THE ROLE OF THE HISTORY AND EPISTEMOLOGY OF MATHEMATICS IN TEACHERS TRAINING

PANEL DISCUSSION

INTRODUCTION

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The theme of "History of mathematics in pre and in-service teacher training" has already been established as one of the six topics of the 1st European Summer University in Montpellier (France) where one panel and many workshops were held. The Proceedings of this meeting contain 7 papers on this theme (IREM, 1993, pp.451-496). Five years later, it was also the subject of chapter 4 of the ICMI Study (Fauvel & van Maanen, 2000). The current contribution will offer the possibility to study evolution of ideas on the theme, as well as some institutional changes. If we are not able to give a complete outline of the situation in each country from which our members come, it is clear that the possibility of communication among the participants in our different meetings is important for both encouragement, and as a way of communication, in order to compare and exchange thoughts and information on projects we are involved with.

The panel was organized to enable 4 communications concerning 4 countries: Italy, Norway, England and France. In order to have a more fruitful interaction, the four participants had to answer the same 6 questions in total, divided into three groups. The aim of the first group was to situate the purpose of investigation in an institutional frame for each country:

1. How is teacher training conceived in your country (pre and in-service teacher training)?
2. What is the official place of the history and epistemology of mathematics in this training?

The following group concerned the theoretical approach of each participant:

1. What are your conceptions about the role played by history and epistemology of mathematics in teacher training? At what level? What goals can you identify?
2. What purposes and what kind of (theoretical or methodological) approaches and practices can you tell us about, that you have witnessed in your country?

The last group was devoted to practices and products with one or two concrete examples where each participant was involved:

1. Were you successful in putting into practice your conceptions and goals?
2. Have you new perspectives for future?

The four participants of the panel were:

-Fulvia Furinghetti (Italy) teaches didactics of mathematics to students in mathematics. She is a member of the Italian Commission of Mathematical Instruction (sub-commission of the Italian Mathematical Union). Fulvia was the HPM chair during 2000-2004.

-Bjørn Smestad (Norway) teaches mathematics to future teachers in primary and lower secondary schools and is the editor of the HPM Newsletter.

-Snezana Lawrence (England) teaches mathematics education to the students preparing to teach in secondary schools. She is the Education Officer for the British Society for the History of Mathematics.

-Evelyne Barbin (France) teaches history of mathematics to future professors of primary

schools and she is co-president of the inter-IREM Committee “Epistemology and History of Mathematics” which organize teachers’ trainings for all the levels.

I - TEACHERS AND HISTORY OF MATHEMATICS IN ITALY

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1 Pre-service and in-service mathematics teacher education in Italy

1.1 Pre-service teacher education

The Italian system of education is centralized. In the recent past there have been a few changes in respect to the situation described in Furinghetti (1998). At present teachers of all school subjects must have a university degree (*Laurea*) obtained according to the 3+2 schema, with significant differences in the curriculum, which depend on the school levels where prospective teachers will teach.

Primary teachers teach mathematics together with other subjects. They obtain the degree in educational departments. In their curriculum there are no courses in the history of mathematics, but in some universities elements of the history of mathematics are introduced in courses on mathematics education.

Secondary teachers obtain their degree in departments as follows. For mathematics the degree for lower secondary teachers (students aged 11-14) has to be in science, or in mathematics, physics, or chemistry. The majority of mathematics teachers in lower secondary school have the degree in science; their university curriculum encompasses at most two mathematics courses and two physics courses. The degree of upper secondary mathematics teachers (students aged 14-19) has to be in mathematics or physics; degrees in other fields such as engineering, statistic, computer science or economics are allowed in certain cases.

In this paper I focus on some features of the curriculum aimed at undergraduate studies in mathematics: this is the only curriculum which may encompass the history of mathematics, while the curriculum aimed at undergraduate studies in physics, science etc. does not encompass courses on the history of mathematics. The mathematics teacher education carried out in mathematics departments may be considered a happy exception in the general panorama of teacher education in Italy, because there is a tradition of interest for primary and secondary teaching inside the community of mathematicians. This tradition, together with others factors, has fostered the development of ‘mathematics education’ as an academic discipline. As a matter of fact, for about the last 50 years, the curriculum of mathematics in Italian universities encompasses special courses addressed to prospective teachers (different in various universities). Possible contents of these courses are:

- critical presentation of mathematical topics linked to mathematics teaching
- foundation of mathematics
- didactics of mathematics
- mathematical laboratories
- history of mathematics.

It is possible that a graduate is appointed as a teacher without a preliminary period of practice in

the classroom; this may happen for any school subject. It is then, a valid concern of mathematics departments to provide courses aimed at these future professionals.

History of mathematics is taught in most universities. Drawing from my experience I would say that usually students like history and sometimes ask to write their dissertation of Laurea on this subject.

1.2 In-service teacher education

There is no regulation or a national plan for in-service teacher education in mathematics, as is indeed the case for other school subjects. There are scattered national initiatives by the Ministry of Instruction and local initiatives organized by groups of teachers, schools, and mathematics teacher associations. In these activities there are sometimes slots dedicated to the history of mathematics.

Because the university courses on the history of mathematics are not compulsory, there are teachers who never encountered the subject in their educational path. Nevertheless, I often met teachers who are attracted by history. At present teachers' interest in history comes not only because, perhaps, they have a personal interest, but because also the guidelines of the national mathematics programs issued by the Ministry of Instruction in 2010 mention explicitly the historical perspective in mathematics teaching and learning. As a consequence, the problem of having some knowledge on the history of mathematics is not only cultural but also professional, as teachers must have the knowledge suitable to cope with the requests of the Ministry.

The ways in which those teachers who did not attended university courses on the history can become acquainted with historical facts are:

- from the historical notes in the textbooks (see Demattè's contribution to the panel *The history of mathematics in school textbooks* in this volume)
- through the local initiatives of re-training courses
- and by reading the books on popularization of the subject.

It happens that after a rather casual approach teachers may feel encouraged to deepen their historical knowledge. Until a few years ago the access to original sources was a problem, because in Italy only a very few 'readers' have been published and original volumes are accessible with difficulties. The availability of important works on the web has made possible the contact with original sources.

2 History of mathematics in teacher education: theory and practice

My approach to the use of history in teacher education has been described in (Furinghetti, 2007; Jahnke et al., 2000). Here I recall the main points and afterwards I briefly describe an example of realization.

2.1 Theoretical frame

My students are students in mathematics whose university curriculum is aimed at teaching. They have beliefs that have been elaborated mainly on the ground of school experience. Among other issues, the beliefs that are of concern are:

- the nature of mathematics
- self as a learner
- self as a teacher
- and the process of teaching and learning.

The importance of prospective teachers' beliefs is due to the influence they have on the future teachers' choices. In this context Megan Frank (1990, p. 12) wrote: "Teachers teach the way they have been taught". This opinion is an old one. Similar ideas are present in Klein's Introduction (1911). According to Klein, when novice teachers have to decide what to do in the classroom they put aside what they learnt at university and go back to their mathematical culture as it was built during secondary school: this fact originates a "doppelte Diskontinuität (p. 2)" (double discontinuity). In the same vein Émile Borel (1907) wrote (p. 657) "[…] une des raisons pour lesquelles l'enseignement secondaire se perfectionne lentement, c'est que l'enseignement que l'on donne ne peut pas différer beaucoup de celui qu'on a reçu" [one of the reasons that make changes in secondary teaching so slow is that what we teach can not differ so much from what we have received]. Gino Loria (1933) put forwards similar ideas in his report on the ICMI inquiry about teacher education around the world.

In my courses for prospective teachers I make my students challenge their existing beliefs for fostering flexibility and openness to different positions. I try to make them reflective practitioners that are able to learn from their practice, as advocated by Jaworski (2006). History, if appropriately used, creates an environment suitable to reflection, because, as (Jahnke et al., 2000) put it:

- Integrating history in mathematics challenges one's perceptions through making the familiar unfamiliar
- The history of mathematics has the virtue of 'astonishing with what comes of itself'
- To walk in the foreign and unknown landscape provided by history forces us to look around in a different manner and brings to light elements which otherwise would escape.

2.2 An example of realization

Algebra is a typical topic in which the gap between what is done in a secondary school and at a university, and the consequent phenomenon of the double discontinuity, are evident. To challenge my students' beliefs about algebra and its teaching I use medieval arithmetic problems. One of them is the following problem 47 in the Medieval treatise *Trattato d'Aritmetica* by Paolo Dell'Abbaco:

> A gentleman asked his servant to bring him seven apples from the garden. He said: "You will meet three doorkeepers and each of them will ask you for half of all apples plus two taken from the remaining apples." How many apples must the servant pick if he wishes to have seven apples left?

The students are invited to solve this problem and to write carefully their solution process. Afterwards, they analyze the written process of a colleague. At the end of the session, the students discuss the findings. In the following I present two examples of processes produced by my students that evidence the possibility of two ways of solving the problem:

> Case A) The student starts from the apples required before passing through the last door. Since the doorkeeper asks half plus 2 apples, the 7 apples are half of the amount less 2. Then, before the last door, the gentleman has 18 apples. The student observes that 18 is (7•2)+4, then, deduces that before the second door, the gentleman has (18•2)+4=40 apples and thus he must pick (40•2)+4=84 apples.
>
> Case B) The student names *y* the apples picked by the servant. Then apples left after meeting the first doorkeeper are (*y*-*y*/2)-2. Repeating this reasoning after the meeting with the second and the first doorkeeper the student writes the algebraic model (a first degree equation in the unknown *y*) of the problem.

We see that two solving paths may be followed, which may be put in relation with the analytic and synthetic methods:

- arithmetic path: from the known (left apples) to the unknown (apples to be picked)
- algebraic path: from the unknown (apples to be picked) to the known (left apples)

The algebraic path is based on the analytic method. To illustrate the different nature of arithmetic and algebraic problems I propose further exercises such as the two reported in the Appendix taken from *Propositiones ad acuendos juvenes* (*Problems to sharpen the young*) by Alcuin of York. In this way I lead my students to reflect on the fact that algebra is not only generalization, not only abstraction, not only using symbols, not only an extension of arithmetic: algebra is a method and the analytic method is its core. Then François Viète's innovation doesn't appearing to come out of the blue, but begins to be seen as a consequence of this way of looking at algebra.

The use of original sources is functional to my aims. Presentation of medieval problems does not have the aim or pretence to be put to students as being "real problems", but the distance from the time from which they originated makes students perceive them not so artificial as, sometimes, real problems appear. The prospective teachers may use medieval problems in classroom and exploit other potentialities, such as the reflection on language, the links with literature and with general history.

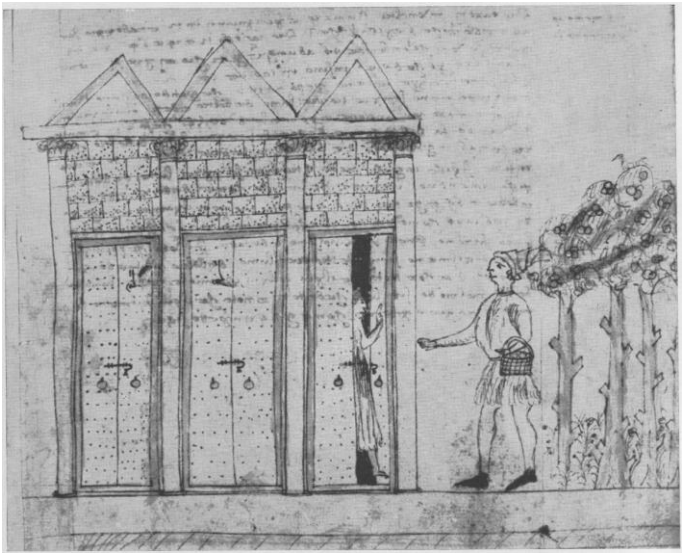

Fig. 1. Illustration of Problem 47 in *Trattato d'Aritmetica* by Paolo Dell'Abbaco

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– For information on the history of teacher education in Italy visit the website designed by Livia Giacardi: http://www.subalpinamathesis.unito.it/storiains/uk/training.php

Appendix. Two problems by Alcuin

1. A snail was invited by a swallow to lunch a league away. However, it could not walk further than one inch per day. Let him say, he who wishes, "How many [years and] days did it take for the snail to walk to that lunch?"
4. A certain man saw some horses grazing in a field and said longingly: "O that you were mine, and that you were double in number, and then a half of half of this [were added]. Surely, I might boast about 100 horses." Let him discern, he who wishes, how many horses did the man originally see grazing?

II - HISTORY AND EPISTEMOLOGY OF MATHEMATICS IN TEACHERS TRAINING IN NORWAY

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1 Institutional Context

There are basically four different ways of becoming a mathematics teacher in Norway (see table). Formerly, there was only one path for teacher training for the whole grade span from 1 to 10, but this has, from 2010, been split into two: 1–7 and 5–10. Both of these pathways last four years and integrate the subjects, pedagogy and didactics into the training. In the 1–7 path, half a year of mathematics (including didactics) is obligatory; in the 5–10 path, one year of mathematics (including didactics) is obligatory for students planning to become mathematics teachers.

Grades (Ages)	**1-7 (age 6-13)**	**5-10 (age 9-16)**	**(5-) 8-13 ((9-) 12-19)**	
Length	4 years integrated subject matter and pedagogy/didactics	4 years integrated subject matter and pedagogy/didactics	4 or 5 years integrated subject matter and pedagogy/didactics	3-5 years of subject matter + 1 year of pedagogy/didactics
Math.	Half year compulsory for *all* teachers	One year (for mathematics teachers)	One year (for mathematics teachers)	One year (for mathematics teachers)
National guide-lines	Yes	Yes	No	No

For the 8–13 education (in theory 5–13), a teacher traditionally completes a bachelor's or

master's degree in mathematics, with one year of pedagogy and didactics added on. This has now been partly replaced by a four or five year education where pedagogy, didactics and the subject are integrated. In both of these variants, at least one year of mathematics is obligatory.

For all the different pathways which teachers can undertake to train as mathematics teachers, only the 1–7 and 5–10 pathways have national guidelines. (Kunnskapsdepartementet, 2010). In the national guidelines, we find the goals of training are connected to "the historical development of mathematics", "the importance of mathematics as a formative subject and its interaction with culture, philosophy and social development" and "how knowledge of mathematics is developed" (my translations).

There is no national system for in-service training. This is left to the schools. Currently, I know of no history of mathematics courses available for teachers.

2 Theoretical Approach

The education of mathematics teachers has at least two goals (Christensen & Nordberg, 2007):

1. to develop students' knowledge of mathematics and their attitudes towards it
2. to develop their ability to *teach* mathematics

Given that many students enter teacher education with misconceptions and a lack of knowledge of key areas in mathematics (Rasch-Halvorsen & Johnsbråten, 2007), there is a danger that goal 1, and in particular developing mathematical skills, will overshadow goal 2 in the minds of the students. On the other hand, that may be all the more of a reason for offering a new approach to learning mathematics.

Concerning the goal 1, all the usual reasons for including the history of mathematics with any group of students/pupils hold. For goal 2, a closer look at teacher knowledge is needed.

I would like to use the model for mathematical knowledge for teaching, developed by Deborah Ball and her colleagues, as a starting point for looking at teacher knowledge (Ball, Thames, & Phelps, 2008). Ball et al are following up on Shulman (1986), and are trying to come to grips with what knowledge teachers use in their teaching. I would like to take part in a discussion on how history of mathematics can contribute to each of these "domains".

I would argue that the history of mathematics can play a part in all of these domains, but in different ways. For instance, in the area of subject matter knowledge:

- *Common content knowledge* is mathematical knowledge that is not special for teachers, such as being able to calculate correctly. In the teaching of teacher students, history of mathematics can play a role just as with anyone else learning mathematics.
- *Specialized content knowledge* is mathematical knowledge primarily necessary for teachers. Ball et al (2008) mention as an example the ability to see a new algorithm and decide whether it is sound. Work on the history of mathematics can help develop such skills, because in history we see mathematics done in very different ways all the time. One example is the work on historical algorithms, see Nikolantonakis and Smestad's paper at this conference.
- *Horizon content knowledge* is knowledge of how the mathematics that pupils are learning now is connected to mathematics that they will learn later and to what they learned earlier. History of mathematics can contribute to seeing the connections between topics, for instance between statistics and probability, and be (as mentioned by Evelyne) a "therapy against scattering".

In the area of pedagogical content knowledge, history also has a part to play:

- *Knowledge of content and students* includes typical misconceptions. History of mathematics sheds light on some of these by showing historical obstacles. The early discussions on probability are examples of this.
- *Knowledge of content and teaching* involves different ways of representing mathematics. Of course, knowing the history of quadratic equations helps to see the different approaches, as mentioned by Michael Glaubitz in this volume.
- *Knowledge of content and curriculum* benefits from a historical perspective of how the mathematics curriculum has developed. For instance, the rise and fall of the "New Math" is an intriguing example of how curricula are influenced by external factors.

In my opinion, the history can play a part in the whole spectrum of teacher knowledge. We could discuss what the different ways of working on history of mathematics can contribute to in the light of these (or other) frameworks for teacher knowledge.

But we should not forget to ask the question "What is 'mathematics' in this context?" If we include history of mathematics in what pupils should learn (that is, we see it as a goal, not just as a tool), subject matter knowledge will include history of mathematics, KCS (knowledge of content and students) will include knowledge of which parts of history of mathematics are suitable for students and so on. That puts high demands on us as teacher educators.

This may not be the way Ball et all (2008) were thinking – the model of Ball is based on classroom experiences, but maybe not on classrooms where the history of mathematics was included in practice. Following up on Uffe Jankvist's comments at the start of this conference: this may be an area in which our perspectives may enrich theories on mathematics education.

Or, instead of clinging to Ball, we could make four goals out of the two we started with:

1a. to develop students' knowledge of mathematics and their attitudes towards it

1b. to develop students' knowledge of the history of mathematics (and their attitudes towards it)

2a. to develop their ability to teach mathematics (also using the history of mathematics)

2b. to develop their ability to teach the history of mathematics.

Most people will agree that 1b is a prerequisite for teaching mathematics using history of mathematics, as well as for teaching history of mathematics, of course. But as Chun-Ip Fung pointed out (Fauvel & Van Maanen, 2000), there is often a question of where to put the emphasis. Torkel Heiede (Fauvel & Van Maanen, 2000) mentions a course of 33 three-hour sessions, which could give an overview of the whole history of mathematics and James Kiernan described a similar one at the previous ESU (Kiernan, 2007). Within the Norwegian half-year course, however, which is trying to cover all the relevant mathematics and math education in perhaps only 40 three-hour sessions, I think we can do no more on 1b than give a glimpse of the history and give the students motivation for further work. That is, we must hope that students will continue learning after becoming teachers. So, what will be their basis for doing that?

Michael Glaubitz' talk in this conference gives reason for hope, in that it may be possible to do useful things without a solid and comprehensive knowledge of the history of mathematics to begin with.

3 Work to be done

In Norwegian textbooks for teacher training, the history of mathematics is presented through the texts for the students to read, with a few exercises here and there. (Smestad, 2010) With one notable exception, there are no ideas in these textbooks of other ways to include the

history of mathematics in teaching. The one exception, the classic "Matematikk for lærere" ("Mathematics for teachers") (Breiteig & Venheim, 1998), includes a list of options, but does not give examples of them. This is a striking contrast to the way most of the textbooks are generally written, with activities and suggestions for teaching.

So I believe that the main problem in Norway is that there are so few resources available to teachers. Teacher education can, sadly, only be a start of a learning process, but this process ends abruptly if teachers have nowhere to look afterwards. There is a tremendous gap between what we hear at conferences like HPM and ESU and what Norwegian teachers have access to. Improving on that situation is important. After doing that, at least it will make some sense to spend energy in teacher education to get teachers interested in the history of mathematics.

Therefore I am working on a wiki (eleviki) to make available materials on history of mathematics and ideas for teaching it. As I discussed in my talk on Monday, I want to make available concrete ideas for teaching, based on ideas from conferences like these and the literature (i.e. Demattè, 2006; Katz & Michalowicz, 2004; Pinto, 2009). In doing this, I also want to take into consideration weaknesses I've found in the Norwegian literature up to now. Thus, I want to emphasize the use of mathematics, not only its development, I want to emphasize mathematics suitable for teaching in primary schools, include mathematicians' motivations, present more original sources, and to link all of these to the easily available Norwegian sources, making them easier to find by teachers. (Smestad, 2010)

In conclusion: in the present situation, Norwegian primary school teachers are not equipped to start including history of mathematics as they finish their teacher education. My goal is that some of them become interested in the subject, and have the means to gradually learn more. In the Norwegian situation, only by gradually improving the situation will the good examples be developed that will help bring more significant change in the future. A successful implementation in schools will be difficult unless there is a certain number of enthusiastic teachers who have already started this process.

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III - THE ROLE OF THE HISTORY AND EPISTEMOLOGY OF MATHEMATICS IN TEACHERS TRAINING – THE CASE OF ENGLAND

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1 Pre-service and in-service mathematics teacher education in England

1.1 Pre-service teacher training in England

There are three routes into teaching that are available to the entrants to the teaching profession in England, all requiring a completed first degree and a grade C in Mathematics and English at GCSE level[1]. These are:

- Postgraduate Certificate in Education, given at the successful completion of the course based at a university for around thirteen weeks, with the remainder of around 120 days spent in two schools in partnership with the university
- School-centered Initial Teacher Training, delivered by a consortium of schools and colleges, where the greatest part of instruction comes from the 'lead' school, and practice is organized in more than one of the schools from the consortium
- Graduate Teacher Training, based in a school (employment-based route) with a university as a school's partner to provide around ten days of training in professional studies (and almost no subject-related content).

All three of the possible routes into teaching last for one academic year; the first two being heavily supported/subsidized by the government through grants and bursaries, especially in such shortage subject as mathematics.

Whilst the teacher education programmes are fairly loosely prescribed by the government, for the universities and schools providing the courses leading to the Qualified Teacher Status, the outcome is deemed to be satisfactory evidence of covering the 33 Qualified Teacher Standards, as defined by the Teacher Development Agency.[2] The pursuit and inclusion of the history of mathematics in teacher training can, in the view of flexibility of designing the training courses, fall under various such standards.[3] Whilst it

[1] GCSE is a basic qualification, granted at 16.

[2] Qualified Teacher Standards for the Teachers in Education are given at the Teacher Development Agency's website – TDA (1), and the same exemplified for the training providers are listed in bibliography under TDA (2).

[3] Some of the Standards which can be possibly seen as offering opportunities for the introduction of the historical element into mathematics education are:

- **Standard 8:** Have a creative and constructively critical approach towards innovation, being prepared to adapt their practice where benefits and improvements are identified
- **Standard 10:** Q14 Have a secure knowledge and understanding of their subjects/curriculum areas and related pedagogy to enable them to teach effectively across the age and ability range for which they are trained.
- **Standard 18:** Understand how children and young people develop and that the progress and well-being of learners are affected by a range of developmental, social, religious, ethnic, cultural and linguistic influences

is, perhaps, easier to identify how one of the standards (standard 14 to be precise), dealing with the 'subject knowledge' can easily be linked with the history of mathematics in both terms of 'history as a tool' and 'history as a goal' approaches (Jankvist, 2009) it can also be used in teacher training courses in multiple ways, for example through 'historical-genetic-principle' (Schubring 1977), or by using the history to enable teachers in education to gain a greater repertoire of pedagogical approaches[4] (Van Maanen, 1997, Jahnke 1996). Barbin (1995, 1996, 1997) argues for the construction of knowledge through problem solving, while Furinghetti (2007) offers the alternative to use the history of mathematics as a 'reorientation' tool. Both of the latter mentioned are possible approaches to build on the students' existent subject knowledge in mathematics as the entry qualifications to the teaching profession include the 'considerable' element of mathematics in a degree (defining the 'considerable' is not always consistent between the institutions). The standards 18 and 19 further offer opportunities for introducing the history of mathematics through a cross-curricular and cross-cultural dimension (Lawrence, 2009; Barta, 1995; Katz, 1997; Grugnetti, 1994; and Proia & Menghini, 1984).

1.2 In-service teacher education

There is no unified view of the in-service education, or as we prefer to call it, continuing professional development, in England but the practice has developed over time to encourage the teachers individual engagement with academic research which, in turn, relies on their practice.[5] As of 2008, the 'historical and cultural roots of mathematics'[6] are part of the entitlement for every child's experience of mathematics; in practice however, the lack of in-service training means that this is usually not a matter of principle but rather a matter of a few individual cases as described in 2.2.

Between 1998 and 2008 the Gatsby Teacher Education Projects, of which the Gatsby Teacher Fellowship was a part, (funded by the Sainsbury Trust and affiliated with the Royal Society), as well as the Royal Society Small Grants for links between schools and industry, offered a small variety of opportunities for teachers to engage with research and include the history of mathematics into their continuing professional development. In fact only one project, by the author, was directly linked to the history of mathematics in the classroom (resources for 11-16 year olds, resulting in the website mathsisgoodforyou.com).[7]

The Royal Institution,[8] although not funding directly teacher development, has for

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- **Standard 19:** Know how to make effective personalised provision for those they teach, including those for whom English is an additional language or who have special educational needs or disabilities, and how to take practical account of diversity and promote equality and inclusion in their teaching.

[4] As reported by Gulikers (2001), 227.

[5] For example, through the foundation of the National Teacher Research Panel, supported by the DCSF (Department for Children, Schools, and Families) during the Labour era.

[6] The National Curriculum, Mathematics KS3 programme of study (11-14 year olds): http://curriculum.qcda.gov.uk/key-stages-3-and-4/subjects/key-stage-3/mathematics/index.aspx and KS4 programme of study (14-16 year olds): http://curriculum.qcda.gov.uk/key-stages-3-and-4/subjects/key-stage-4/mathematics/index.aspx. Accessed 16th October 2010.

[7] All of the Gatsby Teacher Fellowship projects are listed on their website, see GTP.

[8] The Royal Institution of Great Britain was founded in 1799 by the 'Society for Bettering the Conditions and Improving the Comforts of the Poor' (as opposed to the prestigious Royal Society who, some perceived, favoured the well-to-do and established scientists). From 1978, under the guidance of Sir Professor Christopher Zeeman, a mathematician, the Mathematics Master Classes for 13-14 year olds were first

several decades supported the network of teachers who organize the Saturday master classes for the very able pupils. This has, for no explainable reason, been the 'hot-bed' of the development of ideas among teachers on how to use the historical context for their master-classes, and a number of prominent UK-based teachers have, through this involvement, developed a number of historical resources for their classes.[9]

Since Smith's (2004) Review[10] into the teaching and learning of mathematics, and the founding of the National Centre for Excellence in the Teaching of Mathematics (NCETM, founded in 2006) which followed the recommendations of the report, the consensus has been established[11] that the collaboration, networking, awareness of a variety of practices and resources, and the self-study and research in peer groups, are the most valid forms of CPD in mathematics education. To support the development of such a model of practice, the NCETM, immediately upon their foundation, issued a call for proposals for small-scale projects which fit the criteria of collaboration and research, and the project described at the end of this section was one which was supported through the first round of grants between 2006/7 and 2007/8 academic years.

In 2007, the Open University developed a 10-credit course for teachers on the history of mathematics, under the title 'The Story of Maths'. The course's four one-hour programmes were aired in 2008 on BBC4 and later BBC1, with tremendous success,[12] propelling the history of mathematics into an orbit of popular culture, and Professor Marcus du Sautoy, the lead presenter, into something closely resembling celebrity among mathematics teachers. 'The Story of Maths' course now runs twice a year, attracting around 200 students a year, and the resources that were part of the series are sold by popular retailers such as Amazon.

In the summer of 2009, Mathematics became part of the Prince's Teaching Institute programme. The Prince of Wales founded his Teaching Institute in 2001 with English and History as the first subjects, with the underlying aims '…to generate discussion about the specific contribution to education made by English Literature and History and about what constitutes an education in these subjects'[13]. After the interest of teachers expressed at the first Summer School in Mathematics held at the Queen's College, Cambridge,[14] the first continuing professional development event was organized by the Prince's Teaching Institute. This was a day on the History of Mathematics, held on 19th March 2010 at the London Mathematical Society. The oversubscription of the event, and the reports and demands of the teachers on the Mathematics Summer School Programme at the events afterwards, meant that the Prince's Teaching Institute plans to have a similar in-service

established, out of which grew a network of Master Classes throughout the country (now registering around 600 of the teachers/schools that provide these).

[9] To name but a few Martin Perkins and Peter Ransom have been active members of this network. The current list of schools, organizers, and network groups is available from http://www.rigb.org/contentControl?action=displayContent&id=00000001857.

[10] The UK Government commissioned a review into the teaching of mathematics into post-14 mathematics education, the results of which were summarized by professor Adrian Smith in the mentioned report.

[11] In particular see the aims of the NCETM (National Centre for Excellence in the Teaching of Mathematics at https://www.ncetm.org.uk/ncetm/about. Accessed 16th October 2010.

[12] If we are to judge by the reviews in the popular press; for example see http://entertainment.timesonline.co.uk/tol/arts_and_entertainment/tv_and_radio/article4893458.ece. Accessed 16th October 2010.

[13] See 'About us' on the Prince's Teaching Institute website.

[14] June 28th – July 1st 2009.

training day perhaps once a year in the future.[15]

Fig. 1. The participants of the first Mathematics Summer School of the Prince's Teaching Institute, Mathematical Bridge in Cambridge, July 2009.

Teachers who do not undertake either of the above in-service opportunities, may use the two major mathematics education websites, the NRich[16] and Plus Magazine[17], which give a good selection of articles, classroom ideas, and resources for the classroom on the history of mathematics. These stretch all ability levels and all stages of the secondary education, and the University of Cambridge's Mathematical Institute, with the NRich, organize usually three in-service training days for teachers on the various problem-solving approaches and work with the gifted and talented mathematicians. Although not directly linked to the history of mathematics, often the topics of problems and examples are 'rich' in history, giving another opportunity for teachers to get engaged with the subject.

While the above therefore, does not perhaps paint a picture of a unified development of a continuing professional development practice and the place history of mathematics may have in it, it does show an increased trend to favour the topic. It is also evident that there is an increased interest in the subject-matter and an increased awareness of its importance, from the change of the curriculum in 2008 to incorporate the historical and cultural roots of mathematics, to the commitment shown by the national bodies (the NCETM and the Prince's Teaching Institute) to support the history of mathematics as a valid and valuable contribution to the development of teachers at in-service level.

[15] For the short history of the PTA, see http://www.princes-ti.org.uk/AboutUs/Ourhistory/.

[16] See http://nrich.maths.org/public/

[17] See http://plus.maths.org/content/

2 History of mathematics in teacher education: theory and practice

2.1 Theoretical frame

As discussion on the history of mathematics ensued at the National Centre for Excellence in the Teaching of Mathematics[18] in 2007, the theoretical frameworks for incorporating the history of mathematics into mathematics education were examined.[19] On the national level, the introduction of the 'historical and cultural roots of mathematics' in 2008, through the (still valid and current in October 2010) National Curriculum meant that social constructivism as a theoretical framework would or could, to a certain degree, become an element of mathematics education and be placed at the heart of practice in mathematics classrooms across the country. However, there was an obvious lack of training opportunities for teachers in this regard. To bridge this gap, identified both by the Open University and the Prince's Teaching Institute (see the previous section), the two institutions offered courses and one-day events for teachers from across the country. The current state of affairs is therefore that the history of mathematics is a recognized and valid part of in-service training for teachers.

2.2 An example from practice

As already mentioned, the Smith Report (2004) identified collaborative practice as the most crucial and missing link in the pre-2004 provision of continuing professional development in England. The Lesson Study model, as exemplified by the Japanese, American, and Hungarian teachers (Burghes, 2008) was offered as a possible approach.[20] The History of Mathematics and Collaborative Teaching Practice, a project supported by the NCETM through 2006-8, linked the collaboration between teachers to establish a practice of enquiry and research into both the aims and motivations for using the history of mathematics into teaching.[21] In this project, two major benefits for teacher in-service training through the engagement with the history of mathematics have been identified:

- The reorientation – especially of non-specialist and primary teachers (Furinghetti, 2007)
- The mapping of the professional development landscape by teachers to identify their position in this landscape and their possible routes through it (Lawrence, 2008, 2010).

In summary, the history of mathematics in the classroom is becoming a reality through a number of initiatives, and the interest that teachers have in the subject is alive and well. An increased demand for the short courses for the practicing teachers is an encouraging

[18] See in particular the article 'History of Mathematics' on the NCETM portal at https://www.ncetm.org.uk/resources/3245, and the History of Mathematics on-line community on the same portal at https://www.ncetm.org.uk/community/1324 (you must be a member to contribute to this community).

[19] See Lawrence, (2008), (2010), and Kaye (2008).

[20] Although the Burghes wrote the mentioned report in 2008, he has been a leading figure in promoting the collaborative teaching practice (and the Lesson Study) nationally; Burghes was also the first director of the National Centre for Excellence in the Teaching of Mathematics (NCETM).

[21] See the project report at https://www.ncetm.org.uk/files/397060/Final_Report_G002_Simon_Langton_Grammar_School.pdf and the project website at http://www.mathsisgoodforyou.com/lessonstudy/home.html.

trend. The curriculum change, I believe, will not be a change for the worse in this respect if teachers' voices are listened to.

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IV - HISTORY AND EPISTEMOLOGY OF MATHEMATICS IN TEACHERS TRAINING IN FRANCE

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It is difficult to give a complete institutional situation of teachers training in France, specially because things have been changing for some years, and also because the institutional structures (universities, "rectorats") are autonomous (not only for finance). Recently, a question of financial problems also arose, because of which for instance, the summer national universities disappeared some ten years ago.

Until 2010, the pre-service teachers training depended on each IUFM (University Institute for Teachers Training), which exists in each "Academy". These institutes provided training of one year for beginner teachers. When I taught in IUFM of Creteil, the course included 30 hours of teaching on the history of mathematics, but nothing on the subject was evident in some others IUFM. Since 2010, the IUFM discontinued the one-year model of teacher training. The future teachers now have to obtain a master's degree at the universities, where, at the same time, the epistemology and history of sciences appeared in the curricula of such courses. The importance attributed to the teaching of these subjects however, depends on the universities.

Until around twenty years ago, the in-service teacher training depended on the academic structures named "rectorats", and especially on the financial situation of these structures. We learned at the beginning of October 2010 (we did not know that when we were in Vienna) that many rectorats discontinued all teacher training (for secondary teachers) for financial reasons.

1 History and Epistemology in teachers training: new approachs of mathematical thinking and teaching

The history gives a cultural approach to mathematics, by placing mathematics into the context of the histories of sciences, technology, ideas, and societies. But it gives also an epistemological approach of mathematical activity, by emphasizing the part played by the problems, conjectures, experiences, by the rigor, analogy, error and modeling (CII-EHM, 2007).

The theme of the 17th Colloquium organized in Nancy by the Commission Inter-IREM (Epistemology and History of Mathematics) CII-EHM was "La figure et la lettre" (CII-EHM, 2010). Indeed, the history of mathematics permits to analyze the parts played by writing, figures, calculus, symbols, tables or diagrams in the activities of reasoning and proving. From this point of view, the Colloquium organized an event on "Explanation and Proof in Mathematics" some years ago, which was very interesting (Hanna, 2010).

The historical and epistemological approach raise issues related to a reflection on methods and contents of the teaching, and those related to the constructivist conceptions of notions and theories. Specifically, the history is seen here as a tool to study the

processes of rectification of mathematical concepts.

History of sciences can also be used as a tool to enable the multidisciplinary approach to teaching. It was the theme of the last French Summer University organized by the CII-EHM, which took place in IREM of Poitiers in 2000 (CII-EHM, 2003). A good example is the study of motion by Galileo (Barbin, 1997). Of course, it concerns the physics because Galileo researched the law of falling bodies. But this law is also a rupture in the philosophy of nature. Indeed, the *Physics* of Aristotle was a research on the causes of the phenomena of nature, while Galileo was only interested in the effects of the motion, more precisely he studied the relations between the effects, like time, distance and speed, in a given motion. The reason for this interest turns out to had been a technical problem asked by the gunners, on the relation between the angle of the canon and the place where the cannon-ball falls, and this led to the investigation of the trajectory of a canon ball.

The contribution of the history of mathematics in relation to the new curricula is a subject developed in the IREMs. For example, the introduction of a new conception of probability in teaching linked with statistical ideas led the IREMs to new studies on historical relations between probabilities and statistics (CII-EHM, 2004). More recently, the introduction of algorithms in teaching, in relation to the use of computers in the classroom, was an opportunity to analyze the history of algorithms, their role and their importance in the history of mathematics.

The history of mathematics can, therefore, become a tool of criticism about the successive "new curriculums" in mathematics. I would like to mention the scattering of knowledge at both secondary and university levels, the confused part played by the assertions in secondary schools and the problems arising when the mathematics is seen only as a discipline to serve the other disciplines. Today, we can consider that the history of mathematics is a kind of "therapy against scattering", for instance it can be used to understand the relations between algebra, geometry and calculus, to study the "architecture" of mathematics, and to reflect on the "durability" of some mathematical knowledge along the centuries.

2 Purposes and themes for historical and epistemological training: four examples

I would like to give four examples of historical themes studied in the pre-service or in-service teacher training and in courses to mathematics students in Nantes.

The first theme concerns the reciprocal transformations of problems and concepts. The example of the tangent to a curve is especially interesting. We find geometrical propositions in Euclid's or Apollonius' writings, where the curves are conceived as geometrical objects, where the tangents touch the curves in one point (they did not cut them) and where the proofs are developed by absurdum (suppose they cut etc.). In the XVIIth century, the problem became a "method of invention" as related to the study of tangents. For Roberval, the problem became a kinematic one: a curve is the trajectory of a point in motion and the tangent is the direction of the motion. For Descartes, the problem was an algebraic one: the curve and the tangent circle are defined by two equations which must have only one common solution. So the problem of relationship between tangent and its curve is transformed and, at the same time, the notion of a curve to which a tangent is sought is transformed also.

The notion of rectification of concepts can be a way to introduce an idea of a long-term interest to the teaching of mathematics, and to anticipate the future development of the

student teachers.

The second theme is “the proofs and the mathematical methods”, taken by the students of mathematics who want to become teachers. We begin with the axiomatic and deductive proof by Euclid, then we oppose this kind of proof with the algebraic method of Descartes to solve geometrical problems, the infinitesimal methods of Leibniz and Newton, the projective method of XIXth century and the method of “equipollence” of Bellavitis. This is used as a way of making a distinction between the context of proof and the context of invention, which are often confused in mathematical teaching.

The study of epistemological obstacles is dealt with through the extension of the concept of number: rational and irrational numbers, negative quantities and numbers, imaginary quantities and complex numbers, and the construction of real numbers. It is an opportunity to engage and reflect on the positivity of errors in mathematical activity (CII-HEM, 2007).

The last theme is “proofs and algorithms”, which took place at the in-service teacher training proposed by the IREM of Nantes. Firstly we examined the notion of algorithm in history, algorithms on numbers but also algorithms on geometrical figures. Secondly, we read many kinds of written algorithms to understand their difficulty. Thirdly, we arrived to the concepts of algorithms and machines with the example of the machine of Post.

3 Reading and working with original texts

When the French IREM began to organize in-service teacher training at the end of the 1970s, not many original texts were available. As a consequence, the collaboration between IREMs was organized with a purpose of choosing and editing texts which seemed interesting. I can mention some works which came out of this: those on the analysis of Euler, Lagrange and Cauchy, works on geometry texts by Arnauld, Clairaut, Monge, Poncelet and Chasles. The first anthology of these ancient texts appeared in 1987 under the title *Mathématiques au fil des âges*. In the same period, some IREMs edited anthologies on particular domains, like the IREM of Toulouse which edited three booklets on the equations of the first degree, on equations of the second degree, and on equations on the third and fourth degrees.

From 1999, the CII-EHM edited a collection of anthologies of ancient texts on more focused themes, like *La construction des nombres réels dans le mouvement d’arithmétisation* (Boniface, 1999). The title of the last anthology is *L’espérance du hollandais ou le premier traité de calcul du hasard* (IREM de Caen, 2006). It contains original texts of Christian Huygens, and his readers, de Montmort, Jacques Bernoulli, de Moivre and Euler. For each text, there is a historical introduction and commentaries on problems solved by mathematicians. It is therefore, possible to compare solutions, but also make comparison between the different conceptions of the probabilities and their calculus.

The French historian Paul Veyne wrote that “history [has] the virtue of astonishing with what comes of itself”: indeed, the reading of original sources can produce astonishment and give a possibility of a “dépaysement” (Barbin, 1997). Of course, the ancient texts must not be immediately interpreted in modern terms.

I give a course to future teachers of primary schools on the historical relations between numbers and figures. In the practical part of the teaching, students read original texts and the questions lead them to read them with eight purposes:

- to interpret (in historical context)
- to translate (example : Egyptian text)
- to compare translations of an original text

- to compare original texts
- to write in the same manner as somebody
- to pass from one manner to another
- to interpret (through the modern language)
- to compare two interpretations (in historical context or in modern language).

Interpreting an original text inside its historical context is very interesting for teachers or future teachers. Indeed, this process offers a possibility to reflect on epistemological obstacles, on the appropriateness between problems and methods, on the meanings of different solutions of a problem and so, on the meaning of the underlying theories.

4 To form but not to standardize: the introduction of an historical perspective in teaching mathematics

I think that the major role of the history and epistemology of mathematics is to lead the teachers to think, to think mathematically, and to think that their pupils are also persons with an ability to think. From this point of view, any model of introduction of the history of mathematics into the classroom will be harmful, especially if "lessons" of this model replace the historical study and the epistemological analysis.

In 1988, on the occasion of the meetings HPM in Florence and ICME in Budapest, the CII-EHM published a blue book titled in its English version *A case for an introduction of a historical perspective in the teaching of mathematics* (CII-IREM, 1988). This book is also edited by John Fauvel with the title *History in the classroom, IREM's papers* (Fauvel, 1990). This book contains 21 experiences of introducing the history of mathematics into the classroom. I wrote in the foreword to this book, that the reader should not consider these experiences as models of practice; these stories are the facts about practice of teachers who are in a situation which allow them to do research.

What does it mean to introduce a historical perspective into the teaching of mathematics? We do this neither because we can imagine a teaching as an imitation of a historical process, nor because we want to create a new school discipline completely independent of mathematical practice. The reason is that all the historical and epistemological reflections that a teacher comes across through the history of mathematics he/she can integrate in his or her teaching. This can be in order to situate the knowledge in historical context, to explain how and why a concept or a theory was invented, to read original sources, to solve historical problems, or to construct interdisciplinary activities with her or his colleagues.

To propose historical problems to pupils or students is very appropriate, because the problems give the meanings of concepts and theories. The book of the CII-EHM on the history of problems was translated into English by Chris Weeks, under the title *History of Mathematics, Histories of problems* (CII-EHM, 1997).

In 2009, the CII-IREM published a new blue book (which is black and red) tilled *Des défis mathématiques d'Euclide à Condorcet*. There are 9 experiences describing the introduction of the history of mathematics into teaching from colleges (students aged 11-14) to the university level. The authors are Anne Boyé, Renaud Chorlay, Jean-Paul Guichard, Patrick Guyot, Gérard Hamon, Frédéric Laurent, Loïc Le Corre, Dominique Tournés and myself (CII-EHM, 2010). Here also, we made sure to say, that these experiences should not to be considered as models. This is the reason for which each authors tells her or his experience by telling the story in first person, which is not common in French texts on teaching.

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