THE HISTORY OF MATHEMATHICS IN THE CLASSROOM

Some Activities

Hélder PINTO
Faculdade de Ciências da Universidade de Lisboa, Lisbon, Portugal
hbmpinto1981@gmail.com

ABSTRACT

The importance of History of Mathematics in Portuguese school curricula is increasing but there is some difficulty to introduce this topic in the classroom. The material adapted to the level of the undergraduate students is still short and by that it is necessary the production of more useful material that teachers could easily apply to the classroom. This workshop presented various examples of contents of the History of Mathematics adapted to the classroom such as: Euclid's Elements; the Shadow Instrument of the Portuguese mathematician Pedro Nunes; the numeral system of ancient Egypt; the Napier's rods and the Genaille-Lucas rulers.

The importance of History of Mathematics in school curricula is increasing. For example, in Portugal, in the Mathematics program, the History of Mathematics appears as a crosscutting theme throughout the secondary school teaching and that should come along on several and different themes. The idea is not to present the History of Mathematics in a finite and condensate set of classes, but that this subject will be a good support and motivation for the study of other mathematical topics.

However, despite the growing importance of the History of Mathematics in school curricula, there is little literature on this theme applied to the context of the classroom, which difficult the work of teachers in its implementation. In the references at the end of this text, some works containing material related to the History of Mathematics suited to use in the classroom are displayed. One of the most important works in this area is the book "Learning Activities from the History of Mathematics" by Frank Swetz (1994). In this book, the author presents several topics of the History of Mathematics with its direct application to the school context through worksheets with questions, accompanied by texts - such as, for example, short biographies - accessible and appropriate to the skills and knowledge of younger students.

"*We are always looking for good problems to strengthen and broaden our student's knowledge of mathematics as well as to refine concepts taught in the class room. The history of mathematics supplies thousands of useful and interesting problems, problems that are mathematically and pedagogically sound and which, by their historical nature, possess an additional intellectual appeal for students.*"[Swetz, 1994, p.2]

This workshop tried to follow the same idea and presented some proposals relating to the History of Mathematics in the school context (in the classroom and beyond, such for example in an extra-curricular Math Club). We saw how to adapt some topics of mathematics at the school context using:

- the Euclid's Elements (examples of how to use the computer and some interactive content in classroom was presented in the approach to this book);
- the Shadow Instrument of the Portuguese mathematician Pedro Nunes (it can be easily constructed with paper and cardboard in the classroom and be used to measure the altitude of the Sun);

- the numeral system of ancient Egypt (system "visually more attractive" allowing the students to achieve a better understanding of the current numbering system by comparison and contrast);
- The Napier's rods and the Genaille-Lucas rulers that were used for the multiplication of natural numbers (they can also be easily constructed with paper and scissors in the classroom).

In this workshop, worksheets about these last three topics that could be used in the classroom were presented. These worksheets, with some adjustments, are reproduced in the following pages.

The first worksheet actually used in the context of the classroom was about Pedro Nunes and his Shadow Instrument in a 9th Grade class (Portugal, students with 14/15 years), which was very well received by all students. Some students have even constructed some Shadow Instruments of wood in a manual labor discipline, i.e., outside of math class. Also, all students had prepared a written report about the practical activity in order to consolidate the knowledge obtained from this activity. It was a lesson included in the theme of Trigonometry and served to show a possible practical application of this theme. Though simple, this example can show the actual importance of trigonometry in real life; is harder for a student to "see" the mathematics that exists, for example, in a car or in a computer, while on these historical examples, often using more basic math, is easier to see where the mathematics was used in the past to solve practical problems and to understand the role of mathematics as an important factor of the development and progress. From these assumptions, it is easy to believe that, with another level of complexity and sophistication, there are a lot of math in our everyday life.

The Napier rods and the Genaille-Lucas rulers are good examples to show the development of mathematics – though simple and rudimentary, it allows showing that technology, as the mathematics used on it, is/was a science under construction and in a gradual growth. The Napier rods were an instrument built to facilitate the calculation of multiplications. Although they were very helpful, these rods presented some flaws and defects (they required some calculations beyond what was observed directly on the instrument), which "caused" the need to create a better and more efficient instrument. The Genaille-Lucas rulers then arise to solve the flaws previously detected but they still presented some limitations too (not allowing direct multiplication by multipliers with more than one digit) and, because of that, later, these rulers also ceased to be used.

This kind of process in which certain instruments are gradually being replaced by others progressively better and more effective is a good example of how the technology and, in particular, the science associated with it, works. These rods are a good example of how math and science evolves, not only in a theoretical and abstract mode (that many times keeps so many students out from this discipline), but also in a practical way in order to solve concrete problems of real life. Also serves to show that the current calculators and computers were the result of an arduous and demanding intellectual process until reaching the current level of complexity and refinement. Realizing that current technology (such as mathematics itself), although very useful and powerful, is not final and is still evolving could be an important step to humanizing mathematics in the students minds. This multiplication worksheet was used with students aged between 12 and 16 years (in Portugal), under a Mathematics Club, having a very good acceptance.

The numerical system used in ancient Egypt, as in the previous case, can be very advantageous in the classroom to show that mathematics is not a static science. Moreover, using this and other numeric systems of ancient peoples, it is possible to make a parallel with the current decimal numerical system, highlighting their good qualities and the characteristics that made it the dominant system in most of the world. Many times the usual decimal system is presented as a given statement and the proper emphasis to the characteristics that make it very useful is not given. If there are other systems to compare it, many times more difficult to use and understand, it might be easier to motivate students to appreciate the actual number system used in their mathematics education. It seems to me that everything that allows renewing the basic tools of mathematics can be a good activity for several years of schooling (though, I have not yet had the opportunity to use it in the context of the classroom). This activity allows facing, once more, the math as a science in evolution.

In the Euclid’s Elements, examples of how to use the computer to approach this book were shown. In particular, the website (in Portuguese)

http://wwmat.mat.fc.ul.pt/~jnsilva/elementos_livro_1/mat/elementos/index.htm

was shown, where interactive applets of all the propositions of the first book of the Elements are presented. These pages are different from the usual interactive approach to this book because all proofs are constructed from the beginning, i.e., the picture near the proof is done step by step following the explanation. This makes it easier for students to understand the statements which are being proofed in each step, what is being shown and what are the previous results that are being used. There exists already many interactive websites that use the Book I of Euclid's Elements, but all, as far as I know, present the figures that illustrate the proofs in its final form, making it extremely difficult to distinguish, for example, what are the initial data, what are the intermediate conclusions used in the proof and what is the result actually being proved.

This book of Euclid, given its structure and propositional logic, might be a good example to show how mathematical reasoning works. One of the cornerstones of mathematics, without question, is its cumulative nature and the accuracy of their statements and arguments. Showing how, starting from a few given assumptions (five postulates and five common notions), it is possible to construct a set of mathematical propositions of some scale, maybe leading the students to a better understanding of the power of the mathematical "building" which has been assembled over several generations. Furthermore, the brilliant way how the propositions and their proofs are displayed and linked to each other (usually, to proof a certain result, the exactly precedent proposition is used), is a great example of what is a text of mathematics and it is possible to find many similarities with the current math texts.

REFERENCES

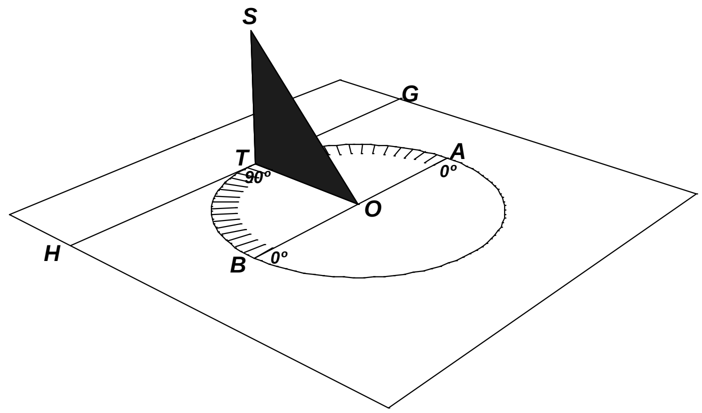
– Katz, V. (ed.), 2000, *Using History to Teach Mathematics, an International Perspective*, The Mathematical Association of America.
– Katz, V., Michalowicz, K. D. (eds.), 2004, *Historical modules for the teaching and learning of mathematics*, Washington DC: The Mathematical Association of America (available in electronic form only).
– Laubenbacher, R., Pengelley, D., 1999, *Mathematical Expeditions: Chronicles by the Explorers*, New York: Springer-Verlag (revised second printing, 2000).
– Laubenbacher, R., Pengelley, D., last revised Jan. 24, 2010, *Teaching with original historical sources in mathematics*, a resource website, http://www.math.nmsu.edu/~history/.
– NCTM, 1969, *Historical topics for the mathematics classroom*¸ Reston VA: National Council of Teachers of Mathematics (31st NCTM Yearbook, reprinted 1989).
– Swetz, F., 1994, *Learning Activities from the History of Mathematics*, Portland: J. Weston Walch Publisher.
– Swetz, F., Fauvel, J., Bekken, O., Johansson, B., Katz, V. (eds.), 1995, *Learn from the Masters*, The Mathematical Association of America.

1 The Shadow Instrument (Pedro Nunes)

Contents: congruent triangles and trigonometry.
Material: cardboard, scissors and glue to construct the shadow instrument; measure tape for the practical activity presented at the end of this worksheet.

The Portuguese cosmographer and mathematician Pedro Nunes was born in 1502 in Alcácer do Sal and died in 1578 in the city of Coimbra. Among his scientific works Nunes presented several measuring instruments for use in celestial navigation. One of these was the Shadow Instrument that measures the angle that the rays of the sun make with the horizontal plan (“the altitude of the Sun”). This was a simple tool, which looked like a sundial, but it had a very ingenious innovation that gave directly the "height of the Sun" through the use of the shadow projected by itself. The measurement of the “height of the Sun” was very important for the ancient Portuguese mariners navigation but can also be used, with a little knowledge of trigonometry, to determine the height of objects (buildings, trees, street lamps, ...) when this is an inaccessible task by direct measurement – just using the measurement of the length of the shadows of those objects. After studying and understanding this instrument, you will make an activity where you practice this second use.

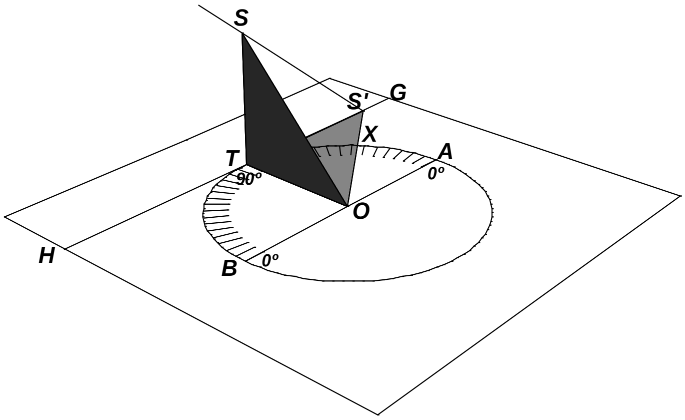
The Shadow Instrument was a plate, usually square, with a circle drawn and a isosceles right triangle set perpendicularly as shown in the next figure (the length of the leg of the triangle was equal to the radius of the circle; the tangent to the circle in T was also marked on the plate).



On the circle, it was still marked the diameter parallel to the tangent GH and the two quarters of the circle closest to the tangent graded from 0º to 90º from that diameter till the point of tangency T.

With this gradation, the angle that the sun makes with the horizontal plan is directly obtained. The Shadow Instrument is used as follows:

- Set the plate horizontally.
- Rotate the instrument until the shadow of the cathetus [*ST*] matches the line tangent to the circle; label *S’* the shadow of the point *S*.
- The intersection of the shadow that the hypotenuse of the triangle makes on the plate with the arc of the circumference between points *A* and *T* indicates the value of the angle that the sun makes with the horizontal plan; label that point as *X*.



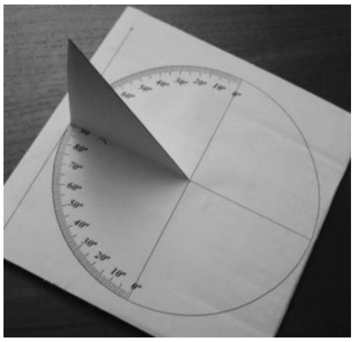
In order to understand and validate the functioning of this instrument, answer the following questions:

1. Show that the triangles [*S'TS*] and [*S'TO*] are congruent and that $\angle SS'T = \angle OS'T$.
2. Show that $\angle OS'T = \angle AOX$.
3. Show that the plan *SS'T* is perpendicular to the horizontal plan.
4. Show that the angle that the sun rays make with the horizontal plan is equal to $\angle AOX$, i.e., equal to the angle marked in the circle by the shadow of the hypotenuse of the triangle.

Practical activity:

Build a Shadow Instrument using the sheet provide at the end of this activity and by following these instructions:

1. Cut the two figures by the segments indicated by the "scissors";
2. Fold the bottom figure by the marks in order to construct an isosceles triangle;
3. Place the isosceles triangle by the opening in the circle;
4. Paste the last construction into a cardboard.



Determine the height of some objects (buildings, trees, street lamps,…) within the school using this instrument to measure the "altitude of the Sun" and a measure tape to evaluate the lengths of their shadows. Register your observations and calculations.

REFERENCES

– Crato, N., 2003, "O Instrumento de Sombras", in *Centro Virtual Camões – Ciência em Portugal, Personagens e Episódios* (http://cvc.instituto-camoes.pt/ciencia/e32.html)
– Estrada, F., Sá, C., Queiró, J., Silva, M., Costa, M., 2000, *História da Matemática*, Lisboa: Universidade Aberta.
– *Gazeta da Matemática n.º 143 – Quinto Centenário do nascimento de Pedro Nunes,* 2002, Lisboa: Sociedade Portuguesa de Matemática.
– Pinto, H., 2009, *História da Matemática na Sala de Aula*, Lisboa: Associação Ludus.

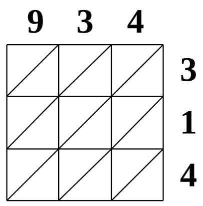
2 The Multiplication (The Napier's rods and the Genaille-Lucas rulers)

Contents: lattice method of multiplication, the Napier's rods and the Genaille-Lucas rulers.
Material: scissors, the Napier's rods and the Genaille-Lucas rulers.

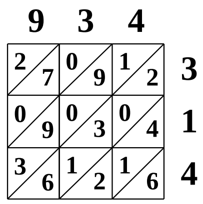
2.1 Lattice Method of Multiplication

At the beginning of the Renaissance, the emergence of various techniques such as, for example, the method of multiplication in lattice led to an increase of ease and speed with which the numerical calculations were made. Let's see, then, how the method of lattice multiplication works.

Assume that you want to do the product 934 × 314. Start to build a table as shown in the figure below.



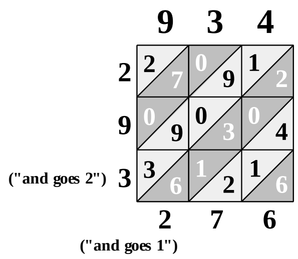
As 4 × 3 = 12, fill the two adjacent triangles that form the square of the column corresponding to the 4 and the row corresponding to the 3 with 1 and 2, respectively. In a similar way, fill up the rest until obtain the table that is presented below



Next, consider the six "diagonals" on the table. So we have that:

- the units digit of the product required is equal to the digit of "first diagonal": **6**;
- the tens digit is equal to the sum of the digits of the "second diagonal": 4 + 1 + 2 = **7**;
- the hundreds digit is equal to the units digit of the sum of the digits of the "third diagonal", while the tens digit of that sum goes to the next "diagonal": 2 + 0 + 3 + 1 + 6 = 1**2**;
- the thousands digit is equal to the units digit of the sum of the digits of the "fourth diagonal" (don't forget to add the digit that comes from the previous "diagonal"), while the tens digit of that sum goes to the next "diagonal": 1 + 1 + 9 + 0 + 9 + 3 = 2**3**;
- the ten thousands digit is equal to the digit of the sum of the digits of the "fifth diagonal" (don't forget, once more, to add the digit that come from the previous diagonal): 2 + 0 + 7 + 0 = **9**;
- the hundred thousands digit is equal to the digit of the sum of the digits of the "sixth diagonal" (note that nothing came from the previous "diagonal"): 2 = **2**.

Therefore, we have that 934 × 314 = 293276.

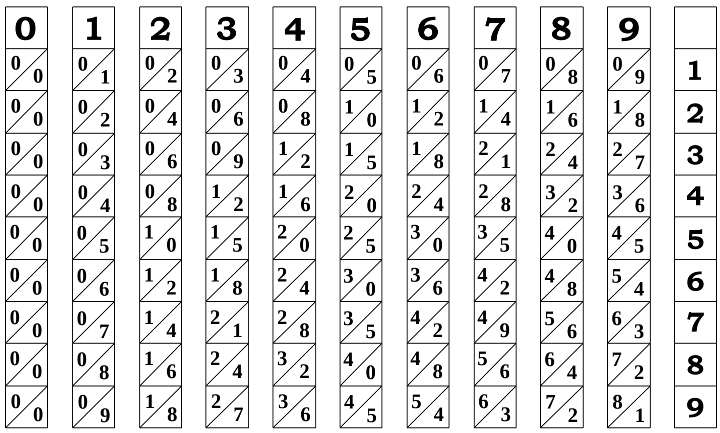


Exercise 1: Determine the numeric value, using the lattice method of multiplication described above, of the following expressions.

1.1 723×149; **1.2** 481×58; **1.3** 451^2.

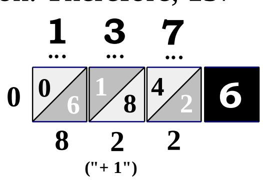
2.2 The Napier´s rods

John Napier (1550 - 1617), famous Scottish mathematician who devoted much of his life researching processes which allow the easiness of doing numerical calculations, noticed that the entries in columns used for lattice multiplication are always filled with multiples of the number that is on top of that column. From this finding, Napier constructed a collection of sticks with these ordered sets of multiples, thereby obtaining the desired lines in the lattice multiplication in a more fast and effective way. Thus, there is a practical tool, usually made of wood or bone, which facilitates the way of performing multiplications which often is designated by Napier's rods.



Let's see how this tool created by Napier works showing how to do, for example, the multiplication 137 × 6.

- Select the rods corresponding to the 1, 3 and 7 and put them side by side so that in the top appears the number 137. On the right of these rods should be placed the stick numbered from 1 to 9.
- To find the product you should "read" the line corresponding to the digit 6.
- Then add up the numbers for each "diagonal", using the same technique used in the lattice method of multiplication. Therefore, 137 × 6 = 0822.



To multiply two numbers with more than one digit, you have to make several partial products and then sum them to obtain the desired product. Take, for example, the multiplication 354 × 628. This product can be seen as 354 × (600 + 20 + 8), i.e., as **354 × 6** × 100 + **354 × 2** × 10 + **354 × 8**.

Therefore, the rods corresponding to 3, 5 and 4 should be placed side by side so that the number 354 appears on the top (again, on the right of these rods put the stick numbered from 1 to 9). To find the product you should "read" the lines corresponding to the digits 6, 2 and 8.

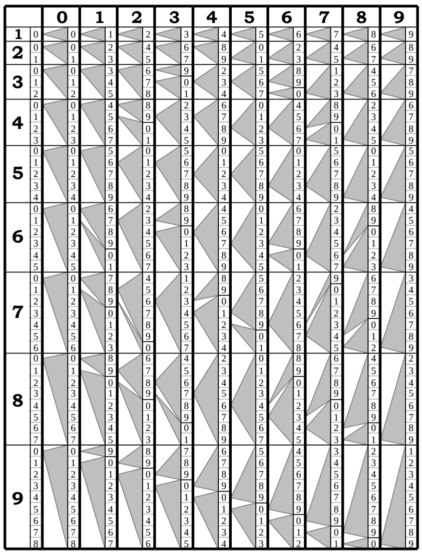
After performing these products with the Napier's rods (354 × 6; 354 × 2 e 354 × 8) is still necessary to "correct" these partial results with the respective powers of 10. So, in the example, we have

Calculations on the Napier's rods	*"Correction"*	*Total*
354 × 8 = 2832	2832 × 1	2832
354 × 2 = 708	708 × 10	7080
354 × 6 = 2124	2124 × 100	212400
		222312

Exercise 2: Using scissors cut the Napier's rods provided at the end of this activity. Use those sticks to determine the following products:

2. 1 305×9;
2. 2 127×83;
2. 3 5016×125;
2. 4 4328×56.7.

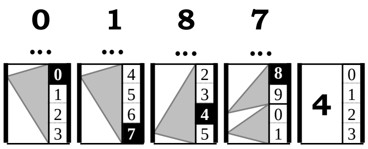
2.3 The Genaille-Lucas rulers



In the late nineteenth century a variation of the Napier's rods was invented where the need of transportation of digits for the next "diagonal" was eliminated (the instrument itself "does" such transportation). Another advantage is the fact that these rulers do not require the user to make any sum; it is possible to obtain the final result only by direct observation of the sticks. These rulers were invented by the French engineer Henri Genaille in reply to the problem posed by the mathematician Edouard Lucas to create an instrument where, indeed, this need to make any intermediate sums was eliminated. Therefore, these rulers are known as the Genaille-Lucas rulers.

Let's see an example of how the Genaille-Lucas rulers work. Think about the product 187 × 4.

- Select the rulers corresponding to 0, 1, 8 and 7 and put them side by side so that on the top appears the number 0187. On the right of these rulers should be the stick numbered from 1 to 9.
- To find the product 187 × 4, you should "read" the line corresponding to digit 4.
- Afterwards, just follow the "triangles" that exist in the rulers to find the value of the required product, beginning on the first digit (from the top) of the first column (from the right). So 187 × 4 = 0748.



Let's see, through the example presented, some justification for the fact that these rulers work properly.

The number of units, 8, comes from 7 × 4 = 2**8**. Note, however, that we still need to carry 2 to the next column. This is done by the "gray triangle" which is near to the digit 8; in fact we have 8 × 4 + 2 = 32 + 2 = 3**4** which means that 4 is the ten digit. Note that the first digit of these column is 2 and that results from the fact that 8 × 4 = 32; the digits below are for the case of transportation from the previous column, just like in this case.

Then follow the "triangle" on the next column (carrying 3 from the tens digit column to hundreds digit column) until the digit 7 that correspond to 1 × 4 + 3 = 0**7** (hundreds digit). Note that the first digit from the hundreds column is 4, which come from 1 × 4 = 04; once more, the digits beneath are for the cases where exist transportation from the previous column. At this step we don't carry anything for the next column and, therefore, the "triangle" at this column sends the user to the first digit, counting from the top, of the column of thousands, i.e., to **0**.

To multiply two numbers with more than one digit with these rulers, you have to perform various partial products and then sum them to obtain the final product using the same process that was indicated for the Napier's rods.

Exercise 3: Using scissors cut the Genaille-Lucas rulers provided at the end of this activity. Use those sticks to determine the following products:

3.1 807×7; **3.2** 129×43; **3.3** 7608×317.

REFERENCES

– Seaquist, C., Seshaiyer, P., Crowley, 2005, "Calculation Across Cultures and History" in *Texas College Mathematics Journal, vol. 1*, 15-31.

– Swetz, F., 1994, *Learning Activities from the History of Mathematics*, Portland: J. Weston Walch Publisher.

3 Number System of Ancient Egypt

Contents: ancient number systems and arithmetic operations.

The need to count is very old and dates back to the time when man settled and began to practice agriculture and livestock (around 10000 BC). Initially, in order to record the information from those activities piles of stones or marks on sticks and bones were often used. Note that these characters were previous to the advent of writing itself (the oldest known form of writing belongs to the Sumerians, a people who lived between 3500 and 2000 BC), i.e., the "signs for numbers probably preceded the words for numbers, it is easier to make incisions on a stick than to establish a well-shaped phrase to identify a number." With time several number systems were created which allowed greater easiness in representing quantities of objects.

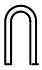
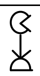
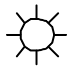
The number system standard today is the Hindu-Arabic system, being used basically all over the world. The name of this system comes from two people: the Hindu people who created it and to the Arabs who used and disseminated it. Although it was known by the Arabs at least since the eighth century, this number system arrived to Europe just in the thirteenth century, mainly by the action of Fibonacci (also known as Leonardo of Pisa) and his book *Liber Abaci* ("the book of the abacus" or "the book of calculation"). Recognizing that arithmetic with Hindu-Arabic numerals is simpler and more efficient than with Roman numerals, Fibonacci traveled throughout the Mediterranean world to study under the leading Arab mathematicians of the time and, in 1202, he published the referred work.

Homework assignment:

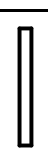
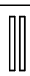
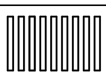
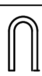
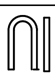
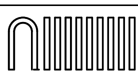
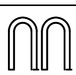
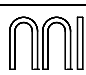
Research the Roman numeral system and write a small text explaining how this system worked (including the symbols used, how each number was represented and so on). Find also places and objects where even today we can find these numbers like, for example, on the kings and popes names, on watches and on the numbering of books chapters.

Other ancient peoples used different symbols to represent numbers and each number system had its own rules. On the next pages we will study the system used in Ancient Egypt as well as some of its essential characteristics.

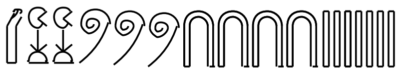
The number system used in Ancient Egypt is at least as old as the pyramids, dating back from 5000 years ago. This system was a repetitive one using special symbols, presented in the following table, for the different powers of 10, from 10^0 to 10^7.

							
10^0 = 1	**10^1 = 10**	**10^2 = 100**	**10^3 = 1000**	**10^4**	**10^5**	**10^6**	**10^7**
"stroke"	"heel bone"	"coil of rope"	"lotus flower"	"pointing finger"	"tadpole"	"kneeling man"	"sun"

To represent the other numbers they used repetitions of these special symbols but none should be repeated more than nine times, which means that it was a decimal system (set of 10 equal symbols were represented by a new special symbol). Thus, we had

1	2	…	9	10	11	…	19	20	21	…
		…				…				…

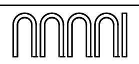
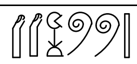
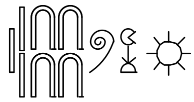
For example, the number 12347 was written as

 .

Remind that the Hindu-Arabic system used today is positional. For example, in the numbers 13, 237 e 351, the symbol "3" don't means always the same quantity: in 13=10+3 represents "3 units", in 237=200+30+7 represents "3 tens", while in 351=300+50+1 represents "3 hundreds".

Exercise 1.

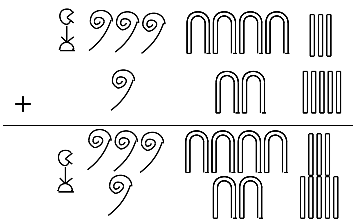
1.1 Complete the following table:

Hindu-Arabic system	*Ancient Egypt system*	*Hindu-Arabic system*	*Ancient Egypt system*
30		3261	
			
752		13125	
			

1.2 What is the largest number that can be written with the Egyptian number system?

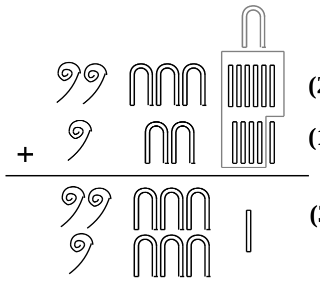
The **addition** was very simple in the Egyptian number system. It was enough to count how many symbols of each type existed in each of those numbers and write the final result with all the symbols of each replacing, if necessary, ten symbols of a given power of 10 by the next higher power. Let's see the following examples:

♦ 1343+125



(1343)
(125)
(1468)

♦ 236+125



(236)
(125)
(361)

Exercise 2.

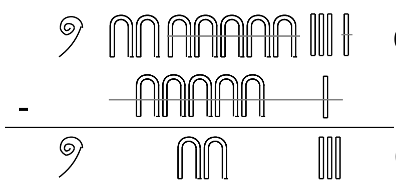
Using the Egyptian number system presented here, calculate the following sums:

2.1 2134+1201. **2.2** 1354+118. **2.3** 1643+357.

For the **subtraction** simply remove from the largest number, the symbols that form the lower number, "replacing" in the largest, if necessary, symbols of a given power of 10 by ten symbols of power immediately below.

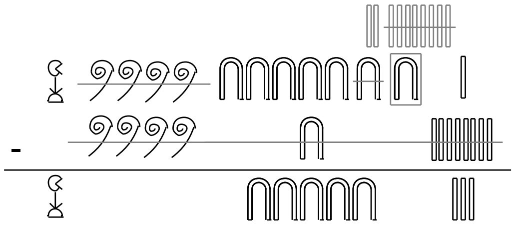
Note that in Ancient Egypt there wasn't the concept of negative number or the concept of zero, and therefore in this context only makes sense considering the cases where the number to be subtract is less than the number to be subtracted. See, for example, the following subtractions:

♦ 174-51



(174)
(51)
(123)

♦ 1471-418



(1471)
(418)
(1053)

Exercise 3.
Using the Egyptian number system presented here, calculate the following subtractions:
3.1 293-122. **3.2** 1562-1303. **3.3** 14523-3128.

The **multiplication** by 10 in this number system is very simple because each symbol represents a power of 10, so each symbol in the representation of the number just needed to be replaced by the symbol of the next higher power.

Exercise 4. Complete the following table:

“n”	𓎆𓎆𓏺	𓍢𓎆𓏺𓏺		𓆼𓍢𓍢𓏺	
“*n*×10”	𓍢𓍢𓎆		𓍢𓎆𓎆𓎆		𓂭𓂭𓍢𓍢𓎆𓎆𓎆𓎆

The remaining multiplications were carried out, in general, by a process of duplation (successive doubling). To better understand from this point on, the usual Hindu-Arabic system will be used. Let’s see an example, the multiplication 15×13. Firstly, it was necessary to double, successively, one of the factors (choose the number 15), recording the values as follows:

1	15 (=15×1)
2	30 (=15×2)
4	60 (=15×4)
8	120 (=15×8)
(16 > 13)	-

The process should stop when the successive doubling in the left column (successive values by which one multiplies the chosen factor) is equal or exceeds the value of the other factor. In the left column values whose sum gives 13 (the other factor) are now reported.

	***1**	**15**
	2	30
	***4**	**60**
	***8**	**120**
Total	13	195

Adding now the values of the right column corresponding to those indicated we obtain the desired final result of the multiplication: 15+60+120=195. This method of multiplying corresponds to the following:

15×13	=15×(1+4+8)	(“left column”)
	=15×1+15×4+15×8	(distributive property of multiplication)
	=15+60+120=195.	(“right column”)

Exercise 5.
Using the successive doubling method, calculate the following products:
5.1 25×8. **5.2** 30×27. **5.3** 312×11.

The method used in Ancient Egypt for the **division** also resorted to successive duplication, in a similar way to that used for multiplication. To understand this method study, for example, the division 91 ÷ 7. Firstly, it was necessary to double, successively, the number 7 (the divisor), recording the values as follows:

1	7 (=7×1)
2	14 (=7×2)
4	28 (=7×4)
8	56 (=7×8)
-	(112 > 91)

The process should stop when the successive doubling in the right column is equal or exceeds the value of the dividend. In the right column values whose sum gives 91 (the dividend) are now reported.

	1	***7**
	2	14
	4	***28**
	8	***56**
Total	13	91

Adding now the values of the left column corresponding to those indicated we obtain the desired final result of the division: 1+4+8=13. This method of division corresponds to the following:

91÷7	=(7+28+56)÷7	("right column")
	=7÷7+28÷7+56÷7	(distributive property of division)
	=1+4+8=13.	("left column")

Exercise 6.

Using the successive doubling method, calculate the following divisions:

6.1 304÷19. **6.2** 345÷15. **6.3** 1457÷47.

REFERENCES

– Gillings, R., 1972, *Mathematics in the Time of Pharaohs*, The MIT Press.