HISTORY AND IMAGE OF MATHEMATICS
AN EXPERIMENT

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ABSTRACT

Is the history of mathematics an educational way for acting on mathematics view? To answer this question in our contribution we discuss the philosophy underlying an Italian book based on historical images. The book is primarily addressed to secondary school students, but also teachers and ordinary people may be potential readers. In the presentation some parts of the book are translated into English. Our discussion is an occasion for investigating the view of mathematics held by people. To this aim we present a questionnaire centred on this subject and analyze some findings coming from the answers we gathered. This analysis allows outlining a project for producing new teaching materials based on the history of mathematics, which encompasses original documents and historical images.

1 Introduction

In (Dematte & Furinghetti, 1999; Furinghetti, 2007) it is discussed the way in which students and teachers consider the development of mathematics. Their views are compared to the socio-cultural orientation, for which the evolution of mathematical concepts is determined by factors inside and outside the discipline, as shown in (Radford, 2006). In this perspective, mathematics is seen as a historical building connected to the context: different forms of mathematics are born, mathematics is used in various professions, is present in everyday life, has relationships with other disciplines, its learning is based on aspects of communication. In (Dematte & Furinghetti, 1999), in particular, some popular conceptions are analyzed: in order to succeed in math, it is better to remember the rules; creativity is not required in the mathematical reasoning; the approximate results in mathematics are not acceptable; symbols +, ×, -, : have been used since before the birth of Christ; etc. Jankvist (2010) illustrates an empirical study on the use of history in a Danish upper secondary class in order to study the students' capabilities at engaging in reflections on mathematics and its history. Liu (2009) analyses connections between the history of mathematics and the students' conceptions. The author describes an annual university course on Calculus which was presented in its historical context. A questionnaire on epistemological aspects of mathematics was administered to students at the beginning and at the end of the course; a similar questionnaire was also used with a group of students who did not attend the course. Results indicate significant influence on the epistemological beliefs of the discipline in the first group but not in the control group.

2 A book

To discuss how to act on the image of mathematics held by students by means of the history, I have written a book addressed to students of the final years of secondary school (16 years old onward), teachers from various disciplines (including history, philosophy, art), or readers who are interested in the popularisation of mathematics (Dematte, 2010).

The book is based on pictures taken from historical sources: pictures have been largely used in history for communicating mathematical ideas, see (Mazzolini, 1993). Words accompany pictures in order to create a unitary discourse and to focus on some aspects. Pictures strengthen what the verbal part say, like in a natural history museum where things and words, verbal and non-verbal communication coexist. Knowledge required for using the book in classroom (or elsewhere) is confined to elementary mathematics.

The book focuses on some relevant ideas of mathematics. Every chapter ends with a discussion ("History for us") about personal beliefs which are connected to the aspects treated in it. This discussion is an opportunity for the reader to reflect on his image of mathematics and it deals with factors that are not always made explicit in the classroom, but influence the personal relation with mathematics. I think it is important to stimulate students' awareness on these factors. Teachers may suggest students they should use the book to start personal insights or to read during holiday periods. It is also proposed to ordinary people to reflect on their vision of mathematics.

The basic reason which has motivated the book is the reflection on pedagogical choices for students who finish secondary school and do not meet mathematics in their study or profession. In any case, for them mathematics could be a cultural stuff and a tool for analysing reality.

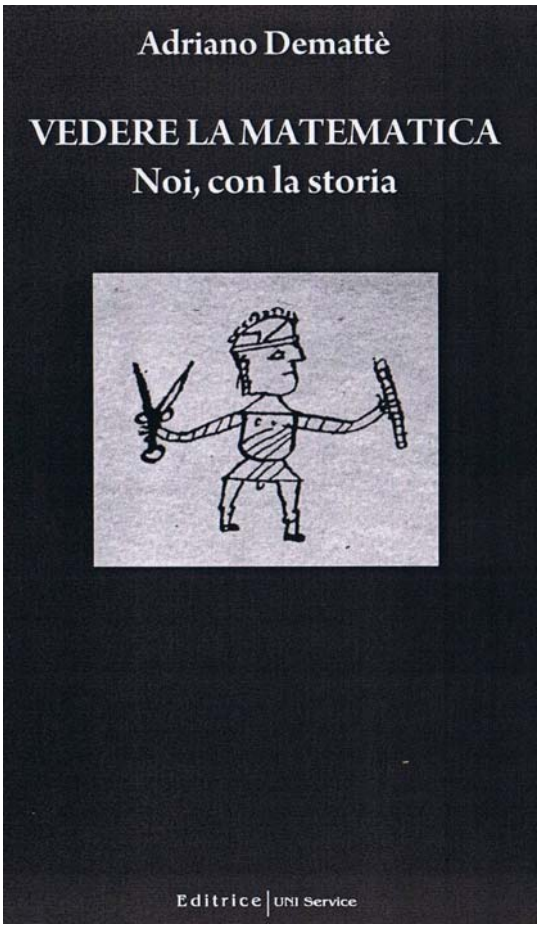

Figure 1. Image on the cover from: Giovanni Agostino Abate, *Giometria de figure quadre*, manuscript, 16th century.

VEDERE LA MATEMATICA
Noi, con la storia
[SEEING THE MATHEMATICS
We, with the history]

CONTENTS (keywords)

Presentation: Fulvia Furinghetti - University of Genoa (aims of the book, method, how to read it)

Image and images…
1. **General view** (images for illustrating relevant ideas in the history of mathematics, m. for everyone)
2. **Role of images in mathematics** (words and things, semiotic systems, geometric figures and definitions, originals, transposition: seeing historical events today)
3. **Suggestions and information in historical images** (people, instruments, diagrams, and drawings)
4. **Mathematics view** (reader's beliefs:
– When I solve a mathematical problem I know that there is only one exact solution
– M. I learn at school is not useful for real life
– Different school subjects have to be learnt separately because they haven't any links
– In my opinion, approximated result of a problem is not acceptable because of mathematical rigor
– When I study m. I always worry not to make mistakes
– I better learn m. if I study alone
– M. I use in everyday life is ancient
– M. and culture are separated)
5. **Something else about the history of mathematics** (history of science: why?, how to select historical

events, m. in every culture, epistemology, hermeneutics)

Figure 2. John of Holywood, image from a 15th c. manuscript
(4. How to write a number).

In the History

1. **The first files of data** (*ishango* bone, objects of clay; **history for us**: numbering is difficult, more and more complex m., connecting with simpler knowledge)
2. **Mathematics for administering a State** (rope, measures, right angle, Pythagorean terns, Herodotus, taxes, *groma*-Roman instrument; m. for a few dominant people, m. as fearing school subject, m. for doing order)
3. **Is mathematics we learn at school ancient?** (Ahmes papyrus, Moscow papyrus, geometry, Babylonia, square root, *Juzhang Suanshu*; ancient m., recent symbols, different manners for doing m.)
4. **How to write a number** (different numeral systems: Egyptian, Babylonian, Maya, Hindu-Arabic, binary, Sacrobosco-John of Holywood; one problem-many solutions, open problems in class)
5. **Does it depend on material we have?** (abacus, quipu, Napier sticks; different instruments-different learning, embodied mind)
6. **Algebra begins** (scribe Jsma-Ja, clay tablet TM 75 G 1693; tablets and modern drafts, contents in tablets and in modern exercise-books)
7. **Mathematics is full of errors** (Ahmes papyrus, "Euclidis Megarensis", to draw an equilateral triangle, *Proofs and Refutations*; errors, inadequacy, to accept own errors)
8. **Pythagoras in China** (*Chou Pei Suan Ching*, bamboo/tree problem; Pythagorean theorem, Pythagorean triplets, arithmetic and geometry, mathematical aesthetics)
9. **A model to imitate** (Euclid's *Elements*, Adelard of Bath, Clavio, Commandino, Spinoza, *Ethica ordine geometrico demonstrata*, fifth postulate; explanation, proof)

Figure 3. Archimedes, Italian stamp
(10. What is geniality?).

10. **What is genius?** (Archimedes, Gauss, Abel, Galois; mathematician-prototype of genius, biographies and mathematics view)
11. **Mathematical knowledge doesn't "accumulate in layers"** (Diophantus, Al-Khuwarizmi, symbolism; historical evolution, easy-difficult, progress-retreat)
12. **Recreational problems** (Alcuin of York, Fibonacci, Calandri, Peano, Carroll, Gardner; problems: m. and narrative, challenge in problem solving)
13. **Authority and knowledge** (reckoning masters, people in images and differences of rank; authority and democracy at school)
14. **Mathematics is culture** (*Margarita Philosophica*, research, specialisation, interdisciplinary themes; ignorance, cultural relevance of m.)
15. **Masters of abacus** (*Margarita Philosophica*, algorists, abacists, reckoning schools; oral and written communication, manipulation of symbols vs. meaningful learning)
16. **Mathematics and trade** (Paolo dell'Abbaco's *Trattato di Aritmetica*, *Treviso Arithmetic*; images and real life, causes of m. evolution)
17. **Geometry for builders** (applications, constructing a rectangle; theorems in action, trial and error, goal of a mathematical reasoning)
18. **Mathematics and politics** (Copernicus, Cardan, Tartaglia, Carnot, Leibniz; necessity of tools for doing m., sources and laws)

19. More recent than we think (symbols for arithmetic and algebra; operations, algorithms, pedagogical choices)

Figure 4. From Paolo dell'Abbaco's Arithmetic, 14th c. (16. Mathematics and trade).

20. Is mathematics the same everywhere? (circle, Archimedes, Liu Hui, Kepler, arithmetic triangle; not only Europe)
21. Problems of priority (Oresme, Fermat, Descartes; misconceptions and diagrams, acknowledging priority)
22. Mathematics and war (problem of ladder, quadrant, measuring for shooting; war and m. growth, ethics)
23. Let's bet everything (gamble and God's will, probability, Laplace, de Finetti; certainty, random, points of view in m.)
24. Calculus (Leibniz, Newton; evanescent quantities in drawings, Calculus as relevant mathematical idea)
25. Mathematics and other sciences (sciences in ancient time, music, golden ratio; interpreting, understanding, explaining)
26. Geometry of position (Euler, topology, Königsberg bridges, Möbius; ontogenesis, phylogenesis, topology in primary school)
27. Beyond infinity (representing infinity, Escher, Peano's curve, Cantor; Hilbert: dignity of human mind, m. and philosophy)

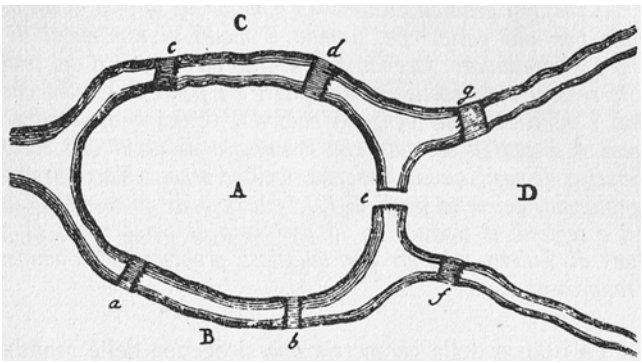

Figure 5. Euler, Königsberg bridges' problem (26. Geometry of position).

28. Etnomathematics (Indian *Sulbasutras*, D'Ambrosio, Gerdes; multiculturalism, m. inside and outside school)
29. Past, present and future (Mandelbrot, fractals, holograms, Morin, where is going m.?; is somebody discovering something in m.?)
30. Imagine a mathematician ([space for drawing], Roman bas-relief, portrait by Beham, Pacioli, Carroll; which m. view in portraits?)

Conclusions (images, history, narrative, mathematical ideas)
References (sites and books)

3 The questionnaire

After reading the book, a questionnaire is proposed (see below): one version for teachers, one for students, one for "common citizens". That one for student is essentially a test of profit which refers to the contents of the volume, although it also includes the requirement of personal opinions. It becomes a "board-book": after reading, students can submit their answers to the teacher and get an assessment on the completeness and relevance of references. In the questionnaire for the teacher, personal opinions about the image of mathematics and the use of history in interdisciplinary perspective are asked. Reflections are also requested to citizens who can, this way, recall their school experience: some parts of the book regard school mathematics and it is assumed that the reader had, right there, his/her most significant experience.

The idea to submit to the reader a questionnaire with open answers will be a way to bring his/her thoughts to specific points, but offering, at the same time, the opportunity to broaden the discourse. The aim is not to collect response data with a statistical value and it is assumed that the way in which the questions are worded can influence the respondents. The book explicitly shows a specific image of mathematics. I am aware that, because of the characteristics of the questionnaire, the reader could be even excessively appeasing. The goal, however, is to raise a debate on the image of mathematics and to suggest that history may be a subject for acting on it. We would like to collect personal opinions from teachers, students, and other readers, who can send their answers (anonymously, if they want).

GREMG – Gruppo Ricerca Educazione Matematica Genova
Project: **"History for discussing the mathematics view"**
[Download of the following questionnaire from
http://www.uni-service.it/vedere-la-matematica.html]
After you read the book *Vedere la matematica*, we ask you some questions. We are interested in knowing your opinion about ideas you have found in the book.
Answer and send the following questionnaire to the e-mail address **storia_e_immagine@dima.unige.it** ('**di**partimento di **ma**tematica **uni**versità di **ge**nova'). If you want, write your name and your personal address.
We ask the teachers who use the book for class to collect students' answers and to send files to the previous e-mail address or to send paper sheets to:

Prof.ssa Fulvia Furinghetti
Dipartimento di Matematica dell'Università
Via Dodecaneso, 35
16146 Genova

We guarantee that personal data will be used only inside the research Project.
Here is the questionnaire for you. Choose:

σ, if you are secondary school student

ι, if you are teacher

χ, if you are neither secondary school student nor teacher

Thank you very much for attention and for collaboration

QUESTIONNAIRE

σ

For **SECONDARY SCHOOL STUDENTS**

1. Kind of school...
2. You read the book:
□ after an internet search
□ thanks to a suggestion of a friend
□ because of a request of your teacher
□ others .....
3. Remember mathematics you have learned at school: do you think that most of it was known before Christ? Choose at least three examples in the book.
4. In the book you don't find an explicit answer but only some hints... Before Christ, could there be somebody who was able to use percentage? Why?
5. About the historical origin of symbols you use for arithmetic, algebra,... before reading the book, what was your opinion regarding the period in which mankind started using them? Was it the same you found in the book?

6. With respect to the content of different chapters, how would you describe human knowledge before Renaissance, if you compare it to that after Renaissance? Did you find something new in the book?
7. "Development of mathematics has been due to the need of applications": do you agree? Why?
8. In your opinion, what is described in the book could help changing school contents? How?
9. Let's do a little "mathematics geography": in your opinion, what is really interesting about mathematics outside Europe?
10. "Nowadays, nobody discovers anything in mathematics": do you agree with this statement? Why?
11. Knowing other school subjects helps better learning mathematics. Why?
12. In your opinion, what *mathematics view* appears in the book? Compare it to your personal *mathematics view*.
13. Briefly describe the three contents which are the most interesting, in your opinion, and explain why you have chosen them.
14. Express your opinion (is the book interesting for you? which contents are more remarkable? which ones less? Have you found some contents you want to deepen? etc.)
15. If you want, write other remarks.

QUESTIONNAIRE

ι

For **TEACHERS**

1. You read the book:
□ after an internet search
□ thanks to a suggestion of a friend/colleague/acquaintance
□ others .....
2. Kind of school where you teach and your school subject
3. In your opinion, is *mathematics view* a relevant pedagogical focus? Why? How can it influence students' involvement? What about other school subjects?
4. Do you consider significant that students know historical evolution of mathematics? Which aspects, first of all?
Do you agree about the mathematics view which appears in the book? Why?
5. In your opinion, what has been or what could be the interest of your students who read or will read the book?
6. Among ideas described in the book, which ones can students better learn?
7. Which ideas described in the book do you agree with? Which ones do you disagree with?
8. Have you found contents you might deepen? If so, which ones?
9. Does the book contain hints for classroom activities? Why?
10. Do you use the book as a pedagogical tool? If so, in which way? If not, why?
11. About 'sensing use' of mathematical symbols, do you think that history can provide teachers pedagogical hints and stimulate students' reflection?
12. Do you think the book has a pedagogical value for teachers teaching other school subjects?
13. If you want, write other remarks.

QUESTIONNAIRE

χ

For readers who are **NEITHER SECONDARY SCHOOL STUDENTS NOR TEACHERS**

1. You read the book:
□ after an internet search
□ thanks to a suggestion of a friend/colleague/acquaintance
□ others .....
2. How do you use mathematics (in your job or in your everyday life or as a topic of interest)
3. What are the most interesting aspects you have found in the book?
4. In your opinion, is knowing historical evolution of mathematics important for one's personal culture? Why?
5. Did you know that some researches about the history of mathematics or about the mathematics view are in progress? What is their relevance, in your opinion?

6. In your opinion, what mathematics view appears in the book? Do you agree or not? Why?
7. The book often reports remarks about school life: compare them to your personal experience.
8. Have you found some contents you want to deepen? Which ones?
9. Have you found some contents which are, in your opinion, uninteresting at all? Which ones?
10. If you want, write other remarks.

4 The workshop

Materials for working groups

- Copy of the book (in Italian)
- Translated contents with keywords (in English)
- Translated questionnaire for readers (in English)
- Following questions for the Workshop
- Answer sheets

Questions for the Workshop

a) In your opinion, can a reader establish meaningful links between mathematics she / he have learned at school and mathematics shown in the book?
b) Can the learning of the history of mathematics change mathematical beliefs? What role could have the book?
c) What's the pedagogical value of historical images?
d) After secondary school (18), many students don't learn mathematics anymore: how can history be a starting point for their autonomous interest in mathematics?
e) Could some historical topics enter class for treating the history of mathematics[1], not only for building new mathematical concepts?
f) Other remarks...

Answers to the questions for the Workshop

a) All participants in the workshop showed almost the same opinion, i.e.: it is necessary that students get more assistance by the teacher and supplementary material (more specific questions, worksheets, exercises, websites); these links can grow if the teacher points out the connection specially in the teaching of a general topic in school and the reading of a linked topic in the book takes place at the same time.

These observations confirm, in my opinion, that it is not easy to establish links between school mathematics and mathematics as shown in the book. In response to the questionnaire, a student wrote: "This book is a very positive book, because it makes math more like the reality that surrounds us". In this statement I see a double value. 1) It shows that the student recognizes that school mathematics has a potential role with respect to reality ("like the reality" could mean "which also refers to aspects outside the discipline," or "which uses a language less specialized, does not include the systematic use of rigorous deduction, refers to global images, etc."). 2) It highlights that student's view of school mathematics and the view suggested in the book are different, so not all that can be said about the former can also be said about the latter, and vice versa. Moreover, note that in the entire questionnaire, students have not used specific examples taken from math class.

[1] "Strong role" of the history of mathematics (Dematté, 2007) or "History as a 'goal'" (Jankvist, 2010).

b) A participant declared he expects it could change math beliefs but under the right circumstances (such as the presence of a well informed teacher), depending on the specific beliefs; yes for: “M. and culture are separated”, “Different school subjects have to be learnt separately”. “M. and culture are separated” (if culture is taken in a broad sense) can be changed because there are many examples in the book that show a connection; chapter 7 can change opinion 5 (“When I study m. I always worry not to make mistakes”). Images might strengthen the idea that mathematics we use is ancient. Therefore not every listed belief can be tackled by reading the book but those which are inherent to mathematics and culture. If we look at the previously pointed out believes, it is doubtful that so many of them can be changed by teaching the history of mathematics, for example the second one (“M. I learn at school is not useful for real life”). Concerning the second, pupils could think it only changed real life some centuries ago.

Every participant made some distinctions: the study of history, and the use of images as well, could act only on some beliefs. It is necessary that the teacher assumes a mediating role.

I agree that some students may believe that mathematics was a useful tool to act on the truth only a few centuries ago. It emerges the educational problem to convince the students that past and present are connected (this way, they could understand the real meaning of the history of mathematics). A student of mine asked me: why do we study history, if we must understand the importance of mathematics we learn today? A question with a suggestion inside.

c) [In the book] There are different kinds of images: 1) There are those meant to illustrate mathematical ideas or problems (Koenigsberg bridges), 2) Pictures of mathematics (*Margarita Philosophica*), portraits, pictures of math activity, 3) Suggestion pictures, not necessarily mathematical (*Divina Proportione*), suggestion of mathematical ideas (basket patterns), 4) Produced pictures (draw the mathematician). Here it is a list of expressions which has been used by participants about the pedagogical value of historical images: attracting students, motivating, enter new situations, answering questions, showing that mathematics is ancient but evolves, giving new perspectives and frames for discussion, starting explorations, suggesting ideas for practical demonstrations, visualizing concepts, stimulating conjectures, supporting or replacing texts, anchoring an idea, starting thinking or talking about a problem, offering a ‘picture of the time’.

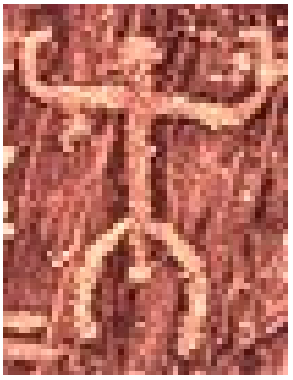

Figure 6.

During the work, a participant pointed out some analogies about postures between the image on the cover of the book and some rock drawings found in the Negev Desert (in figure 6, a similar petroglyph from Valcamonica-North Italy).

I would like to highlight another suggestion created by historical pictures, i.e. their metaphorical value with respect to mathematical reasoning. It is quite easy to imagine that persons in previous pictures (people who are using mathematics in everyday life, like in figure 2 or 4) are acting with a goal. When our students make mathematics, do they work with a goal in mind (a doubt which has to be clarified, a thesis which has to be proved,...), or do they work only for implementing assigned procedures?

d) Some participants agreed that history could be a starting point for autonomous students' involvement in mathematics because, in their opinion, history seems to be interesting for many students and a book like this one could make the reader more susceptible for the history of mathematics in general. Others were skeptical: I don't think it could be very easy; for involving student it could be better to start closer to present-day use of mathematics and from there to go backwards (how did we arrive at the mathematics we use at present? where does school mathematics fit in?...); I think if someone is not really interested in mathematics, history of mathematics can't convince him to start autonomous work in mathematics.

It is really rare that, when they don't learn at school anymore, students are interested in mathematics. It's a pity. How could teachers stimulate their interest? The first step could be to present them resources such as guides for using the web, multimedia, digital or in hard copy books, etc. In my opinion, the problem to involve them is difficult but must be considered.

e) Yes, but in interdisciplinary work or projects, in connection with general history, for instance, in France the revolution in connection with Monge and Condorcet, in England Newton and his time when England became an important country, in the Netherlands in the Seventeenth century when de Witt was not only an important statesman but also a mathematician, in Germany Adam Riese and his time, etc. Biographies of mathematicians could be very encouraging to students. It probably could be an optional subject. Students could use this book to select a topic and study it more in depth.

f) The book could be a good additional schoolbook especially for pupils that do not really like mathematics or for those ones that would like to know more about history of mathematics. A nice thing is that every chapter is like a small story. The teacher can choose the chapter and doesn't need to teach the whole book. Links between actuality and history are very interesting and provide net perspectives. "La storia per noi" gives new ways of looking at pictures and at mathematics. Some of the topics (for example chapter 11) are very useful to change or even to destroy old views, because of provoking. The book looks nice and it could benefit from some activating material. Regarding not previously listed beliefs, another participant says: the book can suggest it is only about man, not woman; I miss some 20^{th} century applications and the danger is that the reader will think mathematics as an old stuff. A popular belief about math is that it is 'finished', that there is nothing left to discover. Then such a book could be even dangerous!

Answers f) complete answers e) and propose a range of ways to use the history of mathematics in itself. The suggestion for interdisciplinary works is really meaningful and expresses the core of the history of mathematics in pedagogy. In my opinion, biographies could also show that the most important mathematicians could be sometimes weak or even could make errors. The opportunity to learn this or other kinds of optional subjects is a way for deepening and for involving less motivated students. The book was just meant to be used for activities in the classroom, but the teacher could also require the student to read it on their own. The questionnaire for students can also be seen as a tool to direct students' attention and to focus some relevant aspects.

In a previous work I have analysed the belief "nobody is discovering new things in mathematics" (Demattè, 2004). Some students of upper secondary school (aged 15) answered oral interview concerning their views of the development of mathematics.

Almost unanimously they declared to believe that mathematics is evolving today, but the examples they produced were certain possible applications to experimental sciences or technology. They didn't refer to the current mathematical research and to the idea that mathematics has developed for internal needs, independent of the demands of applications in other sciences or industry. I agree that students have to know that mathematics is also a modern science. The problem is that mathematical research is often highly specialized. In ESU6, Batya Amit and Nitsa Movshovitz-Hadar have presented examples of activities with students having the aim to bridge the gap between contemporary mathematics and mathematics education, by means of activities linked to the story. This type of educational research should be supported and it could be part of a reflection on mathematics aimed at students, teachers, and people who do not work at school.

Regarding the statement "Mathematical knowledge doesn't 'accumulate in layers'" (#11 in the Contents), a participant pointed out the distinction between the fact that, in a certain moment, there could be 1) ignorance of a previous mathematical topic, 2) conscious choice not to take this topic into account. Depending on the true alternative, interpretation of the statement could differ.

Acknowledgments

So many thanks to all workshop participants, first of all to those who gave their written contributions: Maria Correia de Almeida, Michael Fried, Hannah Hoffmann, Jenneke Krüger, Swetlana Nordheimer, Anne Michel Pajus, Harm Jan Smid. I hope that the ideas of every one of us have been faithfully reported herein. I also wish that my comments could be the subject of further debate.

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