DESIGN AND HIGH-SCHOOL IMPLEMENTATION OF MATHEMATICAL-NEWS-SNAPSHOTS

An Action-Research into *Today's News is Tomorrow's History*

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ABSTRACT

This paper presents lessons from a three year study in the format of an action-research that focused on interweaving, in high-school curriculum, mathematical-news-snapshots (MNSs) which include their historical background. It follows the ESU5 presentation of the rationale for such a study. Parts of the study, in particular a reverse-engineering analysis tracking-back the design principles upon which 10 MNSs were developed for the study, and some of the implementation study results related to the impact on students' perception of mathematics as a living discipline, were presented at ESU6 workshop. They are elaborated in this paper. A sample MNS focusing on Kepler Conjecture is also presented.

1 Introduction

At ESU5, Prague 2007, we discussed the rationale for integrating mathematical news in high-school mathematics teaching, in order to decrease the gap between contemporary mathematics and school-mathematics curriculum (Movshovitz-Hadar, 2008). The proposed pedagogy was interweaving 10-15 minutes especially designed Mathematical-News-Snapshots (abbr. MNSs), based upon their historical background, in the regular teaching of high-school mathematics. We advocated then an empirical study to examine the feasibility of this proposed pedagogy, and to explore its impact on studens' perception of mathematics as a living discipline. This paper presents such a study in the format of a 3 years action-research in which the 1st author acted as a researcher-teacher in her own classes and the 2nd author acted as a design advisor and co-researcher. Parts of this study and some of its results were presented at ESU6 workshop, Vienna 2010, and are elaborated in this paper.

2 The study

2.1 Study design and population

The study reported here took place in three age-group classes at one high-school in Israel. The first author acted as a teacher-researcher in the experimental classes and as an observer-researcher in a few other classes. Three parallel classes, one in each age-group, that learn mathematics at the same level as the experimental classes and their population is of comparable qualities, served as control groups for some of the comparison measurements.

2.2 MNSs design

Movshovitz-Hadar (2008) suggested a classification of mathematical news into five categories with examples many of which can be made accessible to high-school students:

(i) A recently presented problem of particular interest and possibly its solution.
(ii) Long-term open problems recently solved.
(iii) A recently revisited problem.
(iv) A mathematical concept recently introduced or broadened, including concepts that evolved into new areas in mathematics.
(v) A new application to an already known piece of mathematics.

For this study a mathematical result was designated as *suitable* for designing a MNS iff it meets all the following criteria: (i) It was published in the professional mathematics journals in the past 30 years. (ii) This result or its historical background has some connection to high-school curriculum or to another aspect of high-school students' life. (iii) It seems likely that this result can be made accessible to high-school students through a snapshot about this result to be developed based upon the design principles specified later.

Following a literature survey for suitable pieces of news, a series of 10 PowerPoint presentations for a periodical-MNS-interweaving-program was prepared especially for the study. Their contents were based upon the following news:

- The proof of Kepler's Conjecture; (See details below.)
- The discovery of large prime numbers; (See details in Amit et al. 2011.)
- News about the mathematics of Sudoku; (See details in Movshovitz-Hadar 2008.)
- The proof of Fermat's Last Theorem;
- The mapping of the E_8-Group;
- Solution of the Four-Colour problem;
- The linear algebra behind Google search engine;
- The Digits of π ;
- Benford's Law applied to tax-frauds and to election frauds;
- The Fundamental Theorem of Algebra applied to astrophysics.

The design of this series of MNSs was based upon design-principles (abbr. DPs) which were predetermined according to the researchers' math-education perception of best-practice. They came in three types: (i) Mathematics-content related DPs; (ii) About-mathematics-content related DP; (iii) Pedagogy-related DPs.

(i) <u>The mathematics-content related DPs were</u>: Each MNS presentation should expose students to:

1. A new mathematical result (of one of the 5 categories mentioned earlier) published in the past 30 years, and possibly yet unsolved related problem(s) as well;
2. Basics in mathematics from the curriculum, needed to make the news accessible and related to student's presumable state of knowledge;
3. Some advanced and/or extra-curricular mathematics relevant to the news possibly including new concepts and vocabulary as relevant and necessary for basic understanding of the matter.

(ii) <u>The about-mathematics-content related DPs were</u>: Each MNS presentation should include in as much as it is relevant:

4. Elements reflecting the nature of mathematics, such as the nature of definition, proof, and truth; the nature of problem posing and problem solving; search for patterns; generalization;
5. Historical background relevant to the news such as long-term conjectures, failures and success in proving them, and cultural background if relevant;

6. Details about mathematicians:
   a. Details about the special contribution of mathematicians who were involved in the piece of news or in its history;
   b. Personal details about such mathematicians, such as their life time, their relationship with the professional community and with the public;
7. Connections:
   a. Connections to some area within mathematics or to some topic in the curriculum;
   b. Connections between mathematics and other disciplines or to everyday life or to any other students' common interests;

(iii) The pedagogy related DPs were:

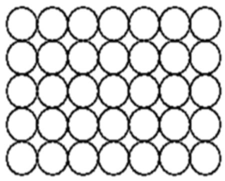
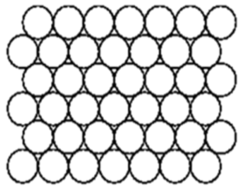
8. Each MNS will be in the form of a PowerPoint presentation consisting of both verbal and visual parts (Photos, diagrams and animated illustrations) to demonstrate, illustrate or represent mathematical ideas, concepts and other elements;
9. The verbal part will take the form of expository statements as well as a Socratic dialogs (Q & A) inviting discourse and student's involvement;
10. The language should be simple and friendly, concrete and non-formal as much as possible, bridging between students' level of mathematics and the advanced level of the mathematical news presented;
11. Duration will be limited to about 15 minutes so as not to take up too much of the ordinary instruction time.

2.3 A sample snapshot: The Kepler Conjecture

The PowerPoint presentation of The Kepler Conjecture MNS consists of 17 slides. Table 1 presents their verbal contents, expected answers to short questions, and some parenthesized notes about the visuals.

Table 1: A Snapshot on: The Kepler Conjecture

Slide No.	**A Condensed Version of The Text on the Slide**
1	**The Kepler Conjecture** A Mathematical-News-Snapshot For High-School
2	**The Cannonball Packing Problem** • In 1590 the British sailor, Sir Walter Raleigh, loading his ship for a voyage, wished to figure out the number of cannonballs a box of a given height could contain. • He assigned the task to his assistant Thomas Harriot (The slide contains a photo of a cannon besides a pile of 3 cannonballs)
3	**Thomas Harriot** • A British astronomer, navigation instructor, and mathematician, who established the first Algebra School. • His mathematical work focused on solving equations having negative and/or complex roots - a field in which he preceded his generation. (The slide contains a photo of Thomas Harriot)

4	**Let us join him in the calculations** The first layer:   **Square Packing** **Hexagon Packing** What is the difference between the two modes of packing?
5	**Let us compare** • Assuming each sphere has a 1 unit radius, and the base of the packing is 10 by 10 square units: • What portion of the base area do the spheres occupy? • We reduce the consideration to the circular cut of the Spheres: • In the square packing? - $25\pi/100$ • In the hexagon packing? - $23\pi/100$ (The Illustrations from slide no. 4 are repeated here as a reminder with frames defining the packing borders)
6	**And in Space?** • How is the second layer placed in each case? – • Consider a sphere in the center of the box, how many spheres does it touch in each case? (12 in both) • Which packing has a better use of the box volume? – Mathematicians consider infinite space. They found that in both cases the spheres take about 74% of the volume (~3/4). (The Illustrations are repeated to demonstrate the 3 different layers, an animated demonstration of counting the number of spheres each central sphere touches, is added to each mode of packing.)
7	**Back to Harriot** • After reaching this solution, Harriot, wondered about a more general problem, an act typical to mathematicians: • Is there a better sphere-packing? i.e., occupying a higher percentage of space? • With this problem he approached the mathematician and astronomer Johannes Kepler. (The slide contains a photo of the young astronomer Johannes Kepler)
8	**Kepler the Astronomer - The Dream** • Kepler was not only an astronomer and a mathematician but also a science fiction writer. • Between the years 1620-1630, Kepler wrote a fantasy called *Somnium* (Latin for The Dream). • It was published by Kepler's son Ludovico, after Kepler’s death. (The slide contains a photo of *Somnium*'s front cover)
9	**Kepler’s Dream-Story** • In the narrative, a student is transported to the moon by mysterious forces. • It presents a detailed imaginative description of how the earth might look when viewed from the moon. • It is considered the first serious scientific treatise on lunar astronomy. (The slide contains a photo of Johannes Kepler in his later years).

10	**In 1611 Kepler argued that:** • Harriot's solution is the best sphere packing (in the 3-d space). It became known as the fruit seller's packing. • Kepler left no proof to his claim. Therefore it became known as: "The Kepler Conjecture". • Although it seems intuitively true, it remained an unproved conjecture for almost 400 years! (The slide contains a photo of a box of fruit packed in 2 layers)
11	**Kepler's Conjecture- Timetable** • 1611: Kepler claims that: No packing of congruent balls in Euclidean three-space has density greater than that of the face-centered cubic packing. • According to Kepler Conjecture, the best packing density is about 74% (~ π/sqrt 18). • 1611-1900: Many mathematicians attempt to prove the conjecture, and fail. • 1900: Kepler Conjecture appears as Problem no.18 in a very important list of problems that will occupy the mathematics community in the 20th century. (The slide contains a photo of a spinning pyramid)
12	**1900: A major mathematical event** • Every 4 years there is an international congress of world mathematicians. In 1900, it took place in Paris. • The opening address was given by David Hilbert, a prominent mathematician of the time. Hilbert presented, a list of 23 mathematical problems he proposed as the most challenging towards the 20th century. • To-date, many problems were solved while others are awaiting their turn… • What was the fate of Kepler Conjecture? (The slide contains a photo of David Hilbert)
13	**1998: Kepler Conjecture proved** • 1998: Thomas Hales a U.S. mathematician claims he has proved Kepler Conjecture, by extraction of all possible packing using super-computers. • He submitted the paper for publication in the most reputable journal: Annals of Mathematics. (The slide contains a photo of Thomas Hales)
14	**1998-2005: The struggle for publication** • The editor nominated a team of 12 notable mathematicians to go through Hales' proof. • In 2003, Fejes-Toth, the chairman of the team, created a historical precedent: • Summing up 5 years during which the team examined Hales' proof, Fejes-Toth wrote that the team did not succeed in proving the correctness of the proof nor would they be able to do it in the future – Due to exhaustion … (The slide contains a photo of Fejes-Toth)
15	**The Approval- The Compromise** • 2005 : On the 14th of November, 7 years after Hales submitted his paper (!), The mathematical-logical part of the proof was accepted and published in 120 pages of Annals of Mathematics, Vol. 162 (3) pp. 1063-1183
16	**A new beginning: FLYSPECK!** • Hales gathered all the other parts including the computational parts and the software involved as part 2 and: • Since 2003: A World-Wide project is on-going aiming at the verification and approval of this part. • The project's goal is to produce. The Formal Proof of Kepler, known by the acronym FPK, nicknamed: Flyspeck, http://code.google.com/p/flyspeck/
17	**The end** but keep updated! Check for more news! (The slide contains a photo of a spinning pyramid)

2.4 Main research question

The main research question was: What is the impact of exposing high-school students to contemporary mathematics employing MNSs, on students' perceptions about the nature of mathematics as a living discipline?

2.5 Cycles of the action-research

It is typical for an action research to consist of various cycles/spirals (Kemmis, 1988; Coghlan & Brannick, 2001, 2005). Our action research took place in two phases, each consisting of 2 cycles. Phase 1 was a design study. (i) The first cycle in it was the MNSs development cycle. The initial step in this cycle was a literature search for a *suitable* piece of news as defined above. The design step itself was based upon the set of *predetermined* principles mentioned above. Following a *field-trial* of each snapshot in one class, a revision step took place according to the observations. Then updates were added if more news had been published meanwhile[1]. (ii) The second cycle took place at the end of the development cycle of all 10 MNSs. It took the form of a reverse engineering study. (See details in section 2.6 below). Phase 2 was an implementation study. It also took place in 2 cycles: (i) Implementation of each MNS in various classes; (ii) Implementation of various MNSs in each class. Qualitative and quantitative data were collected in both phases as will be elaborated below.

Diagram 1 illustrates the relationship among the various cycles

2.6 MNS reverse engineering analysis based upon the Design Principles

Having developed all the PowerPoint presentations based upon the above mentioned Design Principles, a validation process was conducted by assigning 5 experts a reverse engineering task. Their task was to track-back the various DPs in the PowerPoint presentation of each MNS. The purpose was to analyse the outcome of the first cycle of Phase 1, and get a typology of the set of MNSs. (ESU6 workshop participants went through a similar process focusing on Kepler Conjecture MNS, after being exposed to the PowerPoint presentation.)

Each expert received a PPS version of the PowerPoint presentation and screenshots of all the slides of each MNS. Adjacent to each screenshot was a table in which s/he were to insert the design principles of contents and of pedagogy as s/he attributed to each bullet in that slide. Then they were to summarize the occurence frequencies of each DP in each MNS and get the percentage of the total occurence of every DP per MNS, yielding the relative frequency distribution of DPs per MNS. A negotiation session took place to bring the 5 experts into agreement in case discrepancies were found among their results. Following the validation of these experts' analyses, it was possible to compare the inner composition of the MNSs to one another and relate the results to other data collected during Phase 2 of the study.

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[1] For example, in 2008, after the first MNS was developed about Sudoko mathematics, some more news were published: Elser Bauschke and his graduate student Jason Schaad created a computer program that solves Sudoku puzzles using the projection algorithm method.(See http://www.schaad.ca/hpr.html) (Rehmeyer 2008).

Diagram 1: The cycles of the action research

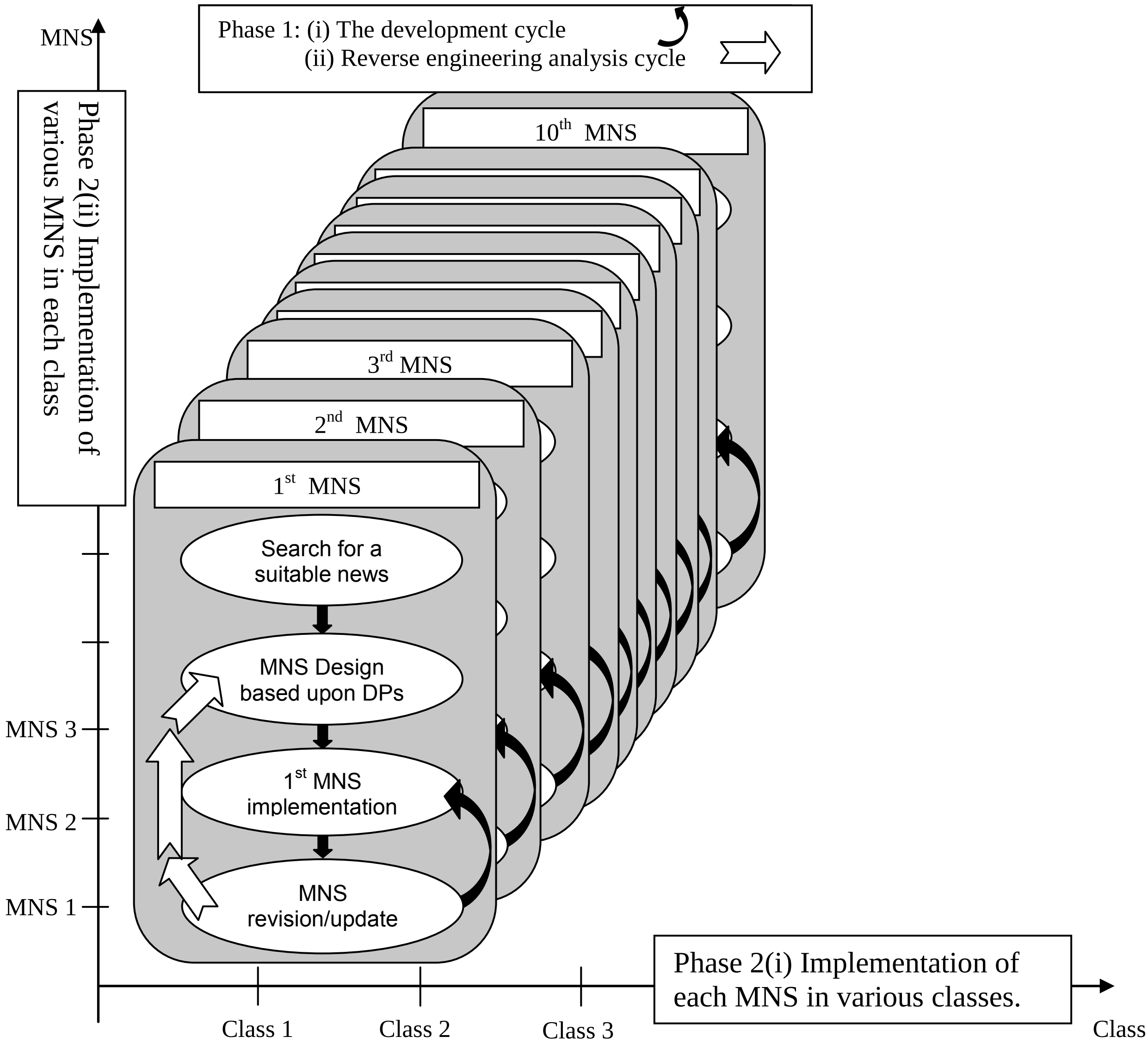


Diagram 2 presents an overall comparison among the 10 MNSs with respect to the relative frequency occurence (%) in each MNS of the mathematics-content related DPs (1, 2, 3), the about-mathematics-content related DPs (4, 5, 6, 7), and the pedagogy related DPs (8, 9). Diagram 3 presents the relative frequency occurrence (%) in the various MNSs of DP 1, 5, 6, namely, of mathematical news, historical background and details about mathematicians.

From Diagram 2 one can see that: All MNSs are pedagogically rich. In all MNSs but one (Sudoko), as the mathematics-content relative occurence decrease the about-mathematics-content relative occurence increase. The about mathematics-content and the mathematics-content DPs split the MNSs into two subgroups: (i) MNSs in which the mathematics-content related DPs are dominant (E.g., E8, PageRank, Sudoku) (ii) MNSs in which the about-mathematics-content related DPs are dominant (E.g., Kepler, Fermat, Primes, 4 Colour).

From Diagram 3, it becomes clear that in each MNS the news is linked to the history of mathematics.

Diagram : An overall comparison among the 10 MNSs with respect to the relative frequency occurence (%) in each MNS of the mathematics-content related DPs (1, 2, 3), the about-mathematics-content related DPs (4, 5, 6, 7), and the pedagogy related DPs (8, 9)

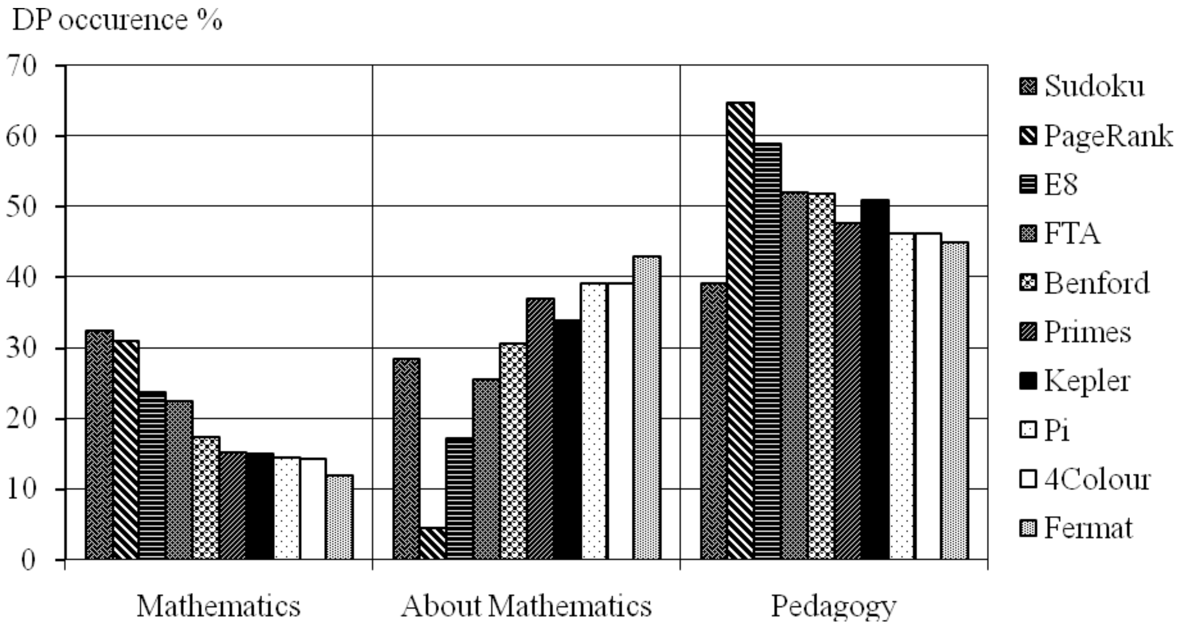
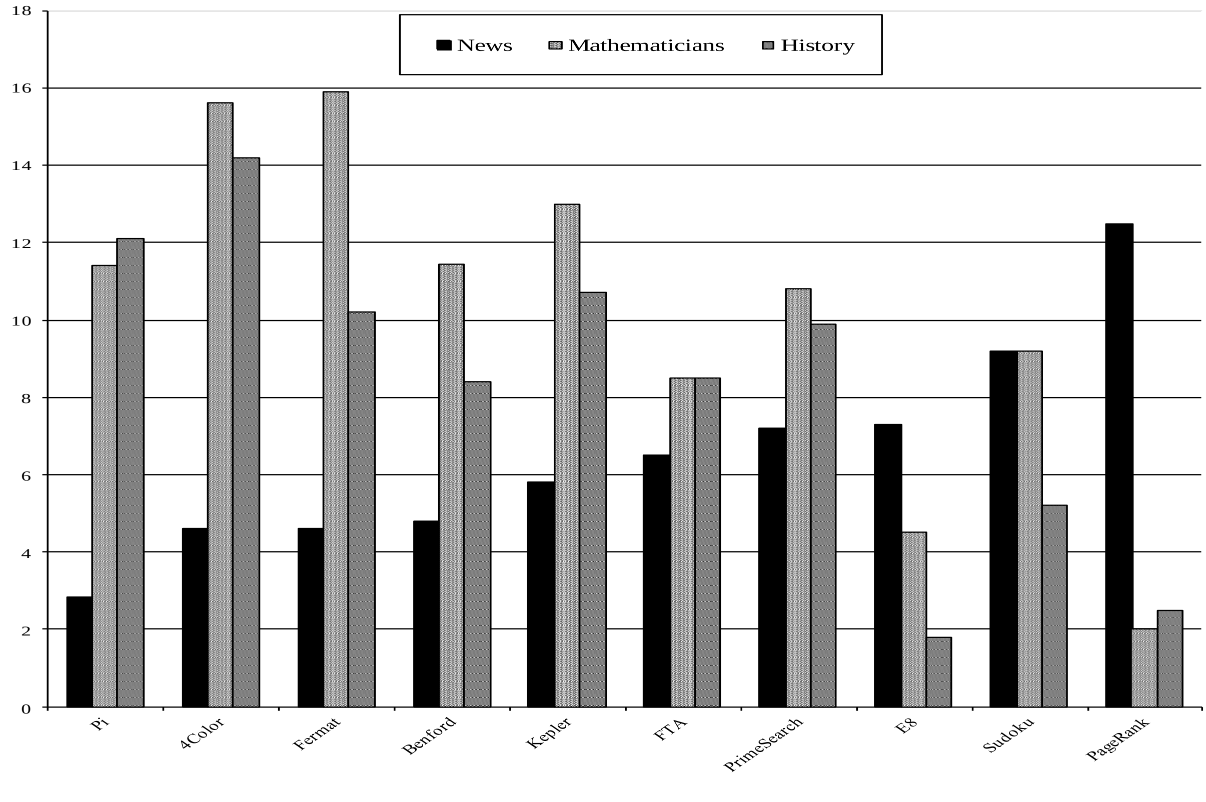


Diagram 3: Relative Frequency Occurrence of: Mathematical News (DP 1), Historical Background (DP 5) and Details About Mathematicians (DP 6), in the MNSs



2.7 Data collection instruments

In each cycle of the implementation study (Phase 2), the following data collection Instruments were used:

(i) Immediately following the implementation of each MNS:

- Students' feedback-questionnaire;
- Group-interview ;
- Teacher's Field Diary documenting special events during the MNS exposition.

The feedback-questionnaire administered after the exposition of each MNS consisted of the following questions:

- If there was anything you found impressive, please state what it was and why.
- If there was anything new you learned from the MNS, please state it.
- If there was anything you found difficult to follow, please state it.
- If there is anything you intend to share with anyone who was not there, please state with who and what.
- To what extent did you enjoy the MNS? Please state why.
- How do you compare this MNS with others you were exposed to?
- Any other comment?

(ii) Before and after the exposition of the whole series of MNSs in 3 intervention classes and in 3 non-intervention control classes:

- Attitudes Questionnaire (28 open-ended items);
- Pre and post individual interviews with 3 students per intervention class.

In the Attitudes Questionnaire students described their interest, attitudes, feelings and opinions about mathematics and learning-mathematics. For example here are a 2 of the 28 items:

- Who of the mathematicians you heard about is your favourite and why? What did s/he do?
- Do you think that school mathematics you learn is different from your parents' school mathematics? If yes - provide details. If no – explain why.

Verbal responses given to the various open-ended items in the Attitude-Questionnaire were analysed qualitatively, and coded for each item separately as follows: A response that expressed a conception of mathematics as unchanging, stagnated, dead-end discipline was marked "Negative". A response that expressed a conception of mathematics as a vivid, active and constantly developing was marked "Positive". If there was no answer or an answer stating more or less that "I do not know", it was marked Neutral. When in the response a student entered a universal generalisation or a strong statement in either direction it was marked "Strongly negative" or "Strongly positive". The five marks per item were gathered for each group and a comparison within groups was performed for the pre-test and the post-test responses.

3 Classroom Implementation – The challenge and some results

3.1 Bridging between school mathematics, contemporary mathematics and its history as a challenge to the teacher

The challenge to the teacher who wishes to implement the MNSs method is threefold: (i) curriculum issues; (ii) system issues; (iii) budget issues. (ESU6 workshop participants discussed these issues).

(i) Curriculum issues

- Teachers are expected to "cover" the curriculum under time constraints. Hence every deviation from the curriculum and any extra-curricular activity is a threat to reaching this goal.
- Many students find school mathematics difficult to cope with. Teachers are responsible for students' success in it. Any extra-curricular activity is a threat to reaching this goal.

(ii) System issues

Teachers and their classroom practice are subject to inspection by the superintendent, the principal, peer teachers, students and last but not least important, by parents. A deviation from the mandatory curriculum and the common mode of teaching invites severe criticism. It puts the teacher in a defensive position.

(iii) Budget issues

- Many schools are not yet equipped with the technical-equipment (computers and projectors) that are required for the implementation, nor are they able or willing to invest efforts in obtaining such equipments. This puts the motivated teacher in a problematic situation. A partial solution to it is adapting the PowerPoint presentation to a frontal exposition using the blackboard.
- Implementation is highly demanding on the part of the teachers. Without an appropriate compensation they are unlikely to take this challenge upon them.

Nevertheless, based upon our action-research teacher's experience, teachers who overcome all those obstacles are likely to find themselves becoming long-life-learners, constantly gaining new knowledge and possibly changing their perceptions regarding contemporary mathematics, its history and their integration in mathematics education. Moreover, students' benefit, as we have seen in our study, and as indicated by the partial results presented in the following section, proves these efforts worthwhile.

3.2 Partial Results

Due to space limitation we confine ourselves to results related to Kepler Conjecture MNS followed by some more general results.

Here are a few selected quotes from the responses 9th grade students gave to the feedback-questionnaire administered following the exposition of Kepler Conjecture MNS.

Expressions of Motivation:

- "The story is interesting, I am curious to hear more about what follows …"
- "I was very interested and enjoyed the thrill and the tension in this snapshot…"
- "I would like to know who will finally complete the FPK project! Really I would like to be informed"

Expressions of feeling of understanding:

- "Some parts were quite complicated but following the explanations it became clear- Now I can say I really understood it!"
- "Quite unusual for me to admit I understood and enjoyed maths… even loved it."
- "The 3-D animations and the pyramid helped me understand everything- - in mathematics it is important to see what happens - - it helps to follow the explanations."

Expressions of being impressed:

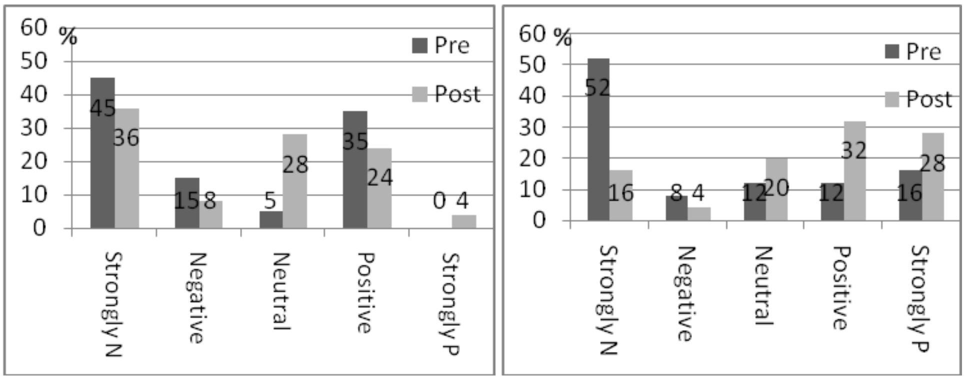
- "One mathematician asks for help from another one and so on: This is new for me that a mathematician needs help from a friend (just like us…)"
- "Harriot seems a clever and interesting man."
- "I was very excited to hear that the proof was accepted."
- "People from all over the world working together – This is some Revolution!"

In general, in all 28 items of the Attitude Questionnaire, a minimal or no change was found in the distribution of the 5 mark between the two administrations of the test in the control group. In the experimental intervention classes, a noticeable shift to the positive side appeared in mostly all the items.

Due to space limitations we confine this paper to results obtained for the three following items only:

(a) Please relate in details to the following statement "Mathematics is a creative profession". Results are summarised in Diagram 4.

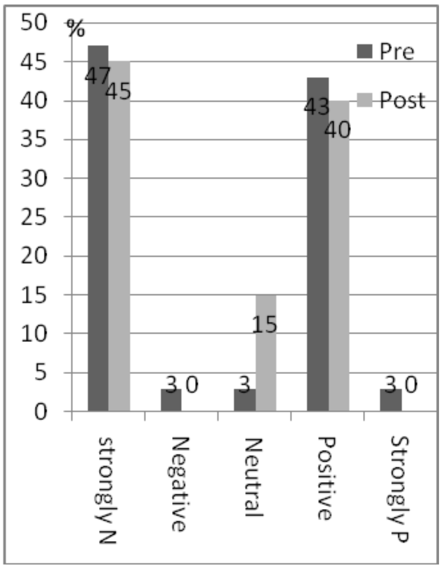
Diagram 4: Distribution of marks for pre-test and post-test responses to item (a) in the control group (left, N=20) and the experimental group (right, N=25)

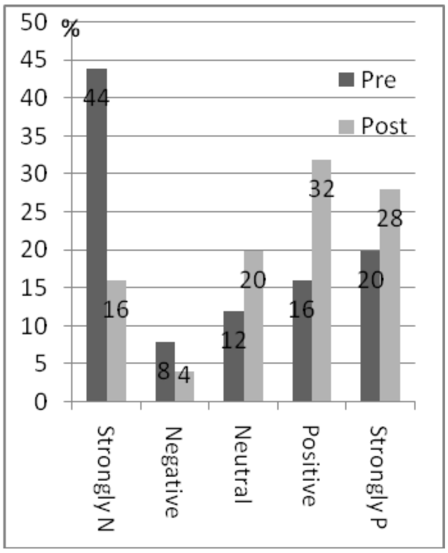


As can be seen from Diagram 4, it so happened that the experimental group started on this item with more strongly negative attitudes (52%) than the control group (45%), and ended up the other way around (16% vs. 36% respectively). On the other end, the experimental group started with more strongly positive attitudes (16%) and this number increased to 28% at the end of the experimental implementation of MNSs, while the strongly positive attitudes on this item of the control group started low (0%) and remained low though a slight increase occurs there too (4%).

(b) What do you find most interesting in school mathematics? Responses are summarised in Diagram 5.

Diagram 5: Distribution of marks for pre-test and post-test responses to item (b) in the control group (left, N=20) and the experimental group (right, N=25)

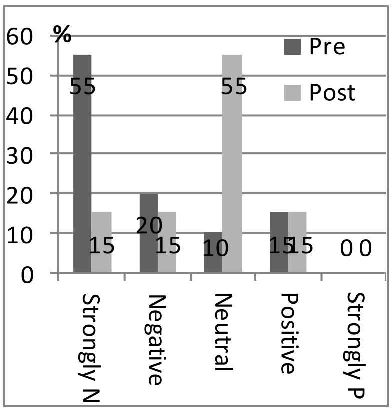


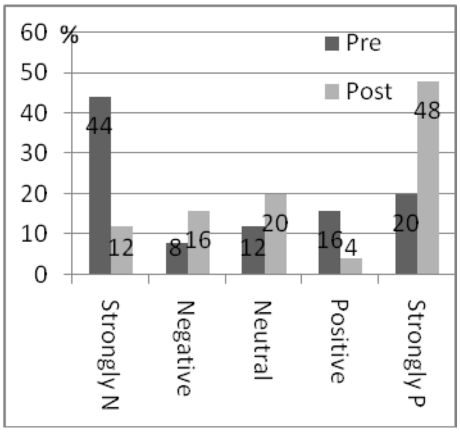


As can be seen from Diagram 5, on this item the two groups started high on strongly negative attitudes but the control group remained almost the same (47% to 45%) while the experimental group decreased largely (44% to 16%). In the rest of the marks there appear also to be very small change in the control group while the change inclined towards the positive side in the experimental group is remarkable.

(c) Would it surprise you to hear that there is nothing new in the field of mathematics? Provide detailed reasons to your answer. Responses are summarised in Diagram 6.

Diagram : Distribution of marks for pre-test and post-test responses to item (c) in the control group (left, N=20) and the experimental group (right, N=25)





As can be seen from Diagram 6, on this item the experimental group started off with 20% students possessing a strongly positive perception. This percent increased to 48% at the end of the experiment. The control group started and ended with no student with a strongly positive oriention and a small percent of positive ones (15%) in both case as well. It is worth noting that the neutral mark in all three items in both groups increased between the pre and the post-tests. This can be explained by the fact that students omitted this question probably as they were tired of the whole questionnaire repeating itself the second time.

We close this section with a few quotes from experimental group students' responses to item (c).

Pre-test	Post-test
• No news because no one is pursuing studies in the field these days. • It is obvious that the entire field is a group of old formulas and drills – so there cannot be any innovations. • In mathematics everything is known - it is the same mathematics all the years. • Mathematics is stagnated in a closed frame and cannot and will not change. • All the answers are known to everything.	• Mathematics is developing all the time. There are many open problems not yet solved, as we saw in the snapshots. • The technology helped many mathematicians to complete proofs and publish new results. Recall the snapshots. • Mathematicians are searching for new solutions- ALL THE TIME • We learnt about many new findings this year. • It is not at all simple yet mathematicians succeed in discover in more and more…

4 Closing Remarks

The proposed pedagogy for introducing mathematics news in high-school so as to decrease the gap between school mathematics and contemporary mathematics assumes that a snapshot is a short intermezzo, taking a small portion of the weekly class-time, and is preferably linked to the curriculum, so that it does not break the flow of teaching the ordinary curriculum. The MNSs developed for this study were not meant to present an in-depth theory, but rather to guide ‘a wander-about’ in the world of contemporary mathematics.

We accept an earlier viewpoint (Movshovitz-Hadar, 1993, 1988, 2006) regarding mathematics education as a meta-mathematical discipline. “...As such, statements about mathematics, with relevance to its learning, its understanding, its use, its teaching and its communication belong to mathematics education” (1993, p. 267). The proof for such meta-mathematical statements is similar to the proof provided by the empirical sciences rather than the common deductive proof in mathematics. In this paper we shared parts of the evidence resulted by our study which support the following general meta-mathematical statement: Employing the pedagogy of interweaving Mathematical-News-Snapshots in high-school mathematics teaching is an efficient strategy for bridging the gap between contemporary mathematics and school mathemtics curriculum. This 'wander-about' in the world of mathematics is intriguing enough to impress students, detailed enough to motivate them to do mathematics, and mind-opening enough to yield a desired image of mathematics as a vivid creative and ever-growing domain.

The reverse engineering analysis of the various MNSs provided eveidence as to a sequel to the former statement: Mathematical-News-Snapshots are deeply rooted in the relevant history of mathematics, the two go hand in hand in leading students to the perception of mathematics as a rich living discipline.

The two assertions are related to a concern expressed during ESU6 by several researchers from various countries, about introducing a historical dimension in mathematics education, and about neglecting the history of mathematics in high school curricula in general and in mathematics textbooks in particular, despite its potential for improving the quality of mathematics education (Tzanakis 2010). Our study contributes towards a solution for this concern. After all, today's news is tomorrow's history.

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