THE HISTORY OF MATHEMATICS IN SCHOOL TEXTBOOKS

Panel Discussion[1]

Anne BOYÉ, Université de Nantes, France cenub@club-internet.fr
Adriano DEMATTÈ, University of Genova, Italy, dematte.adriano@vivoscuola.it
Ewa LAKOMA, Military Univ. of Technology Warsaw, Poland, ewa.lakoma@neostrada.pl
Constantinos TZANAKIS[2], Univ. of Crete, Rethymnon, Greece, tzanakis@edc.uoc.gr

ABSTRACT

Given that on an international scale, there is a growing research interest in the introduction of a historical dimension in mathematics education and actual implementations in the classroom, textbooks and curricula, this 90-minute panel discussion made focus on the following issues:
(a) *The current situation*: What is the international experience on the inclusion of the History of Mathematics (HM) in school textbooks?
(b) *Classification of possible ways to include the HM in school textbooks*: What are the pros and cons, the aims and the methods of the different ways this could be done, or has actually been done?
(c) *Quality criteria, prerequisites and aims of integrating the HM in school textbooks*. Do they exist? Have they been taken into account in writing textbooks that have a historical dimension?

1 Introduction

There is an increasing interest to integrate history into Mathematics Education (ME): Curricula, classroom, books, didactic material, homework, teachers' training. The panel discussion was confined to the introduction of a historical dimension in **textbooks**, this term being understood in a wide sense to include non-conventional forms; e.g. based on ICT. In many countries, there are specific regulations in the mathematics curricula on what this integration really means and how it should be implemented. Some examples were presented, focusing on primary and secondary education (but the discussion touched upon university education and teachers' training, as well).

A short presentation of typical examples in four different countries was given (France, Italy, Poland, Greece; see section 2) and participants were welcome to provide further information based on their own experience, or the situation in their own country, either beforehand and/or on the spot; their contribution was distributed via e-mail to all participants before the Summer University (cf. sections 2 & 3). This helped very much those participants who wished to actively contribute to the discussion.

After a brief introduction to the subject by the coordinator, a short presentation by each panelist along the panel's main themes was given, based on the case of the panelist's country. The follow-up discussion was based on questions among the panelists and the audience and comments made by participants with experience on the subject, who were invited to provide further information (some of them have already sent their comments in advance, as mentioned above). The session ended with the coordinator's summary of the information, opinions and suggestions put forward during the discussion.

[1] Contributions from other participants are explicitly acknowledged at the appropriate place in the text.
[2] Coordinator.

2. The history of mathematics in school textbooks: A short presentation

Each panelist presented the situation in his/her country briefly, illustrating the main points by means of specific examples. In response, to this, as well as, to the documents circulated to all participants, further short presentations came in, illustrating the situation in other countries. Due to space limitations, this section summarizes all these contributions, without details on specific examples. The interested reader is referred to the literature below.

2.1 France[3]

Currently, there is a new math curriculum from the beginning of the primary school, to the end of the " lycée", implemented in primary and middle school (11-14) and the first year of the "lycée" (15-16 year old students). It implies important changes in the math content, especially in secondary school. For instance, an important new item is on "algorithms and programming". School mathematics now has to be useful in modern everyday life and be linked to other scientific disciplines, such as physics, biology, economics, technology, or even the humanities (such as the teaching of history of art in the middle school).

However, there is only a brief reference to the introduction of the HM in the middle school's curriculum: " *it's an opportunity to approach the history of sciences, especially when introducing the notions of number π, square roots, … and measures*"; and there is nothing about the HM in the new curriculum for the first year of the lycée. Very recently, a few comments have been added about the interest in introducing a historical perspective, in the mathematics curriculum for the last two years of the lycée.

Nevertheless, more and more math teachers have recently begun to be interested in using HM, mainly through the work of the IREMs (see however Barbin 2010). But, from now on, the new math teachers will have a compulsory initiation to the history and epistemology of mathematics at the University. Meanwhile, there is a sort of popular infatuation with the HM (even fictionalised), found in films, novels etc. However, it seems that among algorithms, programming, working with other subjects etc, there is no more time for the HM in the French school curriculum. In conclusion, there is no obligation to introduce the HM into teaching; it is left to the teacher's discretion, even if the HM may appear as an "added value". What is encouraged, is to give a historical perspective in special interdisciplinary optional modules, which is a very different point, of course.

In any case, however, one may seriously doubt that the HM is the first criterion of a team of teachers to choose a textbook for their school. Hence, the role of the HM in textbooks depends on the authors; they do as they wish. We have studied most of the secondary math textbooks that are currently used in math classes in France.

In many cases, you can find nothing on history, or only one, or two "pseudo-historical exercises", e.g.: "*Archimedes is a famous ancient Greek mathematician. He was born in Syracuse in 287 BC. and died in 212 BC. How many years did he live?* This is pompously entitled: math and history! The only interest in this exercise is mentioning Archimedes' name. Some other exercises, or activities, have only a superficial historical covering and can be misleading if the teacher is not well informed. But fortunately, in most recent math textbooks there is, usually, more correct historical information, than it used to be; mainly historical notes, introductions, or activities for further study in mathematics. There are also suggestions for homework, especially in middle school. Sometimes, some very interesting

[3] By A. Boyé.

activities can be found, based on original sources. But in fact, they are difficult for a pupil to study alone, and for a teacher with no knowledge of this history to supervise his students.

So, investigating the way the HM is included in school textbooks, the following questions arise in most cases: How have these history-based elements to be used? Just left to the pupils' discretion to read only if they are interested in? To be worked with the teacher in the classroom? In that case, is there any additional resource material to support the teacher's intervention? Are these elements always pertinent to the introduction of a historical and cultural dimension in teaching mathematics? Maybe, they enlighten math teaching, but do they modify the pupils' perception of mathematics? In particular, do they humanize mathematics?

Most teachers will need additional resource material to support their courses, which is often unavailable. Still, if there was no history in math textbooks (as it is often the case), it is likely that the HM would have almost completely banished from math courses in France. At least, teachers and pupils currently have opportunities to ask questions and try to get more information.

Finally, the new technologies provide new opportunities to improve the use of the HM in textbooks. The recent school textbooks propose additional material on the web, for teachers and pupils. Usually, it consists of the solutions of exercises and nothing on the HM. But in one exception, there is a historical timeline from ancient times to nowadays[4], with mathematicians' names all along the timeline, linked to short biographies, the chapters concerned in the textbook, and more information. It is certainly an incitement for using HM, directly linked to the textbook used in the classroom. It is neither closed, nor rigid, so that, even if someone is not well informed on the history of a specific subject one wants to approach, may be, he will find here what he needs. This possibility has to be further explored more thoroughly.

Due to lack of space, we cannot present in detail our analysis of the issues raised above. It has been based on several French contemporary textbooks for primary and secondary school, with an additional brief comparison with the 1960-1970 textbooks, and some of the beginning of the XXth century and made focus on how the existence of historical elements in the mathematics textbooks is influenced by recent fashionable ideas, the curricula's incentive, and in another way, by the IREMs' work.

2.2 Italy[5]

Analyzing Italian textbooks, we can find two fundamental ways of using the HM:

- doing mathematics; i.e. operating for developing mathematical skills, abilities, competencies etc
- reading, in order to know something about mathematics in the past

With respect to the HM, the point of view of teachers and publishers are often the same. In my opinion, this fact is important to understand why the HM appears in school textbooks seldom. Several objections can be raised (summarized in Siu 2006), though I would like to emphasize that teachers consider the HM as a valuable resource; e.g., my colleagues consider the HM an interesting topic, "a good stuff". Some of them read books, collect articles from magazines, and sometimes use the HM with their students: narrations about evolution of a concept (e.g. π), further reading (e.g. biographies), or visiting exhibitions (e.g. in city museums). Publishers sometimes enrich the book with colours, insert images (historical pictures, which are unusual in mathematics texts), historical snippets, etc. to make the

[4] www.repere-prof.hachette-education.com
[5] By A. Demattè.

textbooks more attractive. In summary, both teachers and publishers consider the HM "something more", a way to attract students' attention. Through their agents, Italian publishers contact teachers, who either alone, or in groups choose textbooks for their classes, which almost certainly are approved by the teachers' assembly. Of course, publishers are interested in taking into account teachers' preferences. Hence, although history is "feebly present" both in professional choices of mathematics teachers and in textbooks, it is a potential resource to improve pedagogy of mathematics and enrich textbooks.

Not long ago, some Italian textbooks for upper secondary school, authored by very good mathematicians and researchers in pedagogy of mathematics, included a lot of history. Currently, one of the most popular textbooks is Bergamini et al 2008. Its last edition contains some pages named EXPLORATION. These pages finish with ACTIVITIES, requesting for instance, that students search in the web the history of the *Arithmetic triangle* by using keywords like: Tartaglia's triangle, Pascal's triangle, arithmetic triangle, binomial coefficients. This search in the web is present in every EXPLORATION (see Appendix). However, not all secondary school mathematics textbook contain HM. Some just mention a few words at the beginning of some chapters.

HM is only for reading? In teachers and publishers' view the answer seems to be in the affirmative, which, in my opinion, raises an obstacle for making the HM part of the class: Reading (or speaking) about the HM requires additional time and the teacher either has to subtract it from activities for learning new mathematical concepts or, require from the students more time for homework. Both alternatives are difficult indeed!

On May, 26th 2010 the new "*National directions about specific learning aims*" by the Ministry of Education. For each school subject of the *Scientific Lyceum* (upper secondary school; 14-19 year-old students), "General suggestions and competencies" have been published[6]. It is remarkable that, with respect to mathematics, by means of "copy and paste" of some parts, the program of other types of Lyceums is obtained. Other "pedagogical recommendations" are in the ministerial document; e.g.

> "[the student] is expected to be able to insert different mathematical theories in the historical context, where they have been developed, and he is expected to understand their meaning", or
>
> "The most important moments must be remembered: Greek mathematics, Calculus in 1600, Enlightenment, rationalism and modern mathematics, […] the meaning of postulates, axioms, definitions, theorems, proof […], Euclid's *Elements* in Western Mathematics […], the Euclidean approach should not be only an axiomatic system […] Ruler and compass: their historical meaning in Euclidean geometry […]. The concepts of continuity, differentiability, integrability and historical problems regarding instantaneous speed, tangent straight line, areas, and volumes".

In summary, we can say that although the inclusion of history in Italian mathematics textbooks has a long tradition, it is not common, any longer. The recent national regulations contain suggestions that could serve as important starting points to develop activities focused on the HM as a powerful pedagogical resource and an integral part of textbooks, too.

2.3 Poland[7]

In the last decade, there has been an important change in Poland, in the system of general

[6] http://www.indire.it/lucabas/lkmw_file/licei2010///decreto_Indicazioni_nazionali%20_26_05.pdf
[7] By E. Lakoma.

education for pupils of 7 to 19 years old. The old system "8+4" was progressively replaced by the system "6+3+3": 6 grades in elementary school, followed by 3 grades in gymnasium and finally 3 grades in lyceum. These changes are accompanied by a new system of assessment, applied at the end of each level of education, in the form of a general examination for all students at a given level of education, the evaluation being done by an external institution, according to the regulations of the ministry of education.

The main idea of the new educational changes was to place general education within the framework of the so-called key competencies, which are developed through the realization of the curriculum basis for general education, established by the ministry of education. This curriculum basis includes for each educational level a list of essential mathematical skills to develop and mathematical notions to form, which are considered to be necessary from the point of view of general education and developing the key competencies.

As far as ME is concerned, in this document the HM is absent. However, the HM in ME can be present at the level of the implementation of the curriculum basis.

Looking at the contemporary curriculum proposals for school mathematics in Poland, one can notice that most of them include a fairly superficial attention to history. Similarly, the textbooks proposed to support these curricula tend to include at most a few biographical notes and some rather basic historical information. However, among many proposals for the mathematics curriculum, there is a project called Mathematics 2001, which includes textbooks and other didactical material, with relatively more on mathematics history than other proposals. This project uses the HM as a starting point for didactical situations which can be interesting for pupils and a source of original simple reasoning, readily understandable and helpful for today's pupils.

By presenting some examples taken from textbooks and other didactical material associated with them, the most important issues concerning the role of the HM in mathematics learning at all levels of education, can be discussed from the point of view of both the learner and the teacher.

2.4 Greece[8]

It is the first time in Greece that the (new) mathematics curriculum for compulsory education (officially announced in 2002 and implemented via newly written textbooks since 2007 (Greek Pedagogical Institute 2002) includes many and extensive references to the didactical integration of the HM into the teaching of mathematics. These references vary from the specific teaching objectives, to the didactical methodology and the textbook content. Besides the usual historical snippets, there are many activities of historical content, aiming to provide teaching tools for better understanding the textbooks' mathematical content. However, a critical reading of this historical material reveals the existence of serious errors, obscurities or omissions in the historical comments, which make their use very questionable. Indicative examples can be given to support this conclusion (see Thomaidis & Tzanakis 2010).

In addition, although the official guidelines given by the Greek Ministry of Education follow what didactical research seems to suggest nowadays and emphasize the important role HM can play in ME, their actual classroom implementation is far from being satisfactory. Besides the weaknesses mentioned above, teachers feel unready to follow the guidelines, given that there is no (pre-service, or in-service) training for that and no

[8] By C. Tzanakis.

appropriate accessible resources.

2.5 Denmark[9]

The new reform for the Danish High School system includes history in the mathematics curriculum. To implement the new curriculum, different groups of math teachers write textbook sets that cover the curriculum; HM is treated differently in each set. Each school buys the textbook set(s) it likes, or can afford. In one case, "guest authors" were invited as specialists to contribute to different issues; one of these contributions was a small history chapter[10]. These contributions by "guest authors" are printed in colour to indicate that they are not part of the ongoing presentation of the mathematical subjects treated in the book. In such chapters it is possible to give an account of the development of pieces of mathematics at a historically fairly high level. Associated with a chapter on geometry and trigonometry, there is a chapter on the history of surveying and mapping, treating also the history of the mapping of Denmark in the 18th century and how this led to Wessel's geometrical interpretation of complex numbers. The interaction between practical problem solving, the mathematical tools, and insights into new mathematics are discussed and presented in this 6-page chapter, which is also used by the textbook authors as a suggestion for project work, enriched with mathematical and historical tasks. In this way, it is possible to present a few case studies that in some sense are representative for different aspects of the HM, in a way that is both historically sound and connected to the mathematics of the textbook.

2.6 Israel[11]

An interesting non-conventional new idea is implemented in Israel; namely, to connect the curriculum to some contemporary pieces of mathematics and associated problems dealt with by the mathematics community, and their historical background. A study in the format of an action research focusing on interweaving contemporary mathematical news with their historical background in secondary school curriculum has been conducted over the past 3 years. Parts of the study and some of its results were presented in a workshop (see ch.2.6, this volume).

The key idea is to provide teachers with an on-line access to PowerPoint presentations, each of 15 minutes duration of some *math-news-snapshot* to be implemented once a fortnight in the ordinary teaching. This is in line with the tendency to adapt textbooks to on-line books and the use of ICT in general.

2.7 United Kingdom[12]

In the Curriculum for England and Wales (published September 2008), one of the "Key Concepts" is the *Applications and Implications of Mathematics* and here we find that school pupils should be "Recognizing the rich historical and cultural roots of mathematics." This statement is *statutory* (i.e. it is the law) but no rules or guidance are given in this curriculum document as to how this statement may be put into practice. A similar situation exists in Scotland.

---

[9] By T.H. Kjeldsen (Roskilde University, Denmark)

[10] The same was done with other mathematical subjects as well; e.g. critique and meta reflections on the use of statistics etc. Professionals are invited to write these chapters, i.e. the history chapter was written by a historian of mathematics, the statistics chapter by a statistician and so on.

[11] By Nitsa Movshovitz-Hadar & Batya Amit (Israel Institute of Technology, Haifa, Israel).

[12] By L. Rogers (University of Oxford, UK).

The integration of aspects of the HM into any curriculum requires careful consideration of the national characteristics and 'millieu' of the educational system, the philosophy (or lack of it) of the curriculum-builders, the political and economic contexts prevailing, and of the system of both pre-service and in-service teacher education and training. Borrowing 'ready-made' (or translated) materials from other nationalities is often counter-productive, and pays no attention either to national characteristics, opportunities, needs, local heroes (or villains), or to the particular social opportunities that may present themselves.

New media are available over which governments, official curriculum builders and textbook writers have little or no control, and this provides many opportunities for good (and bad) material to be made available. In the classroom, the teacher is finally responsible and can choose whether or not to pay attention to historical material in printed books. Hence, it is possible to use the new media to make available to teachers appropriately resourced materials that have a clear link with items in their school curriculum, and materials for pupils that show how mathematics is the foundation of, is integral to, and inspires many aspects of our cultural history. Maybe, such "non-conventional" forms of "textbook" material is becoming the 'norm', being more directly accessible to the pupils themselves!

Indicative examples can be given:

- '*Episodes*' with historical notes, references and pedagogical notes and questions for teachers on the NRICH website - a part of the Millennium Mathematics Project (based at Cambridge University); e.g.:

The Development of Algebra 1, http://nrich.maths.org/6485

The Development of Algebra 2, http://nrich.maths.org/6546

- The "*History Corner*" talks on the interactive website of the Association of Teachers of Mathematics (ATM)

For the most recent see:

'Root-two' and irrational numbers, http://www.atm.org.uk/mti/218/root-two.html

3 The main themes in more detail

Both the panelists and several participants were involved in the discussion during the panel session and beforehand via e-mail, and raised several issues pertinent to the main themes of the discussion (see Abstract above). This section summarizes their contributions[13], which have been codified for the sake of brevity and easy reference.

3.1 Classification of possible ways to include the HM in school textbooks

There are many different ways to conceive the integration of the HM in textbooks, some of which have been implemented in several countries, worldwide. Below is a list, divided into groups, which is not exhaustive of course; a similar list analyzed in detail can be found in El Idrissi 2006.

(a_1) Separate chapters, disconnected from the main text: Introductions, Epilogues, Appendices, List of historical remarks etc.

(a_2) History appears as a natural, integral and explicit part of the textbook (see e.g. Hairer & Wanner 1996, Toeplitz 1963)[14].

[13]We acknowledge valuable contributions to this section by (in alphabetical order): B. Amit & N. Movshovitz (Israel), F. Bevilacqua (Italy), T. Kjeldsen (Denmark), L. Rogers (UK), B. Smestad (Norway).

[14]Another tentative attempt by B. Smestad (written in Norwegian) can be accessed at http://home.hio.no/~bjorsme/Sannsynlighetsregning3.pdf

(a_3) History permeates the textbooks, usually integrated implicitly (see e.g. Stillwell 1989).

(b_1) Historical "*snippets*" (see Fauvel & van Maanen 2000, §7.4.1), loosely connected to the main text & course objectives: historical notes with factual information and associated activities, usually in a different format & colors from the main text, possibly with pictures and inserted in a box.

(b_2) Presentation, comments & interpretation of images from books, or original paintings, photographs etc[15].

(c_1) (Short) excerpts from original documents, to introduce a topic, possibly with comments that would help the reader to grasp better the mathematical content, both in its modern form and in its original context.

(c_2) (Worksheets with) exercises, recreational problems and games based on original texts; suggestions for well-defined, optional short projects for further work at home and/or the classroom (including e-sources & the web).

(d_1) Suggestions for further reading at the end of each chapter possibly with historical notes and/or annotated bibliography.

(d_2) A guide to the literature (books, journals, websites) for further reading, research and insights.

(e_1) Connections with some contemporary piece of mathematics and its history: Hints to recent developments, related to outstanding old issues/problems (e.g. recently proved theorems, conjectures, unsolved for a long time and the efforts to answer, solve, or prove them that stimulated important developments), or constituting good examples that can fascinate students, at the same time making clear that mathematics is a continuously evolving science.

(e_2) Presentation, discussion and analysis of errors made in the past and led, or motivated new developments.

(e_3) Links to other sciences (especially those that rely heavily on mathematics) and the arts.

3.2 Quality criteria, prerequisites and aims of integrating the HM in school textbooks

Clearly, the possibilities mentioned above, should serve specific aims, satisfy some minimum quality criteria and have certain prerequisites. Otherwise they produce no positive result. Below is a summary of all these that emerged through individual contributions to and during the panel discussion.

3.2.1 Aims

Historical material in textbooks could serve a variety of different aims, both on inner issues of mathematics and meta-issues (cf. El Idrissi 2006, §1):

(A) *Inner issues of mathematics*

(A_1) To contextualize mathematical concepts.

(A_2) To motivate students go deeper into mathematics and look for more details, through the study of sense-making situations of specific mathematical concepts and methods.

(A_3) To serve as recreational activities.

(A_4) To link school mathematics to mathematics in-the-making, or current new findings & trends in research.

(B) *Meta-issues of mathematics*

[15]In this connection one could add the cover of the text book and/or its title; see El Idrissi 2006 §§2.1, 2.2)

(B_1) To stress and highlight the evolutionary nature of mathematics.

(B_2) To modify students conception of mathematics.

(B_3) To portray mathematicians as creative human beings and more generally, to humanize the mathematics courses.

(B_4) To connect mathematics to other disciplines and the general historical & cultural milieu, hence to help students develop an interdisciplinary understanding of mathematics.

(B_5) To stimulate collaboration of students and teachers in interdisciplinary activities.

3.2.2 Quality criteria

Any implementation of HM in textbooks should satisfy some reasonable, necessary quality requirements:

(a) It should be based on mathematically & historically correct information.

(b) It should serve the objectives of the teaching unit, where the historical material is incorporated.

(c) It should be supported by additional material to help the teacher: extensive bibliography; resource material; specific hints on what the historical material is didactically good for etc.

(d) It should definitely be in harmony with students' capability.

(e) It should avoid "pseudo historical" exercises, or problems.

3.2.3 Prerequisites

Historical elements should constitute hints to motivate and stimulate students & teachers' interest. Hence, it is necessary:

(a) To develop teachers' training on how to profit from the historical elements and to use original texts.

(b) To provide extensive bibliography for further reading by the teacher, suggestions for further optional reading by the student, to look for more details in case he/she wishes so.

(c) To have access to the resource material.

4 Some critical points raised in the discussion

In the previous two sections, examples of the current situation in some countries were presented and concise lists related to the main themes of the panel discussion were given. However, additional points were raised during the discussion, or were communicated beforehand, in the form of *observations*, *claims*, *judgments* and *suggestions of how to make a historical dimension effective and attractive*. They are summarized in this section. We thank all participants, who contributed to enrich the discussion and we explicitly acknowledge their contribution. However, because of space limitations, they are presented as a list, in the hope that they provide insights, which could inspire further work.

4.1 *Observations*

(1) It seems that no real differences among countries are apparent. This concerns mathematics textbooks and their use by schoolteachers, not official regulations (A. Dematтè, Italy).

(2) When there are official regulations in which HM is integrated, then it is not possible to avoid it in ME (T. Kjeldsen, Denmark).

(3) It seems that the two most serious and persisting obstacles against the integration of the HM in teaching, and in textbooks in particular are the lack of time in the classroom and examination constraints; their influence depends on the school system of course (C. Tzanakis, Greece)

4.2 *Claims*:

(1) Many teachers and educators are unsatisfied by math history pills dropped here and there in the textbooks. It seems that a main reason for that is that in this way, the teacher and student are put in the position of a **spectator**, which gets them bored, "….*because math is not a sport that you can follow as a spectator. If you want to appreciate it, you need to practice it*"[16] (F. Bevilacqua, Italy)

(2) The original sources are difficult to understand and the personal stories of great mathematicians are not mathematics. It seems that the HM is something only teachers can turn into a didactic success. Therefore what we really need are separate texts for teachers and appropriate training, showing them what a powerful tool HM can be in the development of various precious goals (F. Bevilacqua, Italy)

(3) Not all historical examples and cases are suitable to develop didactical material for the classroom. Appropriate choice is crucial (F. Metin, France).

4.3 *Judgments & critical issues*

(1) A more in depth analysis of teachers' educational aims and their arguments against using the HM in classroom is necessary (cf. Siu 2006), before writing historically oriented (parts of) textbooks and designing appropriate training. (A. Demattè, Italy).

(2) It is important that teachers should be **motivated** to integrate the historical aspects of the textbooks and HM in general in the classroom; otherwise, they will not do anything in this direction (I. Mavrommati, Greece).

(3) It is important that secondary school teachers are competent in the HM. Therefore, this should form a compulsory part of their undergraduate studies, or in-service training. Unfortunately, this is not the case; university mathematics programs usually do not include history and epistemology courses; pure mathematicians do not consider this as a valuable ingredient of a mathematician's undergraduate studies (R. Bebbouchi, Algeria).

(4) Very often, epistemological issues are implicit or explicit to any historical material. Therefore, it is necessary that the historical material in a textbook is accompanied by epistemological comments for the teacher, for which teachers are usually unaware. This raises the important issue of teachers' training in this area. In general, HM in textbooks should also make focus on epistemological issues as well; it should be conceptual history as well (C. Vicentini, Italy).

4.4 *Suggestions*

(1) What initiatives could the HPM Group undertake to promote the inclusion of the HM in mathematics textbooks? Is it possible that at the end of each meeting, a "manifesto" be translated into different languages and be sent to publishers (and/or universities, research institutions, regional school administrations etcl)? During a Conference, could a panel discussion be devoted to a very specific subject as means to share ideas on writing a hypothetical chapter of a textbook (e.g. on the "Introduction to algebraic manipulation through the HM")? (A. Demattè, Italy).

(2) It necessary to suggest concrete changes in the mathematics curricula and struggle for that (I. Dias, Portugal).

(3) Distinguishing and presenting some appropriately chosen cases as generic examples of how the HM in textbooks could be used in school practice, would be helpful for the teachers in their class work (E. Lakoma, Poland).

(4) Teachers' training in using the historical aspects of textbooks is essential. In this

[16]Gaurav Suri & Hartosh Singh Bal, *A Certain Ambiguity: A Mathematical Novel*, (quoted from the Italian edition, Ponte alle Grazie, 2007, pp.25-26.

connection, small “pieces” of historical material should be used first. (A. Boyé, France)

REFERENCES

- Barbin E. (ed), 2010, *De grands défis mathématiques d’Euclide à Condorcet*, Vuibert, Adapt SNES, France.
- Bergamini, Trifone & Barozzi, 2008, *Manuale di Algebra*-3thedition, Zanichelli, Bologna.
- El Idrissi A., 2006, “L’histoire des mathématiques dans les manuels scolaires”, in *Actes du Colloque EMF* 2006, Faculté d’Éducation, Université de Sherbrooke, Canada
- Fauvel, J., van Maanen, J. (eds.), 2000, *History in Mathematics Education: The ICMI Study*, Dordrecht-Boston-London: Kluwer.
- Greek Pedagogical Institute, 2002, *Cross-Thematic Integrated Curricula of Compulsory Education*, vol. A’, Athens, (in Greek).
- Hairer, E., Wanner, G., *Analysis by its History*, New York: Springer 1996.
- Siu, M-K. 2006, “No, I don’t use history of mathematics in my class. Why?”, in F. Furinghetti, S. Kaijer & C. Tzanakis (eds.), Proceedings of HPM 2004 & ESU 4 (revised edition), University of Crete, Greece, pp.268-277.
- Stillwell, J., *Mathematics and its History*, New York: Springer 1989.
- Thomaidis Y. & Tzanakis C., 2010, “The implementation of the history of mathematics in the new curriculum and textbooks in Greek secondary education” in V. Durand-Guerrier, S. Soury-Lavergne, F. Arzarello (eds) *Proceedings of CERME 6*, Lyon, France, pp.2801-2810.
- Toeplitz, O., *Calculus: A genetic approach*, Chicago: Chicago University Press 1963.

APPENDIX[17]

Recently, I wrote an e-mail to the authors of one of the above mentioned books (Bergamini et al 2008). I introduced myself and this panel discussion and asked for “…. some **motivations** of this choice, as well as the pedagogical **values** you assign to the HM. It could be a valuable contribution, as author of the book, to let us know the virtues HM could have for school textbooks”. Prof. Bergamini answered me, listing some pedagogical “opportunities” and highlighting “methodology”:

“*Opportunities*

-Mathematics contributes in human thought evolution. For instance: Mathematics and democracy (deductive method, proof and birth of democracy in Greek “polis”)

-Mathematics is a cultural building, which is therefore slow, laborious, but powerful, e.g.: Syncopated algebra; from words to algebraic symbols

-Mathematics is not only western. e.g.: The father of polynomials – Al-Karaji’s contribution to algebra

-Mathematics is made by men, with their passions, rivalries, and will to prevail, e.g.: Tartaglia, Cardan and their challenges

-Mathematics contributes to technological growth and is connected with other subjects, also from the historical point of view, e.g.: Triangles on doors, “download triangles” in architecture, Lions’ door in Mycenae.……

[Methodology] *We propose a very short introduction, which give essential information and, after this, hints for deepening and researching; the Internet is a starting point. In our opinion, this approach is better than writing long historical pages to attempt complete explanations. Research and interpretation of information look like correct ways to motivate, and understand. This methodology inspires all our boxes named ‘Explorations’*”.

[17] By A. Demattè.