Theorem in Chinese. The "Out-In Mutual Patching Technique" not only reveals the empirical nature of ancient Chinese mathematics, but also lays the foundation of mathematical proof of ancient Chinese mathematics.

In addition, differing in Western algebraic system invented by Franciscus Vieta for solving equations during 16th century, Tian-Yuan Technique (天元術，Technique of the Celestial Unknown) is a Chinese system for resolving polynomial equations created in the 13th century. The technique is a positional system of rod numerals to represent equations and solving equations by means of oral pithy formulas and manipulation of rods. Students were aware of how ancient Chinese mathematics was built upon an empirical base.

2.6 A contrast between East and West

One of the major goals of the history-based course was to lead students to compare and contrast the diverse approaches between the East and West mathematics. Liu Hui and Archimedes are usually seen as respect representative figures among all. Liu Hui and Archimedes' ways for deriving the area of a circle and volume of a sphere were given as typical instances for students to generate their reflective thinking on various styles of mathematical thoughts. For instance, Liu Hui's circle-cutting method, rearranging infinite sectors of a circle to form a rectangular, heavily relied upon intuition. On the other hand, Archimedes equated the area of a circle to that of a right triangle and proved the result by *reductio ad adsurdum*, which was mostly deductive.

Furthermore, opposing to mathematical approach of Western astronomy, students also realized how ancient Chinese astronomers viewed the movement of celestial bodies in a mythical way. While observing the retrogression of Mars, instead of proposing potential theories, ancient Chinese astronomers saw the phenomenon as a misfortune for the emperor and altered observational information to please the emperor. The man-made data contributed to an acknowledgment of the pattern of celestial motion was lacking and a mathematical model was not available in ancient China, despite its more advanced observational instruments.

3 Results and Findings

For investigating in what way students responded to this history-based course and to what extent their views of mathematics evolved during the course, several instruments, including questionnaires, interviews, student journals and a web-based forum, were employed to achieve the goal. In the beginning of the course, all students' were invited to answer a questionnaire (Liu, 2010) and seven of them were randomly selected to participate in the follow-up interviews to document their initial views of the nature of knowing and knowledge of mathematics. At the end of the course, previous procedure was repeated to compare students' pre- and post-views. During the course, student journals and web-based forum collected students' responses to teaching material, important issues, and personal reflections upon the role of mathematics in diverse civilizations.

Through comparing and contrasting collected data, students demonstrated different epistemological views of mathematics in several aspects: (1) they were able to recognize diverse features of mathematics; (2) thy were more likely to realize how mathematics had interacted with cultural aspects; (3) they tended to comprehend the different mathematical cultures between the East and West.

3.1 Diverse features of mathematics

At the beginning of the semester, most students held a rigid view about mathematics in which the development of mathematical knowledge is mainly tied to daily needs and logical rules. After receiving this history-based course, though the previous two features remained fundamental in their minds, they were able to propose additional characteristics of mathematics. Chuan originally considered that, comparing to the rigid form of mathematics, art allows more space for personal expression. He whereas turned to hold that mathematics could involve personal style:

> *Interviewer*: In your [post-instruction] questionnaire, you said mathematics may contain several forms, such as numbers, graphs and computational equations. What do you mean by that?
> *Chuan*: In the first interview, I thought that mathematics is more formalized, like equations and formulas that we deal with. After taking this course, however, I felt mathematics could have a strong connection with art and science…Compare to mathematics, art is more abstract involving personal style. But mathematics may have personal style too, though it seems to be less visible than art. (Chuan, post-instruction interview)

For abstract mathematics, Chuan changed his initial instrumentalist views and expressed an understanding that though abstract mathematics appears to be irrelevant to daily life, it can be seen as a breakthrough of mathematicians themselves and its value could be justified in the future. Chuan's view was also endorsed by Yu's claim:

> [Abstract mathematics] seems meaningless to us, but that may be helpful to mathematicians. It could be a progress for himself/herself. We cannot assert that it is valueless at this moment just like some painters' works were not appreciated by contemporaries, yet turn out to be priceless. (Yu, post-instruction interview)

Furthermore, contrary to previous superficial understanding about the role of creativity and logic in mathematical thinking, they were able to recognize how creative thinking occurs. For instance, asked about how mathematicians think, Hui were impressed by Newton and Galilei's work and indicated that Newton and Galilei were firstly motivated by an idea and then made potential hypotheses, followed by logic-based verification. In her mind, proof and refutation constitute an alternating mode in mathematicians' thinking.

3.2 How mathematics had interacted with cultural aspects

Before entering this course, few of the students would associate the development of mathematics with culture. They acknowledged the difference existing among diverse countries, but usually referred the distinction to economic status. They were more likely to hold that highly developed countries can create more advanced mathematics than the lower developed regions do. As Hui indicated in the pre-instruction interview, "Europeans seemingly live with more leisure time to focus on philosophical aspects of mathematics". This misconnection of mathematics with economics was then replaced by the relationship between mathematics and culture, as Ling's claim on the post-instruction questionnaire:

> They [mathematics and social culture] are related. The focus of each society may vary. Some stress the exactness of computation to develop science, but others emphasize the use of diagram and ratio in architecture. This is why different mathematics was developed by different societies, making mathematics more colourful. Everything in our life is about mathematics. (Ling, post-instruction questionnaire)

Many attributed the different fashions of mathematics to life styles. Ko considered the birth of mathematics as being related to human's attempt to resolve living problems such as the overflow of Nile River in Egypt and Yellow River in China led ancient Egyptian and Chinese created diverse techniques to find the pattern. Hui also endorse the view that the content of mathematics was deeply influenced by social problem at the time and gave similar examples:

> *Interviewer*: Then, can you tell me why different societal cultures might develop different mathematics?
> *Hui*: In the West, like the overflow of Nile River, [Egyptians] needed to figure out a way to resolve the farming problem. In ancient China, laypersons were required to pay tax on the basis of the farmland area. They therefore had to create arithmetical rules. (Hui, post-instruction interview)

Hui's response was seemingly affected by the growth of geometry in ancient Egypt and arithmetic in ancient China. With the acknowledgment of the intimate relationship between mathematics and societal culture, students tended to comprehend the different mathematical cultures between the East and West.

3.3 Different Mathematical Cultures Between the East and West

By contrasting students' pre-instruction and post-instruction views, the most significant distinction was their awareness of different cultures between the East and West. In the beginning of the course, none of them recognized the mathematical culture may play a role in the development of mathematics. In the post-instruction interview, however, all interviewees demonstrated a moderate understanding of dissimilar fashion of the East and West mathematics. Horng indicated ancient Chinese mathematicians' interest in solving those problems with practical usage restricted the expansion of mathematics, and complicated rules of Tian-Yuan Technique hindered the progress of mathematical knowledge. He also noticed the different ways of interpretation of astronomical phenomena. For instance, ancient Chinese astronomers regarded the alignment of five planets as a bad sign of astrology, but Western scientists treated it as regular and made an effort to look for the pattern. Furthermore, another interviewee Chuan viewed dialectical nature as the major characteristics of Western mathematics:

Interviewer: Is there any significant characteristics for the development of mathematics? Did you learn anything from this course?
Chuan: Taking [ancient] China as an example, the development of mathematics in [ancient] China was quite different from that of the Western. I feel that the thoughts, forms, and styles in the Western were more personal and ancient China was more rigid without much breakthrough.
Interviewer: What factors caused the differences?
Chuan: In the Western culture, I feel that, any school of thought could be criticized by others if its doctrine or research was not persuasive. (Chuan, post-instruction interview)

Chuan's account was attributed to his conceptions that there were several competitive schools of scientific thoughts, such as Pythagorean *vs*. Eleatic schools and Platonism *vs*. Aristotelianism, in ancient Greece, whereas Confucianism played a dominated role in ancient China. This view was also endorsed by Yu's claim:

Interviewer: What factors made the difference between the development of Eastern and Western mathematics?
Yu: Confucianism did not place importance on mathematics. It emphasized the Six Arts…but I forget what they are.
Interviewer: They are rites, music, archery, charioteering, calligraphy, and mathematics.
Yu: Yes!
Interviewer: Mathematics was ranked at the last position.
Yu: Yes! The Western thought viewed mathematics as much more important, like quadrivium. They are……
Interviewer: Arithmetic, geometry, music, and astronomy.
Yu: Yes! (Yu, post-instruction interview)

Though students were unable to indicate the sophisticated relationship between ancient classic philosophy and the progress of mathematics, they appeared to realize the different characteristics of scientific thoughts may determine the paths and directions of later development.

4 Discussion and Conclusion

The present study aimed to investigate in what way and to what extent Taiwanese college students' epistemological views of mathematics had shifted in a history-based general education courses. Though several findings suggest the history-based course did direct students' attention to reflect upon humanistic and cultural issues of mathematics as aforementioned, it was found their conceptions remained at a superficial level at least on two aspects. First, they indicated the potential imitations of ancient Chinese culture in developing advanced abstract mathematics, but were unable to realize the intuition-based inductive approaches employed by ancient Chinese mathematicians may play an indispensable and complimentary role with deduction in mathematical thinking. They tended to judge the value of a mathematical approach in terms of the final outcome it brought about rather than the particular feature it involved. Second, they were less likely to acknowledge how creativity and imagination were involved in the process of mathematical thinking. Liu (2006) reported that, after experiencing a semester of historical approach problem-based calculus course, participating students' focus shifted from mathematics as a rigid product to mathematics as a dynamic and creative process, which were hardly seen in this study. Note that, the course in the present study was a 2-credit elective general education course, differing from that in Liu (2006) which was a 4-credit required course. The former stressed the cultural aspect whereas the latter emphasized problem-solving activities. Students in this history-based course might have an appropriate understanding of the extrinsic nature of mathematics (how mathematics interacts with societal culture), yet showed deficiencies in realizing the intrinsic nature of mathematics (how thinking is processed in mathematicians' minds). Could the two dimensions be well and equally taken into account in a single course? or Should we have to compromise? It might be a challenge for future curriculum development of this kind.

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