Wilhelm Matzka (1798–1891) and His Algebraic Works

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Abstract

This article deals with the topic of my doctoral thesis called Life and Work of Wilhelm Matzka. *The aim of this article is to state some fundamental data of Matzka's life, briefly outline his activity at the University of Prague and his publications, furhter familiarize with his work about the theory of determinants and also with other works about the theory of determinants, that had arisen in Czech countries at that time.*

Wilhelm Matzka (1798–1891) was a full professor of mathematics at the University of Prague. He was born in Lipertice in Moravia, studied at the University of Prague, served many years in Austrian army in Vienna. His pedagogical activities are adherent to Vienna, Tarnow and Prague (Prague Polytechnics and University of Prague). For many years he funcioned at Königliche böhmische Gesellschaft der Wissenschaften *[the Royal Bohemian Society of Sciences].*

He lectured on analysis, algebra and geometry. He was engaged also in other mathematics parts and some special parts of physics. In these areas he wrote textbooks, expert articles and studies. The spectrum of his works was very comprehensive. Complete list of W. Matzka's publications not exist till this time.

In the second part of 19th century it was hardly developed study of some parts of algebra, high attention was attended to the theory of determinants. This problem was very favourite in Czech countries. A lot of les or more original special works had arisen. The first books of the theory of determinants, methodical and popular articles were written.

1 Wilhelm Matzka — Life, Studies, Pedagogical Activity, and Other Activities

Wilhelm Matzka[1] was born on November 4, 1798 in Lipertice[2]. He was raised in Malý Újezd near of Teplice in Bohemia. He received a first education at the primary school in Weisskirchlitz[3] near of Teplice and in Šopka near of Mělník. During the period 1812–1817, he studied at the grammar school in Osek and Chomutov, then, during the period 1817–1819 at the Faculty of Arts in Prague.

Many years he served as a cannoneer in Austrian army in Vienna. He entered the military service with the 2nd artillery regiment in Vienna in 1819. In 1821, he was relocated as a bombardier to the bombardier company also in Vienna. Subsequently, he was promoted

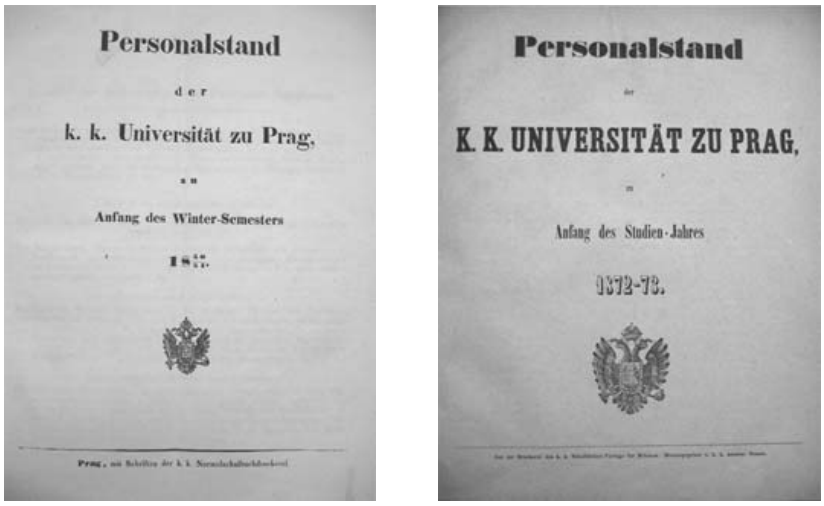
[1] Also written Vilém Matzka.
[2] In German Leipertitz, today Litobratřice in Southern Moravia.
[3] In Czech Novosedlice.

to a cannoneer, chief cannoneer, and then in 1831, to a lieutenant and at the same time he was appointed mathematics teacher in the bombardier company.

W. Matzka visited lectures at the University and Technical Faculty of Vienna to complement and deepen his education at that time. At the University of Vienna, he passed scientific and practical astronomy with Austrian mathematician and astronomer Prof. Josef Johann Littrow (1781–1840), higher mathematics and physics with German mathematician and physicist Prof. Andreas von Ettingshausen (1796–1878), mineralogy with Prof. Friedrich Mohs (1773–1831), and then at the Vienna Polytechnics technology with Prof. Georg Altmütter (1787–1858).

W. Matzka improved his knowledge of mathematics and other sciences and began to lecture as a professor of higher mathematics at the Mathematical and Artillery Staff School of the bombardier staff[4]. He lectured on algebra, analytic geometry, differential and integral calculus and higher mechanics. At this school he taught till 1837. In September that year, he was appointed full professor of elementary mathematics at the newly based philosophical school in Tarnow[5], where he acted until 1849.

In 1843, he passed the rigorous tests at the University of Olomouc and reached the doctorate in philosophy. In April 1849, he was appointed professor of mathematics and practical geometry at the Prague Polytechnics. He entered that position in May of the same year, but his activity at the Polytechnics was very short. After the end of the summer semester of 1850, he already moved to the University of Prague as a full professor of mathematics. He taught there until the summer semester 1871[6]. After Wilhelm Matzka left, František Josef Studnička[7] (1836–1903) took over his place.


Personalstand

der

k. k. Universität zu Prag,

Anfang des Winter-Semesters

Personalstand

K. K. UNIVERSITÄT ZU PRAG,

Anfang des Studien-Jahres

1872-73.

Figure 1 – Front page *Personalstand der k. k. Universität zu Prag zu Anfang des Winter-Semester 1850/51* and *Personalstand der k. k. Universität zu Prag zu Anfang des Studien Jahr 1872/73*. In these documents we can find the first and the last notation about W. Matzka at the University of Prague.

[4]In short: bombardier school or school for cannoneers.

[5]This school was probably higher secondary school which prepared students for study at the University of Krakow.

[6]More about the education at the University of Prague see *Ordnung der Vorlesungen an der k. k. Universität zu Prag 1849/50–1870/71*.

[7]Professor at the Prague Polytechnics, Prague and Czech University. He lectured in Czech and contributed to the enlargement of czech university education and to production of mathematical literature in Czech by his substantial task. More about his life, work and pedagogical activities see Němcová, M., 1998, *František Josef Studnička (1836–1903)*, Edition Dějiny matematiky, Volume 10, Prometheus, Prague.

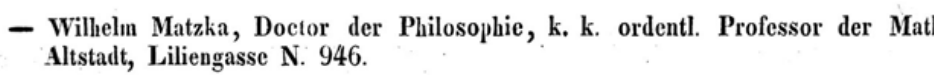

— Wilhelm Matzka, Doctor der Philosophie, k. k. ordentl. Professor der Mathematik. Altstadt, Liliengasse N. 946.

Figure 2 – The first notation about W. Matzka is in document of the University of Prague *Personalstand der k. k. Universität zu Prag zu Anfang des Winter-Semester 1850/51.*

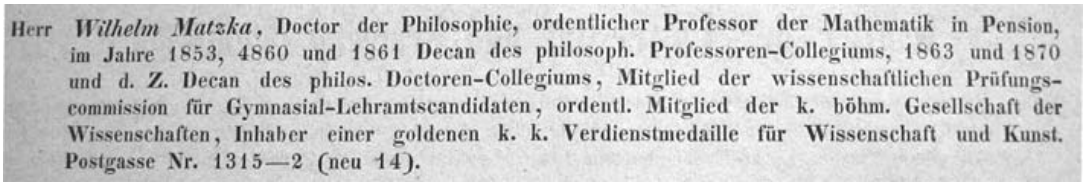

Herr *Wilhelm Matzka*, Doctor der Philosophie, ordentlicher Professor der Mathematik in Pension, im Jahre 1853, 4860 und 1861 Decan des philosoph. Professoren-Collegiums, 1863 und 1870 und d. Z. Decan des philos. Doctoren-Collegiums, Mitglied der wissenschaftlichen Prüfungs-commission für Gymnasial-Lehramtscandidaten, ordentl. Mitglied der k. böhm. Gesellschaft der Wissenschaften, Inhaber einer goldenen k. k. Verdienstmedaille für Wissenschaft und Kunst. Postgasse Nr. 1315—2 (neu 14).

Figure 3 – The last notation about W. Matzka is in document of the University of Prague *Personalstand der k. k. Universität zu Prag zu Anfang des Studien Jahr 1872/73.*

According to the new universities organization law from September 30, 1849, academic senate systematized the university. The academic senate consisted of rector, vice rector, four deans of professor staff of faculty, four vice deans of professor staff of faculty and four deans of doctoral staff of faculty. The University of Prague was divided into four faculties at that time — Faculty of Theology, Faculty of Law, Faculty of Medicine and Faculty of Arts. Professor staff[8] was at the head of each faculty and voted a dean from their members for one year, who was a vice dean next year. Further of the professor staffs were doctoral staffs, that also voted their deans[9].

Wilhelm Matzka was the dean of professor staff of the Faculty of Arts in years 1853, 1860 and 1861, and he was the vice dean of professor staff of the Faculty of Arts in years 1852, 1854 and 1862. He was the dean of doctoral staff of the Faculty of Arts in years 1863, 1870 and 1873. He was a member of library committee of the University of Prague in school years 1865/66–1868/69.[10]

W. Matzka attended the special care to the instruction of teachers of mathematics and physics at secondary schools. He noticeably influenced the level of mathematics educations in Czech countries. He was a member of the committee for secondary school teachers of mathematics in Czech countries from the beginning of the fifties of the 19th century.

Scientific activity of Wilhelm Matzka founded acknowledgement also in academic circles. In 1845, he was invited to *Königliche böhmische Gesellschaft der Wissenschaften* [the Royal Bohemian Society of Sciences] and at the beginning of 1850, he was elected its regular member. The Royal Bohemian Society of Sciences was the only one scientific institution in Czech countries at that time. To become its regular member meant an important position, proving the appreciation and acknowledgement of the "scientific" work of the person. He held an office as a cashier of this society also more than 30 years. He was graced with gold medal in science and art in the same year (1850).

Wilhelm Matzka died on June 9, 1891 in Prague.

2 Wilhelm Matzka — University Lectures and Publications

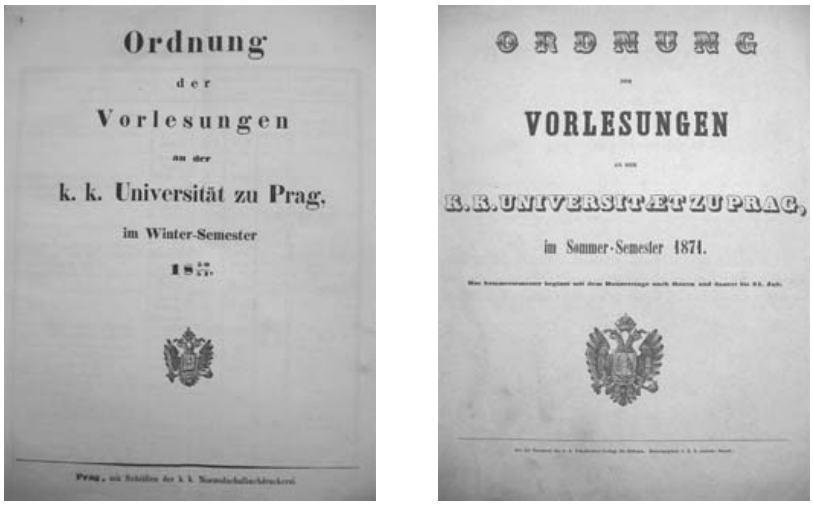
Wilhelm Matzka taught in German. The fact is that during his activity at the University of Prague, the level of mathematics teaching raised. He lectured above all on differential and

[8]Professor staff ruled the Faculty directly. It was formed by full and adjunct professors. Professor staff was one part of teacher staff and its the most important part. Teacher staff was more general and it was also formed by privat dozents and other teachers.

[9]More about the University of Prague see Kafka F., Petráň J., 1995–1998, *Dějiny Univerzity Karlovy I.–IV.*, UK, Karolinum, Prague.

[10]See archival materials of the University of Prague *Personalstand der k. k. Universität zu Prag.*

integral calculus, two- and three-dimensional analytic geometry, plane geometry, stereometry, algebraic analysis, spheric trigonometry, mathematical physics and analytical mechanics. Rarely he dealt with calculus of probabilities, numbers theory, goniometry, higher equations and some special parts of physics.



Ordnung

der

Vorlesungen

an der

k. k. Universität zu Prag,

im Winter-Semester

18$\frac{50}{51}$.

ORDNUNG

DER

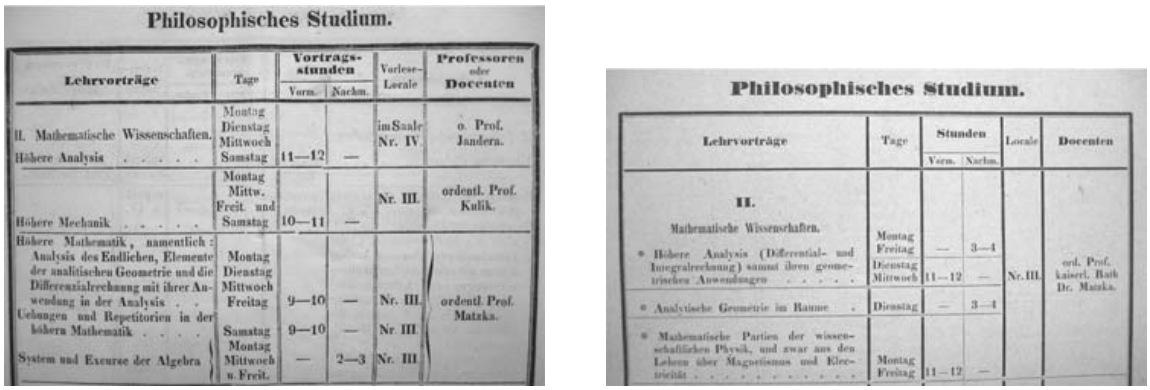
VORLESUNGEN

AN DER

K. K. UNIVERSITÄT ZU PRAG,

im Sommer-Semester 1871.

Figure 4 – Front page *Ordnung der Vorlesungen an der k. k. Universität zu Prag, im Winter-Semester 1850/51* and *Ordnung der Vorlesungen an der k. k. Universität zu Prag, im Sommer-Semester 1871.* In these documents we can find a list of W. Matzka's lectures at the University of Prague in the first and the last half of his acting.



Philosophisches Studium.

Lehrvorträge	Tage	Vortragsstunden Vorm.	Nachm.	Vorlese-Locale	Professoren oder Docenten
II. Mathematische Wissenschaften. Höhere Analysis	Montag Dienstag Mittwoch Samstag	11—12	—	im Saale Nr. IV	o. Prof. Jandera.
Höhere Mechanik	Montag Mittw. Freit. und Samstag	10—11	—	Nr. III.	ordentl. Prof. Kulik.
Höhere Mathematik, namentlich: Analysis des Endlichen, Elemente der analitischen Geometrie und die Differenzialrechnung mit ihrer Anwendung in der Analysis . .	Montag Dienstag Mittwoch Freitag	9—10	—	Nr. III.	ordentl. Prof. Matzka.
Uebungen und Repetitorien in der höhern Mathematik	Samstag	9—10	—	Nr. III.	
System und Excurse der Algebra	Montag Mittwoch u. Freit.	—	2—3	Nr. III.	

Philosophisches Studium.

Lehrvorträge	Tage	Stunden Vorm.	Nachm.	Locale	Docenten
II. Mathematische Wissenschaften.					
* Höhere Analysis (Differential- und Integralrechnung) sammt ihren geometrischen Anwendungen	Montag Freitag	—	3—4	Nr. III.	ord. Prof. kaiserl. Rath Dr. Matzka.
	Dienstag Mittwoch	11—12	—		
* Analytische Geometrie im Raume	Dienstag	—	3—4		
* Mathematische Partien der wissenschaftlichen Physik, und zwar aus den Lehren über Magnetismus und Electricität	Montag Freitag	11—12	—		

Figure 5 – The list of W. Matzka's lectures in the first and the last half of his acting (winter half 1850/51 and sommer half 1871) at the University of Prague, in *Ordnung der Vorlesungen an der k. k. Universität zu Prag, im Winter-Semester 1850/51* and in *Ordnung der Vorlesungen an der k. k. Universität zu Prag, im Sommer-Semester 1871.*

In these mathematical, also unmathematical, areas he wrote textbooks, expert articles, studies, historical, methodical and popular works.[11] The spectrum of his works was very comprehensive. He published except works from mathematicial areas also works about optics, mechanics, geodetics or astronomy.[12]

[11] A complete list of W. Matzka's publications does not exist till this time. To compile one, is one part of my doctoral thesis called *Life and Work of Wilhelm Matzka*. I start with information in bibliographic distionaries Poggendorff, J., C., 1904, *Biographisch-Literarisches Handwörterbuch*, Johann Ambrosius Barth, Leipzig, and Würzbach, C., 1867, *Biographisches Lexikon*, Druck und Verlag der k. k. Hof- und Staatedruckerrei, Wien. I check, specify and complete these information. I know nearly 60 works so far, the author of which is W. Matzka.

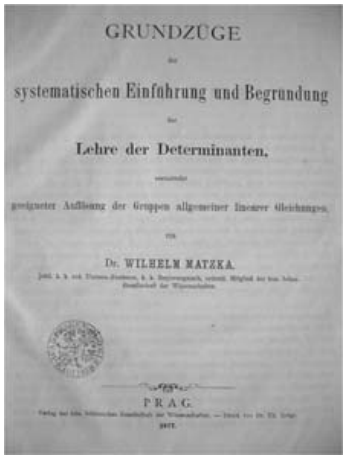
[12] In these unmathematical works he was probably affected by his studies in Vienna with the astronomer Prof. J. J. Littrow and with the physicist Prof. A. von Ettingshausen.

All of W. Matzka's works are in German. He published his works in: *Abhandlungen der königlichen böhmischen Gesellschaft der Wissenschaften*[13] [Discourses of the Royal Bohemian Society of Sciences], *Sitzungsberichte der königlichen böhmischen Gesellschaft der Wissenschaften*[14] [Protocols of Assemblies of the Royal Bohemian Society of Sciences], *Archiv für Mathematik und Physik*[15] [Archive for Mathematics and Physics], *Journal für die reine und angewandte Mathematik*[16] [Journal of Pure and Applied Mathematics], *Astronomische Nachrichten*[17] [Astronomic News], *Annalen der Wiener Sternwarte* [Annals of Vienna Observatory], *Annalen der Physik und Chemie*[18] [Annals of Physics and Chemistry]. Some works are issued separately.

3 Wilhelm Matzka — the Theory of Determinants

In the second part of 19th century high attention was attended to the theory of determinants. This problem was very favourite in Czech countries. A lot of les or more original special works had arisen. The first books of the theory of determinants, methodical and popular articles were written.

Wilhelm Matzka published a work in area determinants theory called *Grundzüge der systematischen Einführung und Begründung der Lehre der Determinanten, vermittelst geeigneter Auflösung der Gruppen allgemeiner linearen Gleichungen* [Principles of the Determinants Theory by the Help of Appropriate Solution the System of Linear Equations]. It was a work written in German published by the Royal Bohemian Society of Sciences in 1877.


GRUNDZÜGE

systematischen Einführung und Begründung

Lehre der Determinanten,

Dr. WILHELM MATZKA.

PRAG.

Figure 6 – Front page of W. Matzka's work about the theory of determinants.

[13] German magazine issued by *Königliche böhmische Gesellschaft der Wissenschaften* [the Royal Bohemian Society of Sciences]. The contribution to science of this magazine was publication of works, that were read in session of this society.

[14] German magazine issued by *Königliche böhmische Gesellschaft der Wissenschaften* [the Royal Bohemian Society of Sciences] from the beginning of the 19th century. It is one of the oldest special science magazine in Czech countries.

[15] Also called *Grunert's Archiv* [Grunert's Archive], according to its founder, who was German mathematician Johann August Grunert (1797–1872). This magazine, founded in 1841, dealt with mathematics, physics and astronomy. It belonged to excellent magazines and is still being issued.

[16] Also called *Crelle's Journal*, according to its founder, who was German mathematician August Leopold Crelle (1780–1855). This magazine, founded in 1826, is one of the oldest mathematical magazines and is still being published.

[17] Also called *Schumacher's Astronomische Nachrichten* [Schumacher's Astronomic News], founded in 1821 and called according to its founder, who was astronomer Heinrich Christian Schumacher (1780–1850). This is the oldest astronomical magazine in the world and is still being published.

[18] Also called *Poggendorff's Annalen*, according to its founder, who was German physicist Johann Christian Poggendorf (1796–1877). This magazin, founded in 1824, is still being issued under the title *Annalen der Physik* [Annals of Physics].

The work has sixty-one pages. Four of them are preface, further the text is divided into four paragraphs.

The author was engaged in elimaniton of unknowns from a system of linaer equations in the introduction. At first he characterized the form of the system of linear equations of several unknowns. Unknowns were marked by letters $x, y, z, t, u, v, w, \ldots$[19], their coefficients were marked at the same order by letters $a, b, c, d, e, f, g, \ldots$, and the known of the right side of this equation was marked m. The general form of equation was $ax+by+cz+dt+eu+fv+gw+\ldots = m$[20] and the system of linear equations having the same number equations as unknowns was:

$$\begin{aligned}
a_1x + b_1y + c_1z + d_1t + e_1u + f_1v + \ldots &= m_1\\
a_2x + b_2y + c_2z + d_2t + e_2u + f_2v + \ldots &= m_2\\
a_3x + b_3y + c_3z + d_3t + e_3u + f_3v + \ldots &= m_3\\
a_4x + b_4y + c_4z + d_4t + e_4u + f_4v + \ldots &= m_4\\
a_5x + b_5y + c_5z + d_5t + e_5u + f_5v + \ldots &= m_5\\
a_6x + b_6y + c_6z + d_6t + e_6u + f_6v + \ldots &= m_6\\
\ldots
\end{aligned}$$

Further is the general instruction how to solve this system of equations by the help of *subtractive method*. The author supposed that in every equation are all unknowns, and so all coefficients are different from zero. He imaginary merged the first equation with the second, the second with the third, the third with the forth, ..., and finally next to the last equation with the last equation, in one pair. From each pair of equations he eliminated one, the same, unknown.[21] In this way he deduced the determinant of the second order: From the first pair of equations he eliminated the first unknown x.[22] The diference, which he got from the first two pair numbers, $a_1b_2 - a_2b_1$[23], W. Matzka marked according to Laplace, by the help of marking minuend to parentheses (a_1b_2)[24]. He called this term according to the same mathematician *resultante* or newly according to Cauchy *determinant of the second order*, which was generally defined by $a_1b_2 - a_2b_1 \equiv (a_1b_2)$.

After eliminating unknown x he got the second system of linear equations, which had this form[25]:

$$\begin{aligned}
(a_1b_2)y + (a_1c_2)z + (a_1d_2)t + (a_1e_2)u + (a_1f_2)v + \ldots &= (a_1m_2)\\
(a_2b_3)y + (a_2c_3)z + (a_2d_3)t + (a_2e_3)u + (a_2f_3)v + \ldots &= (a_2m_3)\\
(a_3b_4)y + (a_3c_4)z + (a_3d_4)t + (a_3e_4)u + (a_3f_4)v + \ldots &= (a_3m_4)\\
(a_4b_5)y + (a_4c_5)z + (a_4d_5)t + (a_4e_5)u + (a_4f_5)v + \ldots &= (a_4m_5)\\
(a_5b_6)y + (a_5c_6)z + (a_5d_6)t + (a_5e_6)u + (a_5f_6)v + \ldots &= (a_5m_6)\\
\ldots
\end{aligned}$$

[19] He called them also in this order *the first, the second, the third,...* unknown.

[20] From this general form he got individual equations. He added ordinal number to coefficients, also to the number in the right side. He wrote it down and called it pointer or index.

[21] At first he ensured by both equations the same coefficient. He multiplied one equation with coefficient from the second, and vice versa. He subtracted these equations from each other, always the first from the second. These coefficients were automatically subtracted.

[22] He multiplied the first equation with a_2, the second equation with a_1, and subtracted them from each other. For coefficients by unknowns $y, z, t, \ldots$ he got $a_1b_2 - a_2b_1, a_1c_2 - a_2c_1, a_1d_2 - a_2d_1, \ldots$

[23] This term he got for coefficients by unknown y. It was put together by the cross multiplication from coefficients a_1, a_2 and b_1, b_2, that are under each other, if you like, from coefficients a_1, b_1 and a_2, b_2, that are beside of each other.

[24] He said that it is the simplest and optimal mark.

[25] He used determinant of the second order there, which he difined as term in parentheses, to make this system of linear equations simpler.

Next he eliminated the second unknown y from this system of linear equations and he got determinant of the third order. By the third unknown z he got coeficient $C = (a_1b_2)(a_2c_3) - (a_2b_3)(a_1c_2)$[26]. He used the definition of determinant of the second order and made next changes:

$$\begin{aligned} C &= (a_1b_2)(a_2c_3 - a_3c_2) - (a_2b_3)(a_1c_2 - a_2c_1) = \ldots = \\ &= a_2[(a_1b_2)c_3 + (a_2b_3)c_1] - c_2[(a_1b_2)a_3 + (a_2b_3)a_1] \\ (a_1b_2)a_3 + (a_2b_3)a_1 &= (a_1b_2 - a_2b_1)a_3 + (a_2b_3 - a_3b_2)a_1 = \ldots = \\ &= (a_1b_3 - a_3b_1)a_2 = (a_1b_3)a_2 \\ (a_1b_2)a_3 + (a_2b_3)a_1 &= (a_1b_3)a_2 \end{aligned}$$

He used the term $(a_1b_3)a_2$ in formula $C = (a_1b_2)(a_2c_3) - (a_2b_3)(a_1c_2)$ and he got $C = a_2[(a_1b_2)c_3 - (a_1b_3)c_2 + (a_2b_3)c_1]$. Trinomial $(a_1b_2)c_3 - (a_1b_3)c_2 + (a_2b_3)c_1$, formed from 3×3 coefficients $a_1, a_2, a_3; b_1, b_2, b_3; c_1, c_2, c_3$, W. Matzka marked according to Laplace $(a_1b_2c_3)$ and called this term according to Cauchy *determinant of the third order*, which was generaly defined by $(a_1b_2)c_3 - (a_1b_3)c_2 + (a_2b_3)c_1 \equiv (a_1b_2c_3)$.

Further in the book is in the same way deduced the determinant of the forth and the fifth order. In the book is also a historical part, that is about determinants by the mathematician Gabriel Cramer[27] (1704–1752). After this introduction there comes a chapter which establisches the basic properties of determinant; and finally a chapter which treates of the solution of linear equations. One part of the second paragraph is about correct notation of determinant's elements into columns ans lines when usual rules are used.[28] Next we can find short demonstrations of connection between this new notation and Laplace's notation.[29]

The best expert of the theory of determinants was the Scottish mathematician Thomas Muir (1844–1934). During the period 1906–1930 he created a work called *Theory of Determinants in the Historical order of Development*. This work has five volumes and there is a survey almost of all works about determinants, that are in chronological order from Leibniz (1693) until 1920. There are brief reports of these works and emhasize their mutual connections. In the third volume of this work is nearly one and half page about W. Matzka's work, which was described above. T. Muir wrote about it:

> What is fresh in this interesting memoir is the mode in which the student is introduced to determinants and becomes acquainted with their fundamental properties. The set of equations... is proposed for solution, and by multiplication and subtraction x is eliminated between every adjoinig pair of them, the opportunity being taken to give a definition of a determinant of the second order and to use Laplace's notation for the same...

Lots of mathematicians attended to the theory of determinants and their applications at schools during the 19th century in Czech countries.

The first czech textbook which was specialized in principles of determinants theory was *Determinanty a vyšší rovnice* [Determinants and Higher Equations], which was published

[26] Determinants of the second order were marked by terms in parentheses.

[27] He introduced a rule for solution heterogeneous system of n linear equations with n unknowns in the postskript in his work *Introduction á l'Analyse des Lignes Courbes algébriques*, Genéve, 1750. There he used terms that we called determinants today and the method is called Cramer's rule today.

[28] It is marking of determinants, which is used today, when the matrix of elements is written between two vertical lines. This notation is from Cayley from 1841. W. Matzka did not use the term of matrix!

[29] Determinant of the second order $(a_1b_2) = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$, determinant of the third order $(a_1b_2c_3) = \begin{vmatrix} (a_1b_2) & (a_1c_2) \\ (a_2b_3) & (a_2c_3) \end{vmatrix} = a_2 \cdot \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$, etc.

by Matin Pokorný, a secondary school teacher, in Prague in 1865. It was designed for the secondary schools. The work has 133 pages. The first part, about determinants, has 40 pages and the second part, about higher equations, has 93 pages. In the second chapter the author established determinants[30] and their basic characteristic. Next there are examples[31] how to get determinants from the second to the fifth order. There are other characteristics and their proofs and operations with determinants in next parts.

Karel Zahradník published elementary textbook for secondary school called *Prvé počátky nauky o determinantech* [The First Beginning of Determinants Theory] in Prague in 1879[32]. This textbook has 48 pages and is above all about determinats of the second and the third order, their basic characteristics, operations with determinants and their applications (by solution of system of linear equations, geometry, etc.). In this textbook we can find, in comparison with W. Matzka's and M. Pokorný's textbooks, not only general examples, but also practical examples with concrete numbers.[33] Further K. Zahradník published his work called *O determinantech* [About Determinants] in Brno in 1905. This textbook was especially for high technical school students. Next he published also Czech and Croatian lithographic mimeographed.[34]

František Josef Studnička wrote a lot of textbooks, mathematical and popular articles about deteminants and their applications. For example elementary textbooks which were especially for university students: *O Determinantech* [About Determinants] (1870, Prague) and *Úvod do nauky o determinantech* [The Introduction to the Determinants Theory] (1899, Prague).

Eduard Bartl, the professor at German real school in Prague, is the author of the textbook *Einleitung in die Theorie der Determinanten zum Gebrauche an Mittelschulen sowie zum Selbstunterrichte* [The Introduction to the Determinants Theory for Using at Secondary Schools and also for Self-taught People]. This German written book was published in Prague in 1878 and it was used at German secondary schools in Czech countries.

The classical German textbook was *Theorie und Anwendung der Determinanten mit Beziehung auf die Originalquellen* [The Theory and Applications of Determinants] which was published by Richard Baltzer in Leipzig in 1857. This textbook has 129 pages and it was a model for M. Pokorný's textbook *Determinanty a vyšší rovnice* [Determinants and Higher Equations]. It is dividend into two parts. The first part Theorie der Determinanten [The Theory of Determinants] has 34 pages. The second part Anwendug der Determinanten [Applications of Determinants] has 95 pages and is above all about the solution of system of linear equations, functional determinants and some other special applications of determinants.

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[30] For notation of determinant he used $\begin{vmatrix} a_{1,1} & a_{1,2} & a_{1,3} & \dots & a_{1,n} \\ a_{2,1} & a_{2,2} & a_{2,3} & \dots & a_{2,n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n,1} & a_{n,2} & a_{n,3} & \dots & a_{n,n} \end{vmatrix}$ or $\sum \pm a_{1,1}a_{2,2}\dots a_{n,n}$.

[31] There are not practical examples, but general examples, such as example for determinant of the second order $\begin{vmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{vmatrix} = a_{1,1}a_{2,2} - a_{1,2}a_{2,1}$, determinant of the third order $\begin{vmatrix} a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,1} & a_{2,2} & a_{2,3} \\ a_{3,1} & a_{3,2} & a_{3,3} \end{vmatrix} = a_{1,1}a_{2,2}a_{3,3} - a_{1,1}a_{2,3}a_{3,2} + a_{1,2}a_{2,3}a_{3,1} - a_{1,2}a_{2,1},a_{3,3} + a_{1,3}a_{2,1}a_{3,2} - a_{1,3}a_{2,2}a_{3,1}$, etc.

[32] It was published in Croatian in Zagreb one year earlier.

[33] The second chapter start with the "definition" of determinant of the second order $\Delta_2 = \begin{vmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{vmatrix}$. The author wrote that this determinant has $2^2 = 4$ elements, two lines and two columns, two terms $-a_1b_2$ and $+a_2b_1$. Next is general example $\Delta_2 = \begin{vmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{vmatrix} = (a_1b_2) = a_1b_2 - a_2b_1 = a_1b_2 - b_1a_2$, and practical example $\begin{vmatrix} 3 & 5 \\ 2 & 7 \end{vmatrix} = 3 \cdot 7 - 2 \cdot 5 = 11$.

[34] *O determinantima. Predavanja u nimskom semestru godine 1897/8.* Zagreb, 112 pages, and *O determinantech. Přednášky z vyšší mathematiky I. běh, část úvodní.* Brno, 1903–1904, 62 pages.

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