A HISTORICAL APPROACH OF THE FUNDAMENTAL CONCEPT OF MEASUREMENT

Measuring "Time", in Portuguese Textbooks for 5th and 6th grades

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ABSTRACT

In this article we present a part of a PhD degree project on: "The fundamental concept of Measure: epistemological and pedagogical aspects related with the first six years of schooling".
It aims at analyzing the historical approach of the fundamental concept of Measure as presented in Portuguese mathematics textbooks for the 2nd Cycle of Basic Education. In particular, it intends: 1) to explain why do we look at the mathematical concept of Measurement as not only an elementary concept but, and above all, a fundamental concept in mathematics; 2) to identify a historical approach, in textbooks, in as much as it contributes for promotion and enrichment of learning the concept; 3) to analyze activities (problems and exercises) suggested in textbooks and requiring Measurement. Our specification of the concept of Measurement will be, in this article, "Time".

1 Introduction

Facing the persistent bad results in mathematics, achieved by Portuguese pupils, in the past years - in fact, Portugal is listed, in some international comparative tests (PISA 2000, 2003, 2006) in the last places of the analysed countries - the Portuguese Ministry of Education decided to take concrete measures, namely, implementing a national *Action Plan for Mathematics* on very precise areas: Mathematical Teacher Training, Mathematics Curricula and Mathematics Textbooks.

In what respects to Mathematical Teacher Training, a national in-service teacher training programme was implemented: it is designed for elementary school teachers (grades 1 to 6) [1] and it intends to deepen teachers' mathematical, curricular and pedagogical knowledge. In this programme, the importance of the History of Mathematics is cleared assumed.

As for the Mathematics Curricula, new Programmes for Basic Education were designed. The Mathematics Program for the 2nd cycle (used until 2008/2009) suggests the use of activities with some historical data, in order to *"help students to understand the relationship between some mathematics historical facts and some problems that man has sought to solve"*. It is also an aim of this programme to develop "interest in Mathematics historical facts". On the other hand, the new Mathematics Program for Basic Education (MPBE, that just started to be implemented in 2009/2010) strengthens the importance of History of Mathematics to understand Mathematics as an element of human culture and to realize that Mathematics is an alive science in continuous evolution.

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[1] Portuguese Educational System has 3 Cycles of Basic Education and the Secondary Education. The 1st Cycle has 4 years of education; the 2nd one has 2 and the 3rd one has three years. The first and the second Cycles of Portuguese BE constitute the so-called Elementary Education, when children are 6 to 12 years old.

Concerning Mathematics textbooks, the Portuguese government established, in 2006, a system for textbooks accreditation: scientific commissions were formed having the responsibility to certificate textbooks (Despacho n.º 25190/2009); in addition, the assessment criteria have also been defined by law (Lei nº 47/2006 de 28 de Agosto). Therefore, all textbooks have to be certified by a group of specialists; then, in schools, teachers decide on the textbook that will be chosen to use in their schools for a period of six years.

Although the Mathematics Programme for the 2nd Cycle of Basic Education (particularly the new MPBE) suggests several didactical resources, the truth is that textbooks still play a significant role both inside and outside the classroom, and despite being, officially, recognised as "a tool for pupils' use", it is a fact that elementary teachers, as mediators of textbooks (Johnsen, 1993; Van Dormoien, 1986), tend to rely on them more than in any other curricular source (various Portuguese studies and reports reveal that 80% to 90% of teachers use it always or almost always). This situation in Portugal does not, therefore, differ from the situation in England, German or France (understood as, probably, the most influential educational systems in Europe). Haggarty and Pepin (2002) refer that teachers use textbooks regularly to prepare lessons or in classroom. In summary, mathematics textbooks seem to dominate what pupils learn, since they are mediators of the meaning and content of which is defined at the Mathematics Program (Chopin, 2004; Apple, 1993; Castell et al, 1989) and being the teachers' primary aid organizing lessons and structuring the subject matter it may well turns out to be a substitute of the Mathematics Program itself.

Even though the Mathematics Programmes emphasize the importance of History of Mathematics in Mathematics Teaching, following, at least theoretically, some research suggestions, its implementation (in textbooks and in classroom) seems to be far from being accomplished and/or appropriated.

We shall begin this article by presenting our definitions about what we call elementary and fundamental concepts, in mathematics, explaining our reasons for establishing such a distinction.

We will take the specific mathematical concept of Measure, and in particular, the concept of "Time". We will make a brief introduction to its history that will emphasize the fact of Measure emerging over time, its genesis being inherent to human activity and it being presented in many areas of mathematics as well as in several other areas of knowledge, both in the school context and in society.

Then we will present a set of criteria to analyze the most used mathematics textbooks for 5th and 6th grades, in Portugal, as well as the results of the analysis of activities (problems and exercises) suggested by textbooks and requiring measurement.

2 Elementary vs. fundamental concept

Concepts play a key role in the construction of mathematical knowledge. Fischbein (1993) defends that a mathematical concept expresses an idea, an ideal and a general representation of a class of objects with common characteristics. Furthermore, a definition of a mathematical concept, accurate and appropriate to the level of education is also the fundamental structure of mathematical knowledge (Wu, 2007).

In our opinion elementary and fundamental have different meanings, although a concept may be, at the same time, elementary and fundamental.

A mathematical concept taught and learnt at elementary level is an elementary concept.

For instance, the concepts of: number (excluding complex numbers); line; perimeter; area; mean; mode; angle; set or measure are, in our point of view, elementary concepts.

On the other hand, we say that a concept is fundamental if it emerges over time, its genesis is inherent to human activity and it is presented in many areas of mathematics as well as in several other areas of knowledge, both in the school context and in society, in general. For example, measure, count, limit or relation are fundamental concepts.

This meaning that we defend for fundamental concepts is clearly different from the one used by Ma (1999) when she states that Elementary Mathematics is fundamental mathematics, since it constitutes the foundation of this discipline and contains the rudiments of many important concepts in more advanced branches of this knowledge. In fact, in our opinion, there are fundamental concepts that are elementary but there are, certainly, others which are not. And, on the other hand, there are elementary concepts that are not fundamental.

For what was presented we consider Measure as an elementary concept, but above all, a fundamental one. Its history and relevance in mathematics teaching provide the evidence to make this assumption.

2.1. Why “Time”?

“Time”, as a specification of Measurement is, in our opinion, considered a fundamental concept.

It is deeply connected to life - we hardly understand human life without time. Measuring time is a human necessity well documented by sources from Antiquity and is a multiple dimension concept: philosophical, scientific, human and scholar.

History, both of Mathematics, Science, Technology and Humanity shows us that the evolution of time measurement, of instruments for measuring time, of new units to measure time as well as the need to "democratize" the right time to implement different rhythms of life and to measure time accurately, have different reasons including religious, political, social and scientific.

The philosophic facet of “Time” highlights that the perception of this concept is different for each person; it is a subjective concept which perception is significantly affected by both actions and state of mind.

“Time” as a scientific concept is related to sequencing and is a physical magnitude present in the definition of other concepts and magnitudes (like velocity, angle and meter). Nevertheless, in what respects to the learning and teaching of “Time” it seems that it is reduced to “*read the clock. However, this teaching involves only social-conventional knowledge, which is often arbitrary”*(Kamii, 2003).

We acknowledge that “Time” is one of the most problematic magnitudes to teach since it is intangible and continuous and is difficult for children to perceive:

- the idea of *time interval* and of *recorded time*;
- the relations between the units to measure time (and its relations with more complex concepts);
- the different representation about time (circle or timeline);
- the range of ways of telling the same time;
- the estimation of time and operation with time units.

But we also acknowledge that these difficulties may prevail through years and influence the understanding of other mathematical concepts extremely related with

Measure and “Time”.

On the other hand, “Time” is also an elementary concept because it is taught at elementary level. For that reason it is important that elementary teachers, mathematics programmes, textbooks and other students’ resources are aware of these relations between history – development of Measurement – and pedagogical aspects to teach Measurement, and in particularly, “Time”. They must be conscious, for example, that measuring time (as any other magnitude) is based on approximations, estimations and should not be treated, as an exact concept.

3 Mathematics Textbooks

A textbook of Elementary Mathematics, being a resource to support pupil’s process of learning, is also a vehicle for mathematics culture, fundamental concepts and methods of mathematical knowledge, which, having a consistent content sequence, promotes thinking and integrates that knowledge in various human activities.

Textbooks, playing a contextualized view of pedagogic discourse, reveal the interpretation that the author has of the discipline program, reflecting the importance that he/she attaches to the content, knowledge and techniques, regarding the objectives defined by the educational system. They are, therefore, individual projects to an educational system for all. Choppin (2004) argues that a textbook should be the privileged support of educational content, the depository of knowledge, techniques or skills that a social group believes needs to be communicated to new generations.

Moreover, textbooks, as bridges between curriculums and educational actors, reflect and legitimize the traditions of national culture. Apple (1993) asserts that the curriculum is never simply a neutral assemblage of knowledge, somehow appearing in the texts and classrooms of a nation. It is always part of a selective tradition, someone’s selection, some group’s vision of legitimate knowledge.

Thus the mathematics textbooks seem to dominate what students learn, as we previously acknowledged, since they are mediators of the meaning and content of which is defined at the mathematics program and being the teachers’ primary aid to organizing lessons and structuring the subject matter.

3.1 History of Mathematics and Mathematics Teaching

Several well-known authors refer the importance of using the History of Mathematics in Mathematics Teaching and point several reasons to use it. Jones (1989) states that the History of Mathematics, combined with mathematical knowledge allows us to teach the "whys" in Mathematics ("Chronological whys," "logical whys" and "teaching whys"), to understand and to motivate students by the nature and role of mathematics. Klerk (2004) argues that the context of science and technology in the teaching of mathematics allows to integrate the History of Mathematics and to emphasize the close relationship with nature and human culture. This vision of Mathematics, intrinsic to the evolution of science and humanity, motivates students to a particular subject and demonstrates the evolution of this science and shows the human side of Mathematics. It softens mathematics and motivates students in their study (Klerk, 2007). *With such historical approach one can guide students to the point of realizing that there is a relationship between mathematical matters … and the wide field of reality … Mathematics does not stand in isolation, but* *forms part of a much bigger reality relating to different real world contexts.* (Klerk, 2004)

In Struik' (1997) opinion, History of Mathematics helps us to appreciate our cultural heritage, understand the trends in Mathematics Teaching and helps to increase students interest. Using History of Mathematics in Mathematics Teaching facilitate to understand the learning difficulties, it improves instruction and creates a classroom climate of search and investigation; it humanizes Mathematics (Avital, l995). Swetz (1995) defends that knowing History of Mathematics helps us to understand the Mathematics evolution.

One the other hand, Sui (2000) argues that

perhaps it can be added that not only does the appropriate use of History of Mathematics help in teaching the subject, but that in this age of "mathematics for all", History of Mathematics is all the more important as an integral part of the subject to afford perspective and to present a fuller picture of what mathematics is to the public community.

3.2 Criteria

As we said at the beginning of this article, we are presenting a small part of a PhD degree project. It is our purpose to analyse the most used textbooks in Portuguese public schools, for Elementary Mathematics. To do so, we defined a set of criteria based on different suggestions provided by Sá et al (1999), Portuguese Ministry of Education (PME)[2] (2006), Rezat (2006) and Ponte (2007), defining main domains (table 1) to analyze mathematics textbooks. Other authors provided some ideas to analyze textbooks, in general: Chopin (1992) and Gérard and Roegiers (1998).

Sá et al	PME	Rezat	Ponte
Scientific	Scientific; linguistic	Content	Scientific; didactical
Pedagogical	Curriculum suitability	Linguistic	Text; illustrations
Technical	Programme suitability	Visual	Citizenship construction
	Pedagogical/didactical	Pedagogical	Editorial
	Technical	Situational	

Table 1. Suggested categories to analyze mathematics textbooks

These are necessary criteria to analyse any mathematics textbook, yet not sufficient to analyse Elementary Mathematics textbooks and the approach of fundamental concept of Measurement. Therefore our set of criteria is divided into three main groups: Scientific; Methodological (Presentation of the Measure, The learning of Measure development, Illustrations) and Measure Purpose, based on didactical, historical and scientific issues about Measure.

Two of the aims of this article are two criteria from Methodological aspects and from Measure Purpose:

1) to identify a historical approach, in textbooks, in as much as it contributes to promote and enrich the learning of the concept;

2) to analyze activities (problems and exercises) suggested by textbooks and requiring Measurement.

We will also consider, to analyze the historical approach of Measure in textbooks three of the four categories defined by Siu (2000):

A for anecdotes

[2] We assumed the PME's suggestions because they are defined by law, although those criteria refer to textbooks in general, and not specifically to mathematical textbooks.

An authentic and amusing or instructive anecdote, which carries a massage can be used as a catalyst. Eves, quoted by Siu (2000), argues that

these stories and anecdotes have proved very useful in the classroom - as little interest-rousing atoms, to add spice and a touch of entertainment, to introduce a human element, to inspire the student, to instill respect and admiration for the great creators, to yank back flagging interest to forge some links of cultural history, or to underline some concept or idea.

B for broad outline,

Useful to overview a topic or even of the whole course at the beginning, or to give a review at the end. Siu (2000) refers that, ideas in the history of the subject

can provide motivation and perspective so that students know what they are heading for or what they have covered, and how that relates to knowledge previously gained.

C for content

This category establishes a relation between culture, mathematics and its history, showing the evolution of concepts and promoting an instructive discussion about the content to be taught.

Although the study of the History of Mathematics has an intrinsic appeal of its own, its chief raison d'être is surely the illumination of mathematics itself ... to promote a more mature appreciation of [theories]. (Edwards, in Siu, 2000)

4 Analysis

We analysed the most used textbooks in the 2nd Cycle of Basic Education in Portugal during 2009/2010 - official information, provided by the Portuguese Ministry of Education:

Cycle	Grade	Title	Publisher
2nd	5th	Matemática 5	Porto Editora
	6th	Matemática 6	Porto Editora

Table 2. Analyzed textbooks (not certificated)

4.1 Historical approach

We can find “Historical Notes” in the beginning of all topics in the analyzed textbooks. However, they are not more than simple historical information - indicating events, facts and curiosities. None of these notes have any kind of connection, at least not referenced throughout the text, with the approach of the content on each topic. There are some interesting “Historical Notes”, yet not pedagogically or mathematically explored. We can find, at the beginning of the *Direct Proportionality* topic in the 6th grade textbook, an interesting “Historical Note” relating this concept with society, Portuguese Mathematics History (Pedro Nunes – whose work is extremely connected with Measurement and “Time”) and his studies about Euclides. But, once again, there is no more references to this note in the topic (fig. 1).

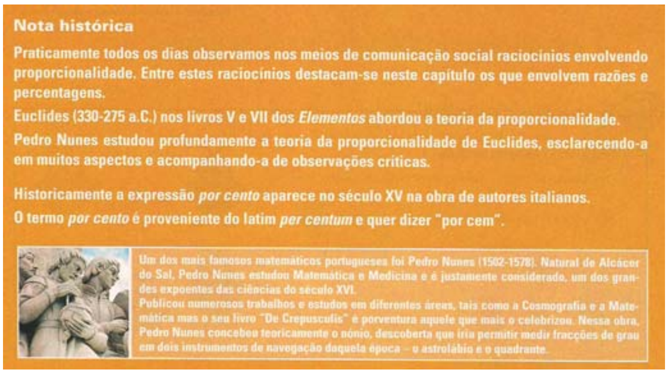

Figure 1. Historical note to introduce the *Direct Proportionality* topic
(in Neves, M.A. et al (2008). Matemática - 6º ano. Vol. II, p.38. Porto Editora)

Another example of a not explored historical approach, making no connection between its information and Measurement, in the 5th grade textbooks, is about numerical systems. Three different numerical systems (Egyptians, Babylonians and Chinese) are presented nevertheless with no further explanation or any purposed activity to find more about this theme, or to relate it with Measurement and "Time" (fig. 2).

Figure 2. Historical note to introduce the *Integers Numbers and decimal numbers* topic
(in Neves, M.A. et al (2008). Matemática - 5º ano. Vol. I, p.30. Porto Editora)

Other example of using History of Mathematics related with "Time" (fig. 3), is presented to introduce the chapter about *Angles*, in the 5th grade. This "Historical Note" induces student to a misconception: the division of a circle into 12 equal parts with 30º each (and later the idea of the accuracy of the length of a day and the concept of angle development – and the relation between hours and degrees) without contextualizing historically that conclusion.

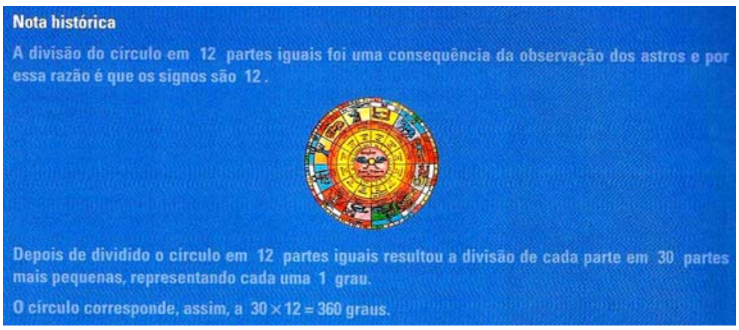

Figure 3. Historical note to introduce the Angle chapter
(in Neves, M.A. et al (2008). Matemática - 5º ano. Vol. III, p.3. Porto Editora)

On the one hand, when considering a circle, with 360 º, divided into 12 equal parts (in this specific case relating months and years; but also with hours in the case of the clock) may raise some confusion related to:

- distinction between the true "Time" and the "Time" defined by Man;
- understanding the concept of angle (the relation between degrees and minutes; for example: when considering or representing a 360 degrees full turn of the minute hand, then 360 degrees are equal to 60 minutes, and typically 90 degrees, represented by a quarter-hour, 15 minutes will be associated. This is the idea that textbooks may develop in students when representing angles in watches. Later, students will learn that one degree is 60 minutes. This may lead to a misconception).

On the other hand, the signs of Zodiac, presented in the fig. 3 *no longer correspond to the actual positions of constellations* (Holford-Strevens, 2005) and here are historical indications that relate the sexagesimal system and the division of the day into 60 parts in their sub-multiples

The arithmetic of ancient Babylon was based on the number 60; accordingly, astronomers (despite the existence of double hours) divided the natural day into 60 parts, these parts in turn into 60^{ths}, and so on. (Holford-Strevens, 2005).

One other example, relating the concept of angle with clockwise movement, is presented at the beginning of the *Angle* topic (fig. 4): pupils are asked to look at a picture and notice that the balance pointers describe an angle.

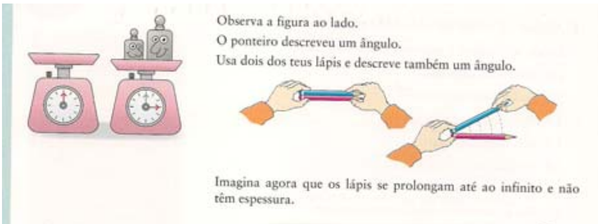

Figure 4. Introducing the concept of angle in the Angle chapter
(in Neves, M.A. et al (2008). Matemática - 5º ano. Vol. III, p.10. Porto Editora)

4.2 Activities

According to Mathematics Programme for 2nd Cycle of Basic Education, textbooks should contain numerical problems involving Measurement, and in particularly "Time", demanding students to select the appropriate instrument and/or unit. However, pupils are not required to do so in context and there are no worked examples (only one exercise, in the 5th grade, asking the difference between two children's ages in years and one more, in the 6th grade, asking "how many minutes are" – relating minutes, hours and fractions, see fig. 5). Once again it is implicit the connection between the position of the hands of a clock and the angle concept.

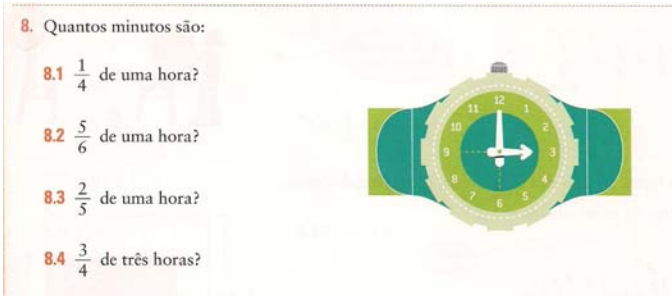

Figure 5. Exercise in Numbers and Calculus Topic
(in Neves, M.A. et al (2008). Matemática - 6º ano. Vol. II, p.11. Porto Editora).

Textbooks present some other exercises using Measure as a context (with different magnitudes) but in most of them the use of Measurement instruments is not necessary. Among a small number of exercises or activities requiring measurement we can find few soliciting the use of rulers, compasses or protractors (to construct geometric figures, to measure the length of objects and angles or the measures of an architectural project), but no other measurement instruments.

The 6th grade textbook introduces *Relative Numbers* with a linear representation of "Time", using negative numbers to represent years (fig. 6), without a reasonable explanation for that.

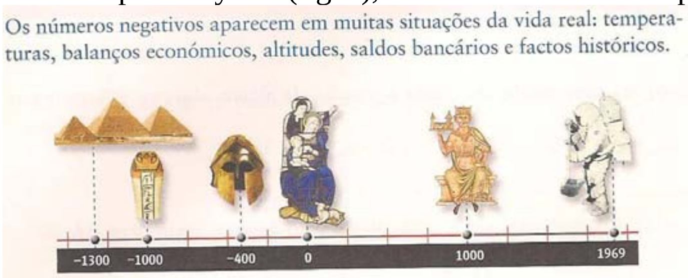

Figure 6. Example to represent negative numbers in real life
(in Neves, M.A. et al (2008). Matemática - 6º ano. Vol. III, p.74. Porto Editora)

In *Statistic* topic, textbooks for 5th and 6th grades, include several exercises with line charts for recording observations that evolve over "Time" (in hours or days of the week), as the Mathematics Programme recommends (Fig. 7). Though, those situations do not differ much from each other, in fact, activities/exercises (posing interpretation questions) are repetitive and there is no explanation of why using this type of graphic representation.

Figure 7. Line chart recording temperature over "Time" (hours)
(in Neves, M.A. et al (2008). Matemática - 5º ano. Vol. I, p.72. Porto Editora)

Some bar charts and pictograms are presented as well, recording observations evolving over days of the week. Still, some confusion, between discrete and continuous notions, may rise if to different types of graphic representation (discreet and continuous) of the same unit, day, are presented without explanation of this difference (Fig. 8).

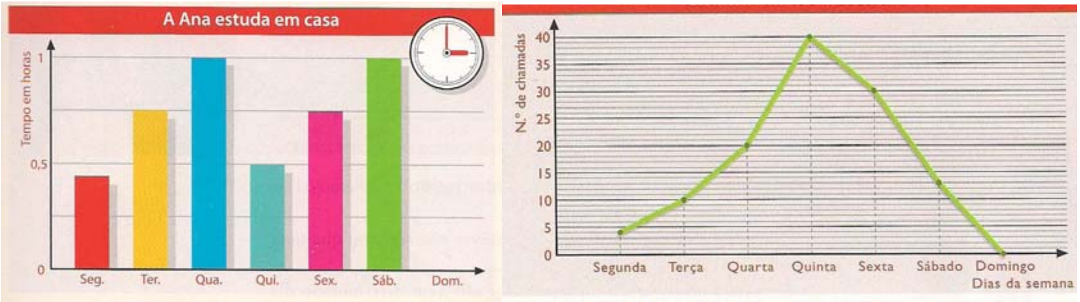

Figure 8. Bar chart recording “Time” of Ana’s study (in hours) at different days and line chart recording the number of phone calls over days
(in Neves, M.A. et al (2008). Matemática - 6º ano. Vol. I, pp. 74 and 73. Porto Editora)

5 Some conclusions

The analyzed textbooks usually focus on telling “Time” rather than on developing the concept, which is not defined and is addressed in two topics: *Numbers and Calculus* and *Statistics*. It is clearly assumed by the textbooks’ authors that “Time” is not considered a content, but an example, a context, a way to teach other contents (graphic representation, angles). Although this position may not be criticized, presenting certain situations related to the concepts of Measurement and “Time”, whether as historical, as a content or context, should have a purpose.

The most used textbooks in 5th and 6th grades present a superficial view of the History of Mathematics, using it merely as an informal “wrapping”, quite far from Siu’s *B-broad outline* purpose, and there are examples of bad historical information, such as the example presented at fig. 3, that presents misconceptions.

It seems clear that Measurement is a powerful concept to use History of Mathematics and history of other fields (social, science, technology) – as we showed at section 3.2. – enlightening the evolution of this concept and “Time”, promoting an instructive discussion and relating them with other concepts. However, the “Historical Notes” presented in the analyzed textbooks don’t involve children in the concept construction and don’t explore other dimensions of “Time”, such as its subjectivity or its scientific facet. We continue therefore to attend to social-conventional knowledge of the time.

The exercises and activities presented in textbooks present “Time” in a context situation. However they are repetitive and don’t require the selection of appropriate measuring instruments and its use (it brings up the clock as the sole instrument for measuring time). Estimation, in Measurement, is not considered in those exercises. It is clear that pupils are not involved in the construction of the concept. “Time” is given as the independent variable (fig. 7 and 8) and there are no textbooks that present activities where children could experience the transitivity and the unit iteration in the measurement of “Time”.

These conclusions emphasize the need to use and develop other sources to prepare lessons and work in class. Pupils should be involved in activities with History of Mathematics related to the concept of Measure and, in particular, “Time”, since elementary school. Also textbooks (and teachers) should present problems and exercises requiring the measurement of time from different perspectives and examples involving children in the construction of the concept and of the mathematical knowledge.

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