CLASSROOM EXPERIENCES WITH THE HISTORY OF MATHEMATICS

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ABSTRACT

Even though the history of mathematics is required in the lesson plan in Austria there are only rare possibilities to carry out historical projects or to discuss developments. By strenuous efforts as a grammar school teacher the author has realised some historical projects in the classroom, however. This paper will give a brief outline of the projects on the history of mathematics and focus on the problems and results. In doing so, classroom experiences with finished projects as tools for an approach to cultures and arts are introduced and an overview of the following topics is given:

- Adam Ries
- Austrian mathematicians
- Leonhard Euler
- Euclidian geometry
- Georg von Vega and logarithms
- School leaving examination - past, present, future

The paper will also present an excerpt of the pupils' work.

1 Introduction

The general part of the mathematics curriculum for secondary schools in Austria does call for historical considerations and developments in mathematics. The original text is shown below. The first part refers to the lower grades (ages 10 to 14), the second part concerns the higher grades (ages 15 to 18).

Historical considerations (lower grades):

Students are to gain insight into the development of mathematical terms and methods with the help of suitable topics. They should become acquainted with selected renowned historical mathematicians. Mathematics is to be presented as a dynamic science and its contribution to the development of western culture should be demonstrated. The significance of mathematics in the present should be stressed in the lessons.

Cultural-historical aspects (upper grades):

The essential role of mathematical findings and achievements in the development of European cultural and intellectual life makes mathematics an indispensable part of general education.[1]

These aspects are called for with regard to historical considerations: the development of terms and methods, famous mathematicians, mathematics as a dynamic science, and the importance of maths for the development of Western culture.

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[1] *„Historische Betrachtungen: (Unterstufe)*
Den Schülerinnen und Schülern ist an geeigneten Themen Einblick in die Entwicklung mathematischer Begriffe und Methoden zu geben. Sie sollen einige Persönlichkeiten der Mathematikgeschichte kennen lernen. Die Mathematik soll als dynamische Wissenschaft dargestellt und ihre Bedeutung bei der Entwicklung der abendländischen Kultur gezeigt werden. Die Bedeutung der Mathematik in der Gegenwart soll in den Unterricht einfließen.
Kulturell - historischer Aspekt: (Oberstufe)
Die maßgebliche Rolle mathematischer Erkenntnisse und Leistungen in der Entwicklung des europäischen Kultur- und Geisteslebens macht Mathematik zu einem unverzichtbaren Bestandteil der Allgemeinbildung.“ (http://www.bmukk.gv.at/medienpool/789/ahs14.pdf)

According to the Mathematics curriculum, teachers must deal with a great number of topics in a very short time, and they do feel the pressure of this lack of time. To make matters worse, several teaching hours have been cut during the last years. I remember my first years (30 years ago) as a teacher at the grammar school when I taught 34 maths lessons a week over the eight grades. Currently, mathematics is taught only in 28 lessons, a reduction of six valuable lessons. Despite this smaller numbers of lessons, there has been no reduction in the subject matter, however. In fact the contrary is the case. Many additional topics have been added, e. g. the use of CAS-systems, which is really up to date and increases the standard of maths-education, but it requires much more time for teaching. It is not surprising, therefore, that teachers resist the introduction of a program of history of mathematics. And most teachers may well ask: "Where do I find the time to teach history?" According to Fried: *"You do not need any extra time. Just give a historical problem directly related to the topic you are teaching, tell where it comes from; and send the students to read up its history. In this case the teacher is not forced to find extra time for extra material in an already overloaded program, and students are not forced to find extra time for extra homework".*[2]

But for teachers it is not so easy to set historical problems for their pupils because of the way teachers are trained. At university, students are not required to listen to lectures on the history of mathematics or physics, and unfortunately there is not even a chair for the history of science in Vienna.

Fortunately, however, many young colleagues are really interested in the history of science and they are keen, for example, to connect science with ancient languages or arts. And it is really remarkable that the history of mathematics is offered to the students at the Vienna's UT.

I did my doctoral thesis on "Austrian mathematicians …" 20 years ago and I am quite enthusiastic about teaching the history of mathematics. According to Fried I like setting historical problems directly related to the topic, but during my 30 years of teaching I have come to realise that this is very difficult indeed. In fact, I did it partly by way of story-telling (e.g. fractions, number systems, trigonometry), and my experience is that story-telling is very motivating for pupils to enjoy mathematics. Even if the story-telling becomes more and more complex, the primary need for understanding the simple, archetypical stories remains strong nevertheless.

2 Projects

2.1 Adam Ries

The first modest "Adam Ries project" was carried out with 10-year-olds (1^{st} grade) and it was directly related to the four basic arithmetical operations with natural numbers. The pupils learned "calculating on the lines". As an example, the addition of the numbers CCCVII, DCLXXVIII on the lines is shown below:

[2] Fried: Can Mathematics Education and History of Mathematics Coexist? In Science & Education 10 p. 394, 2001.

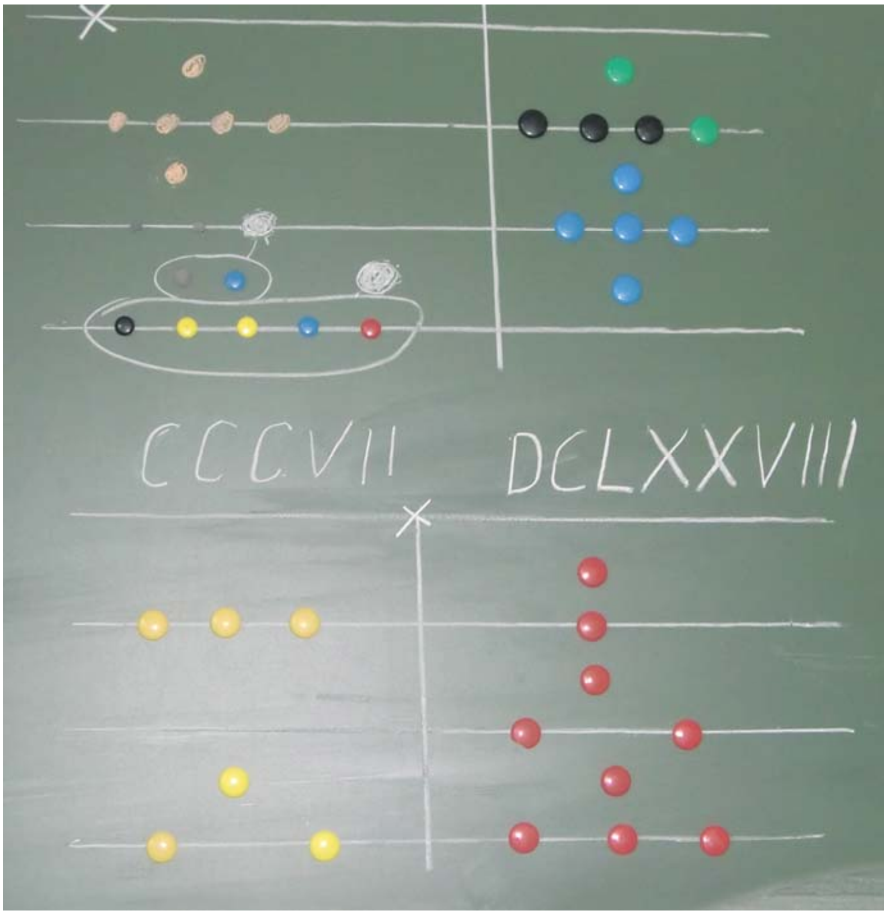


Figure 1: Calculating on the lines with beads (picture made by the author)

The lines show the place value from bottom to top. Beads in the interspace increase the value fivefold. As soon as there are five beads on the line, four are removed and one is moved into the interspace. Two beads in the interspace must be replaced by one bead on the next higher line.

Only 30 additional minutes were needed. As homework the pupils prepared a poster exhibition and short talks.

Another “Adam Ries project” was carried out with 13-year-olds (4^{th} graders). A power-point presentation about the life and the reckoner books was done by the pupils on their own.

This project required only two lessons. One lesson was used to introduce the pupils to the school library’s department of the history of maths, and the second lesson was dedicated to searching the internet for suitable sources for the preparation of a journey to the Adam Ries town of Annaberg-Buchholz in Sachsen-Anhalt, Germany. During the actual nine-hour bus ride a CD about Adam Ries’ life and work was shown, and during their stay in Annaberg-Buchholz the pupils visited Adam Ries reckoner-school. There they were graduated to “Historian reckoner”. The powerpoint-presentation was made, amongst other things, as a summary of the journey.

With this project mathematics was done outside of school, which helped to drum up interest in the history of mathematics. It was a great pleasure for me to see that the pupils were able to pass “Adam Ries” on to colleagues, friends and parents. Undoubtedly this project was a considerable contribution to the pupils’ general education and even contained elements of social work.

2.2 Austrian mathematicians

With regard to Austria's millennium celebrations in 1996 this project was interdisciplinary. It concerned a group of 13-year-olds (4th graders) in the subjects mathematics, arts and technical drawing. The objective of the project was to find Austrian mathematicians and artists and to make a connection to the corresponding periods. The pupils worked together in groups to present the lives and works of famous Austrians. With great detail, an exhibition of the lives and works of Hermann von Kärnten, Johannes von Gmunden, Christoph Rudolff, Paul Guldin, Georg von Vega, Johann Blank, Kurt Gödel, Olga Tausky Todd was presented and a paper was published.

Figure 2: Picture from the exhibition (made by the author) "Austrian mathematicians and artists"

The project required about two weeks (16 lessons) in all three subjects. As sources books from the school library were used exclusively and considerable work was done as homework. This project made the pupils recognise the creative nature of mathematical enquiry. They gained an insight into the various techniques of research as well as the analysis and synthesis of mathematical history, and they learned to grapple with the study of biographies and the history of the subject.

2.3 Leonhard Euler

This project was conducted with 16-year-olds (6th graders) and it required ten additional lessons. On three afternoons the pupils worked together in groups of up to five people, researching "Euler" in the library and on the Internet.

They started with looking for "primary resource sites" like: Euler's Introductio and they found a number of "mutating sites" – which provided links to and advice on topics such as number theory and others. In addition, "accumulative and list sites" were very helpful as well. As a result of the project the pupils published a paper and a power-point presentation. An excerpt of the content of the paper is shown below:

1707	Born on April 15in Basel. Father: Paulus Euler – pastor. Mother: Margaretha Brucker
1713	Began the education at the Latin grammar school in Basel.
1720	Attended lectures at the University of Basel.
1722	School leaving exams. Attended mathematics lectures by Johann I. Bernoulli.
1723	Gained his master degree. (In his thesis he compared Newton's and Descartes's systems of natural philosophy.)
1726	Publication of his first mathematic paper in Leipzig.
1727	Appointment to the Academy of Sciences of St. Petersburg.
1731	Full member of the Academy and Professor of Physics.
1733	Professor of Mathematics .
1734	Collaborator within the scope of the Russian general map. January 17: marriage with Katharina Gsell, daughter of a painter originating from St. Petersburg.
1735	Collaboration in the administration of the Geographical Department of the St- Petersburg Academy.
1738	After suffering a nearly fatal disease he went blind on his right eye.
1741	Acceptance of the invitation of Frederick the Great of Prussia to join the Berlin Academy.
1745	Euler's father died in Riehen.
1746	Director of the mathematical class of the Berlin Academy.
1749	First personal meeting with Frederick the Great.
1750	The last of his 13 children was born. Euler took up his mother to Berlin.
1755	Euler becomes an elect member of the Paris Academy of Sciences.
1762	Invitation of Catherine the Great to return to St. Petersburg.
1766	Weak eyesight on the left eye. Euler returned to the St. Petersburg Academy and spent the rest of his life in St. Petersburg.
1771	Blindness.
1773	Death of Euler's wife.
1776	Marriage with the half-sister of his first wife.
1783	Death on September 18 due to an apoplectic stroke.

Figure 3: translated pupil's paper excerpt of Euler's life and work

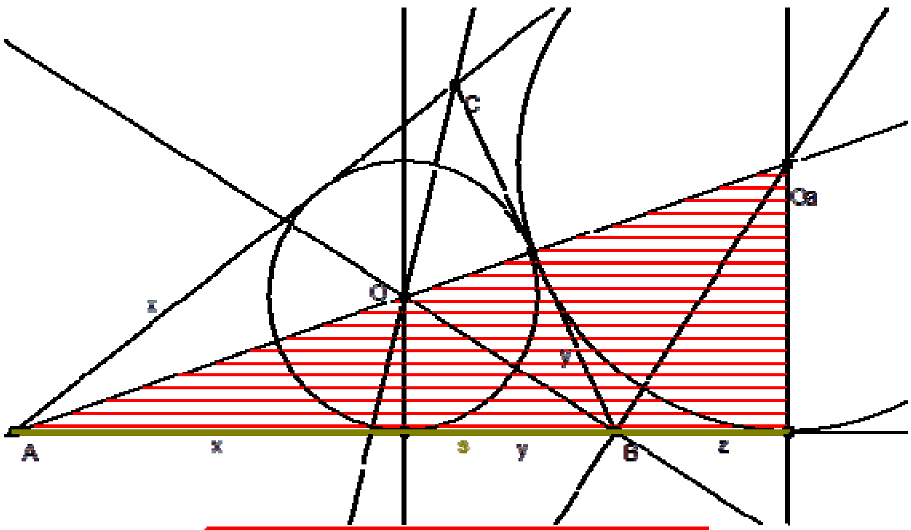
Furthermore Euler's Algebra, the seven bridges of Königsberg as well as Euler's autograph were studied among others.

The students learned that a line determined by any non-equilateral triangle which passes through the orthocenter, the circumcenter and the centroid is named after Euler. They proved this with congruent triangles by making use of the well known fact that the centroid divides the median line in a ratio of 2:1. Furthermore, they constructed the Feuerbach-circle and proved it.

I think that because of the project the pupils obtained a historically sensitive picture of Euler's work and an increased understanding and appreciation of constants, variables and functions. Furthermore, they received an impression of the number theory and of problems they had never heard before. The success of the "Euler-project" motivated the pupils to do a further project, and they chose the project described in the following paragraph.

2.4 Euclidian Geometry

As sources they used these well known works about geometry: Coxeter, Baptist as well as the newly published Scriba and an English translation of the "Elements". Elementary theorems were proved and the corresponding drawings were done with the "EUKLID" software. As an example, the geometric proof of Heron's formula for the area of a triangle is shown below:



$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

Figure 4: Proof of Heron's formula constructed with "EUKLID": similar triangles, intersect theorems, s=(a+b+c)/2, x=s-a, y=s-b, z=s-c.

Plane geometry according to Euclid was not entirely new to the pupils, but as I watched them during the project I noticed their reactions and compared them with what Davis said in "The Education of a Mathematician" *"I loved the theorem proving portion of the textbook, and I loved to work the theorem proving problems that were set… If there was any one thing that hooked me on mathematics it was the approach to geometry of Euclid and the Greeks."*

Firstly, I noticed that the students were not able to offer geometrical proof. They were not aware that they had been doing Euclidian geometry ever since they entered grammar school. By studying the postulates and definitions in the "Elements" they did not see the necessity of definitions at first, but after studying some of the propositions repeatedly they were convinced of the basics of Euclidian geometry. Subsequently they took much pleasure in geometry, and even “fell in love” with it. They particularly liked drawing with the “EUKLID” software. They appreciated the accuracy of the software – as opposed to drawing by hand – and were happy to see that the sides of a triangle are really tangents to the inscribed circle. By constructing the Feuerbach-circle their happiness increased when they observed that the corresponding nine points are really on the periphery of the circle.

The students’ work exceeded the requirements of the curriculum, and they got a better insight into geometry than during standard lessons. They learned about the Nagel-point, the Gergonne-point, the Simpson-line and other phenomena. The standard of the students’ work was almost on a par with that of a “pre-scientific” paper. With a few corrections and only slight extensions it could be used as a project work for the school leaving certificate.

As this project was carried out with students from different classes, they embraced the opportunity to speak with their colleagues about their approach to geometry.

The success of the project is underlined by the fact that two years later one of the students chose this topic as the foundation of his “pre-scientific paper”.

Probably the following two projects will be realised in the future.

2.5 Georg von Vega and logarithms

VEGA was born in Slovenia and he spent most of his life in Vienna. He is well known in Slovenia, as his numerous memorials show, but he is not so famous in Austria, although a military barracks and a street in Vienna have been named after him. I think it is really worth studying VEGA’s calculation of logarithms and his calculation of 130 decimal places of π.

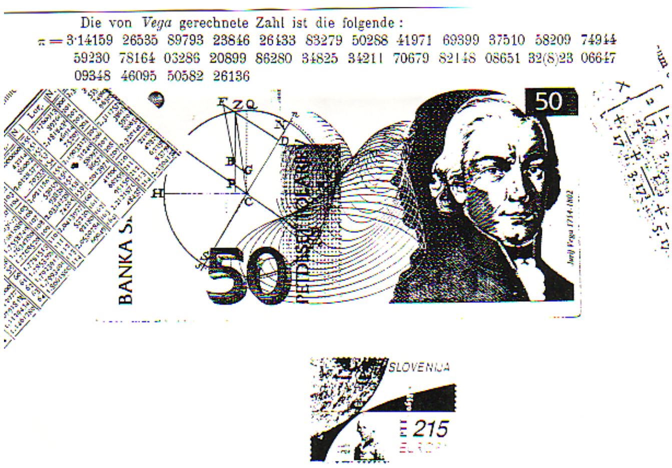


Figure 5: Slovenian Tolar bill, stamp dedicated to Vega, π, logarithmic table
(picture made by the author)

My idea is to hand out some original documents and books to 16-year-olds with the aim of making them work out a presentation of Vega's biography and especially about his logarithmic tables. Moreover, the pupils should be able to explain Vega’s method of the calculation of logarithms.

24. 3. 1754	- christening ceremony
1767 - 1775	- grammar school - Ljubljana
1775	- engineer
7. 4. 1780	- artillery
18. 11. 1780	- teachership of mathematics in the school for artillery
1782	- publication of his first book
1783	- publication: „Logarithmische, trigonometrische,und andere zum Gebrauche der Mathematik eingerichtete Tafeln und Formeln“
1. 4. 1784	- promotion
1. 3. 1787	- professor of mathematics ("Bombardierkorps")
1787	- marriage - Josepha Swoboda
1788	- war
1789	- siege of Belgrad - changed the mortar-batteries
1791	- son Heinrich was born
1793	- major
	- daughter Maria Theresia Regina was born
	- member of the academy of science in Erfurt
13. 10. 1793	- attacke - "Weißenburger Linien" - peace
10. 11. 1793	- conquest of Forts Louis - attest
1793	- publication: „Logarithmisch trigonometrisches Handbuch“
1794	- Stuttgart
	- writing member of the royal british society of science in Göttingen
1794	- publication: „Thesaurus“
1795	- Mannheim
11. 5. 1796	- knight of the Iron Cross
1796	- son Franz was born
7. 7. 1800	- wife Josepha died
22. 8. 1800	- baron
1802	- lieutenant colonel
26. 9. 1802	- Vega´s dead body was found in the Danube

Figure 6: Vega’s biography

LOGARITHMISCHE,
TRIGONOMETRISCHE,
und andere
zum Gebrauche der Mathematik
eingerichtete
TAFELN UND FORMELN.
Herausgegeben
von
GEORG VEGA,
Unterlieutenant und Lehrer der Mathematik
bey dem K.K. zweyten Feld-Artillerie-Regiment.
WIEN,
gedruckt bey Johann Thomas Edlen von Trattnern,
kaiserl. königl. Hofbuchdruckern und Buchhändlern.
1783.

Figure 7: Vega’s logarithmic tables
(scan made by the author)

2.6 The school leaving exam – its past, present and future

This is the second project I hope I will be able to carry out. Pupils will be asked to study tasks of the school leaving exams of Einstein and of BRG Wiener Neustadt (from previous exams). These tasks will be provided to the pupils.

*„**Prüfungsarbeit in Geometrie:** 19. September 1896 von 7:00 Uhr bis 11:00 Uhr*

***Erste Aufgabe:** In einem Dreieck mit Umkreisradius r = 10 verhalten sich die Höhen wie 2:3:4. Berechne die Winkel und eine Seite.*

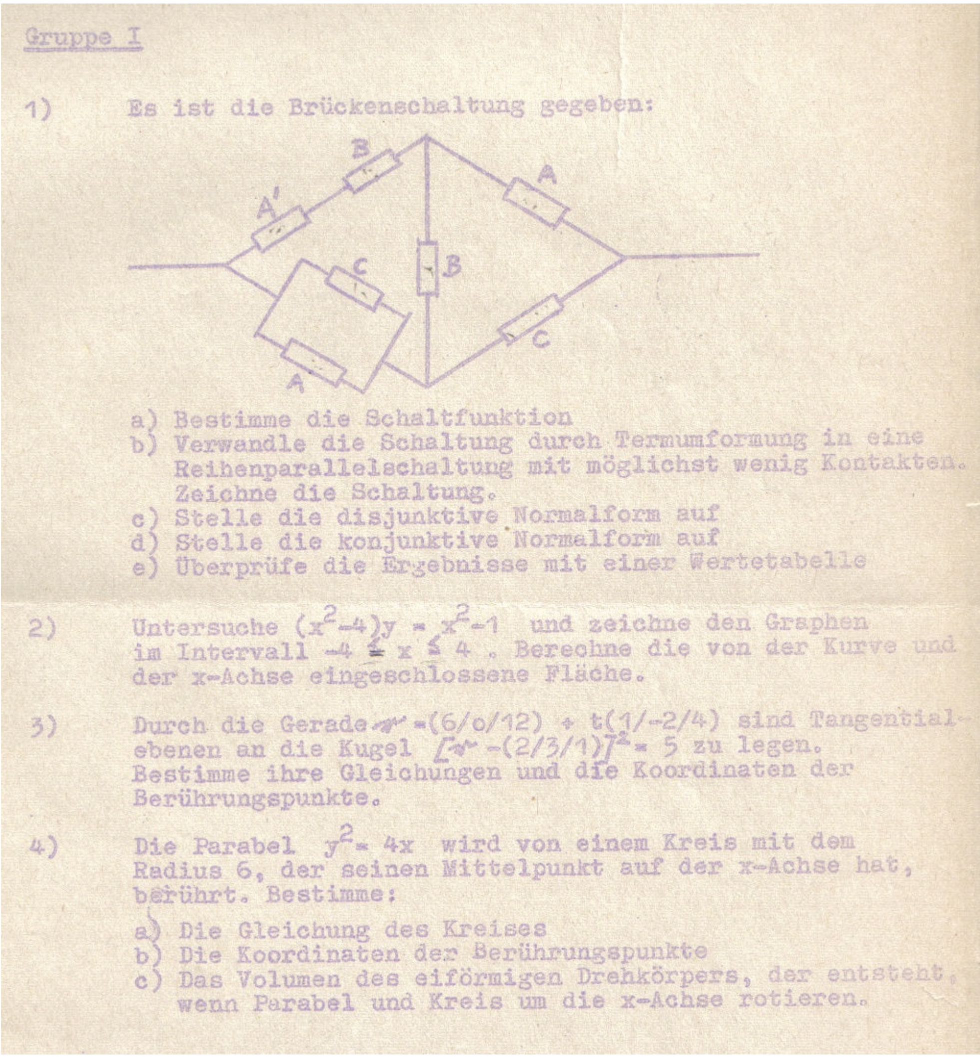
***Zweite Aufgabe:** Gegeben ist ein Kreis mit Radius r dessen Mittelpunkt im Ursprung O eines rechtwinkligen Koordinatensystems liegt. Man zeichne senkrecht zur x-Achse Sehnen in diesen Kreis. Die Kreise, für die diese Sehnen Durchmesser sind, berühren die Ellipse mit den Halbachsen* $r\sqrt{2}$ *und r, und erst wenn der Abstand p ihrer Mittelpunkte von O einen gewissen maximalen Wert überschreitet, hört die Berührung auf. Man beweise diesen Satz und bestimme den Maximalwert von p.*

***Prüfungsarbeit in Algebra:** 21. September 1896 von 9:30 Uhr bis 11:30 Uhr*

***Aufgabe:** Von einem Dreieck kennt man die Abstände l, m, n des Mittelpunktes des einbeschriebenen Kreises von den Ecken, man ermittle den Radius* ρ *des einbeschriebenen Kreises* $l = 1 \;\; m = \frac{1}{2} \;\; n = \frac{1}{3}$.“[3]

Figure 8: Einstein's school leaving exams

[3] Elemente der Mathematik Vol. 56 No. 2. , 2001, 45-54.



Gruppe I

1) Es ist die Brückenschaltung gegeben:

a) Bestimme die Schaltfunktion
b) Verwandle die Schaltung durch Termumformung in eine Reihenparallelschaltung mit möglichst wenig Kontakten. Zeichne die Schaltung.
c) Stelle die disjunktive Normalform auf
d) Stelle die konjunktive Normalform auf
e) Überprüfe die Ergebnisse mit einer Wertetabelle

2) Untersuche $(x^2-4)y = x^2-1$ und zeichne den Graphen im Intervall $-4 \leq x \leq 4$. Berechne die von der Kurve und der x-Achse eingeschlossene Fläche.

3) Durch die Gerade $\vec{x} = (6/0/12) + t(1/-2/4)$ sind Tangentialebenen an die Kugel $[\vec{x} - (2/3/1)]^2 = 5$ zu legen. Bestimme ihre Gleichungen und die Koordinaten der Berührungspunkte.

4) Die Parabel $y^2 = 4x$ wird von einem Kreis mit dem Radius 6, der seinen Mittelpunkt auf der x-Achse hat, berührt. Bestimme:

a) Die Gleichung des Kreises
b) Die Koordinaten der Berührungspunkte
c) Das Volumen des eiförmigen Drehkörpers, der entsteht, wenn Parabel und Kreis um die x-Achse rotieren.

Figure 9: BRG 1975 school leaving exams

It would make sense to do this project in the last class of grammar school but there is the problem of only three maths lessons a week, some of which will have to be cancelled due to tests and projects in other subjects. Experience has shown that the average number of weekly lessons is rather closer to two. So I am not very optimistic as far as this project is concerned, but I will keep it in mind. I am convinced that in view of the future centralised version of the leaving examination it is important that pupils "remember" the tasks set in the "old" version in order to see and judge the developments.

3 Conclusion

I think that with the help of the projects I have presented the pupils did not only learn about history from the different topics, but they were also confronted with different approaches to historical problems and they saw ways and methods that made more educational sense than modern ones.

The requirement in the national curriculum "to become acquainted with historical mathematicians" was no doubt fulfilled. By studying the biographies, the students learned more details about the spirit of the age and historical motivations. During the “Euclidian Geometry project” they came to understand the importance of the ancient Greek mathematicians in the development of European culture.

For triangle geometry, which is a part of the curriculum at nearly every level in Austrian schools, many relevant historical problems could be found. The "Adam Ries Project" showed that the history of mathematics is an essential part of general education. From Euler's "Introductio" they got a good idea of the genius of this very important mathematician. While working with the "Introductio", the students' understanding and appreciation of functions increased significantly. Studying these historical mathematical texts can be compared with studying the great works of prose and poetry in literature. There is no doubt that this method, which – according to Fried (2001, p.401) - is called "radical accommodation", helps to humanise mathematics.

Furthermore, the students learned to understand important developments from the Middle Ages to the present time as laid down by the "Austrian Mathematicians' Project". The students had never heard about the biographies and works of some of the Austrian mathematicians before, and so this project contributed considerably to their appreciation of their native land and its people.

Finally, this project meets the demands for general education in the national curriculum. Simultaneously, the students learned that human beings thought differently in the past and that, by implication, they will think differently in the future as well.

This glimpse at my projects, which do not claim to be perfect, just offers a very basic framework for teaching the history of mathematics. I think a desirable objective for the future would be to analyse the development of mathematics in the context of its function in culture and politics.

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