Contribution of Czech Mathematicians To Probability Theory

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Abstract

The paper discusses three groups of contributions of Czech mathematicians to probability theory in the 19^{th} and the first half of the 20^{th} century, namely the contributions dealing with the foundations of probability theory (Karel Rychlík, Otomar Pankraz), contributions dealing with interpretations of probability theory (Bernard Bolzano, Tomáš Garrique Masaryk, Emanuel Czuber, Otomar Pankraz, Václav Šimerka) and contributions to the development of probability theory as a mathematical discipline (Emanuel Czuber, Bohuslav Hostinský). The aim of this discussion consists not only in a historical overview, but above all in the motivation of teachers to reappraise the usual approach to probability theory education and to find a way how to make probability theory accessible to everybody as one of the most interesting and important mathematical disciplines with a close relation to our daily life. The common feature of all discussed contributions is the conception of probability as conditional: probability does not mean throwing an absolutely ideal dice; what is really substantial is a probability that something happens under certain conditions.

1 Introduction

In the school mathematics, probability theory often seems to be identified with throwing dices and coins or drawing balls, with artificial examples without any connection to reality, and for most students (and perhaps also many teachers) it is therefore an unloved discipline.

On the other hand, we are surrounded by randomness: consider an organic world (tissue cells, vegetations, people themselves, ...), inorganic world (molecules of gas and liquid, crystals, ...), random meetings or accidents, illnesses and chance for healing and surviving, defects of materials, failures of railway systems, etc. Every day we are faced with various hypothesis about our surroundings and about ourselves: for example, global warming, human evolution, psychological processes, reasons and causes of our illnesses, credibility of historical events, partners, friends, etc. We constantly search grounds for them and ask, to which degree these grounds support the hypothesis in question and to which degree we can believe it. Even if we have solid measurements or observations, our evidence is always restricted and entails the validity of a hypothesis only partially, with some probability. Theory of probability is therefore substantial e.g. for physics, biology, medicine, engineering, humanities, as well as for our everyday life. It is a great task for us, mathematics teachers, to perceive it, to devote adequate space to probability theory in education and to persuade our students that it is one of the most interesting and important disciplines, inseparably connected with our lives.

We hope that the discussion of the contributions of Czech mathematicians to probability theory helps the readers to find the way to master this task. In various context we shall see that the mathematicians mentioned in the present paper conceived probability theory as a substantial tool for scientific and philosophical cognition. They also seem to be aware of inadequacy of unconditional probability for real applications and of importance of *conditional probability* as a fundamental concept of the theory.[1]

2 Contributions dealing with the foundations of probability theory

Let us briefly recall that from the point of view of pure mathematics, an important milestone was represented by Kolmogorov (1933).[2] Here an axiomatization of probability theory was given in today sense and up to some exceptions, it has generally been accepted. It also led to the acceptance of probability theory as a "true" mathematical discipline. Soon after its publication several reviews appeared; in mathematical papers it started to be cited in 1934. The theory is usually considered established when it gets into textbooks. In this case the first textbook that incorporated Kolmogorov's axioms into the exposition was Cramér (1937).

2.1 Karel Rychlík (1886–1968)

In the Czech lands we can observe an immediate reaction to Kolmogorov's axiomatics. Karel Rychlík, professor of mathematics at the Czech Technical University in Prague and private associate professor at Charles University in Prague, promptly recognized the significance of Kolmogorov's work. Shortly before the beginning of the winter semester 1933/34, he canceled the originally announced lecture on linear algebra at Charles University and replaced it by the lecture *Introduction to probability calculus (from the axiomatic point of view).* Only one year after Cramér, Rychlík (1938) published the textbook *Introduction to Probability Calculus* based on axioms for probability distribution in a set field corresponding to the system proposed by Kolmogorov. Not only made it Kolmogorov's axiomatic probability theory available to students soon after its birth but it put the two current theories abreast: the theory of Kolmogorov and a bit older frequency theory of von Mises. Rychlík accepted the later in relation to reality and spent enough space to show its usefulness for practical applications.

2.2 Otomar Pankraz (1903–1976)

Rychlík's assistant at the Czech Technical University in Prague, Otomar Pankraz, was also interested in the development of probability theory. In 1939 and 1940, he published a couple of papers dealing with probability axioms. Inspired by Reichenbach (1935), Pankraz criticized Kolmogorov's theory for introducing probability as a one-argument function $P(A)$ only, leaving a conditional probability (a two-argument function) to an additional definition:

$$P_A(B) = \frac{P(A \cap B)}{P(A)}$$

Pankraz argued that it was just the *conditional probability* that should have been the fundamental concept of the whole theory, and introduced the axiomatics based on the conditional probability.[3]

If probability theory should not be a mere mathematical theory far from the reality, this opinion seems to be quite reasonable. It is *conditional probability* that corresponds to our

[1] Lecture slides are available at the web page: http://euler.fd.cvut.cz/~hyksova/lectures.

[2] For the discussion of the predecessors, see the paper of Lambalgen (1996).

[3] Let us remark that such an approach was also advocated by Popper (1959) and Hájek (2003).

experience; unconditional probability seems rather artificial: no dice is perfect, no board is absolutely flat, every event occurs *under certain conditions*. With the words of Bruno de Finetti: *Every prediction, above all every probability evaluation is conditional; not only by a mentality or psychology of the individual in question, but also — and above all — by the degree of knowledge...* (de Finetti, 1974). In a completely non-mathematical world, the main hero of the movie Pianist says: *I'm sure I could be a movie star, if I could get out of this place*. In other words, his probability of becoming a movie star is high, but *conditionally* on his escaping from certain place.

As a motivation for his axiomatics, Pankraz considers so-called *randomness propositions* of the form: *An event E occurs* $\Leftrightarrow$ *one of elementary events of a set C occurs before it*. Here C is an arbitrary set that represents a set of possible causes of an event E. In other words, when E occurs, we know that some of the events from C must have occurred before but we do not know with certainty which one. For example:

Hypothesis H — one specific element of C:

- Erroneous calculation of the structural engineer
- Erroneous opinion of the geologist
- The site manager did not keep the project
- The supplier provided bad material
- The neighbor damaged the subsoil when extending a cellar

...

Available evidence E:

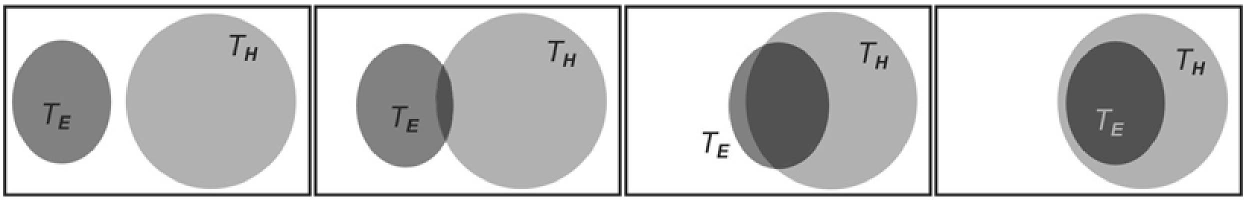
The question is, which one of the possible causes actually led to it; each cause represents a hypothesis and we are interested in the degree to which this hypothesis follows from our restricted evidence. In other situations, randomness propositions may concern predictions about future events, or they need not necessarily run on the time scale; we may be interested for example in eventual causes of some physical or biological phenomenon.

Let us remark that the mentioned cases illustrate the difference between *deductive* and *inductive logic*. In the former one, the premises logically entail the conclusion. The later one was established with the aim to deal with inductive conclusions that are not fully guaranteed by premises. The specification of a measure of the degree to which an evidence E supports a hypothesis H is called *inductive* (*logical*) *probability of H supported by E* and it can be expressed by

$$P(H \mid E) = \frac{P(E \wedge H)}{P(E)} = \frac{P(E \mid H)\, P(H)}{P(E)} \text{ for } P(E) \neq 0. \qquad (1)$$

Note that if $E \Rightarrow H$, i.e., if the domain of truth T_E of the evidence E is contained in the domain of the truth T_H of the hypothesis H, $T_E \subseteq T_H$, then $P(E \wedge H) = P(E)$ and

$P(E \wedge H)/P(E) = 1$. If, on the contrary, $E \Rightarrow \neg H$, the domains of truth of H and E are disjoint, $T_E \cap T_H = \emptyset$, $P(E \wedge H) = 0$ and $P(E \wedge H)/P(E) = 0$. Still, there are many more possibilities for the relation between domains of truth of E and H. Intuitively, the greater part of T_E is contained in T_H, the higher is the degree to which E entails H, and it is reasonable to identify this degree with the size ratio $p = |T_E \cap T_H| / |T_E|$, where $|\cdot|$ is a suitably selected set measure, and take use of the correspondence to probability calculus:



$T_E \cap T_H = \emptyset$	$\emptyset \subset T_E \cap T_H \subset T_E$	$T_E \cap T_H = T_E$
$P(H \mid E) = 0$	$0 < p = P(H \mid E) = \dfrac{P(E \wedge H)}{P(E)} < 1$	$P(H \mid E) = 1$
$E \Rightarrow \neg H$	$E \Rightarrow_p H,\ p = \dfrac{\lvert T_E \cap T_H \rvert}{\lvert T_E \rvert}$	$E \Rightarrow H$

As before, it is meaningless to speak about the *probability of a hypothesis*, we can speak only about its *probability based on the given evidence*. As we shall see in the next section, the described conception is termed *logical interpretation of probability*.

3 Contributions dealing with interpretations of probability theory

In simplified words, for a pure mathematician, probability is a real function over a σ-algebra with values in the interval $[0, 1]$ and satisfying certain axioms, which lead to a nice theory. Nevertheless, this explanation is not satisfactory for philosophers and all other scientists who would like to use probability theory in the real world. Therefore they are trying already for a long time to find an answer to the seemingly simple question, namely what the probability *really is*, how to interpret it.[4]

Recall that two main groups of interpretations are usually distinguished, namely *epistemological interpretations* identifying probability with the degree of our knowledge or belief, and *objective interpretations* that consider probability as feature of the objective material world, independent of the individual, without any relation to human knowledge or belief.

In this paper, we will restrict our attention to the first group. In Czech lands we can find remarkable contributions to both types of epistemological interpretations, namely to logical interpretation that identifies probability with the *degree of rational belief* and can therefore be understood as an extension of deductive logic, and subjective interpretation that identifies probability with the degree of belief of a particular individual.

3.1 Logical interpretation

As the main representatives of logical interpretation are usually considered William Ernst Johnson, John Maynard Keynes, Ludwig Wittgenstein, Harrold Jeffreys and Rudolf Carnap, who dealt with it in between 1920's and 1950's. Recently also a 1886 book by Johannes von Kries and another 50 years older contribution of Bernard Bolzano started to be again appreciated.[5] It is remarkable that still in the first half of the 20$^{\text{th}}$ century the last two names were often cited and they were considered important. Nevertheless, later came the contributions written in English into the foreground. In addition to the mentioned authors, there

[4] A detailed survey of various interpretations can be found in the book of Gillies (2000).
[5] See e.g. papers by Heidelberger (2001) or Hykšová (2006).

are several more who are much less famous or almost forgotten, yet deserve our attention: Tomáš Garrigue Masaryk, Emanuel Czuber and Otomar Pankraz.

BERNARD BOLZANO (1781–1848)

Philosopher, mathematician and theologian Bernard Bolzano, native of Prague, incorporated probability calculus into a religious textbook published in 1834, in order to defend the Holy Scripture against attempts to shatter the belief or more precisely, to predict the decay of Christian belief. From the mathematical point of view, more interesting seems to be the book *Wissenschaftslehre* (1837) where Bolzano builds probability theory as an extension of deductive logic. He considers a relative validity of a proposition H with respect to propositions A, B, C, ... as the size ratio (compare 2.2)

$$\frac{|\text{set of all cases where besides } A, B, C, D, \dots \text{ a proposition } H \text{ is true}|}{|\text{set of all cases where all propositions } A, B, C, D, \dots \text{ are true}|},$$

which he calls *probability* and uses probability calculus for operations with it. Note that it coincides with the conception of probability as the degree of justification of a hypothesis H on the basis of the evidence $E = A \wedge B \wedge C \wedge \cdots$,[6] as mentioned above. If we denote with $m(X)$ the measure for the set of the cases where a proposition X is true, we obtain

$$P(H \mid E) = \frac{m(H \wedge (A \wedge B \wedge C \wedge \ldots))}{m(A \wedge B \wedge C \wedge \ldots)} = \frac{m(H \wedge E)}{m(E)} \text{ for } m\,(E) \neq 0.$$

Remark that the “inconspicuous” dots in the expression of the evidence $E = A \wedge B \wedge C \wedge \ldots$ express exactly the core of the problem we are faced whenever we deal with real situations. We are mostly unable to name all premises such that their truth guarantees the truth of a hypothesis in question. For example, consider a hypothesis: $H \equiv$ *at 17:30 I will be at home and have a dinner*. The validity of this hypothesis is conditioned e.g. by the premises $E_1 \equiv$ *no traffic jam occurs*, or $E_2 \equiv$ *the chief will not want any additional work*. Still, we can write only a *probability implication* $(E_1 \wedge E_2) \Rightarrow_p H$, since there can always appear another event that prevents us from being at home at 17:30. For example, we can get stuck in a lift, so an additional premise E_3 should exclude it, and we obtain $(E_1 \wedge E_2 \wedge E_3) \Rightarrow_{p'} H$, etc; the dots remain always at the end: $E = E_1 \wedge E_2 \wedge E_3 \wedge \ldots$.

Bolzano's contribution to probability theory was cited for example by Emanuel Czuber (1923) and several participants of the conference *Erste Tagung für Erkenntnislehre der exakten Wissenschaften* that took place in Prague in 1929 (P. Frank, F. Waismann, W. Dubislaw; their contributions were published in the first volume of *Erkenntnis*, a publication series of the Vienna Circle whose program declaration was read just at the Prague conference). In the introduction to the new edition of *Wissenschaftslehre*, J. Berg compared the theories of Bolzano, Wittgenstein and Carnap and highly appreciated Bolzano's contribution by denoting him the first philosopher who drew up the concept of inductive probability.[7]

TOMÁŠ GARRIGUE MASARYK (1850–1937)

It is not well known that the first president of the Czechoslovak Republic was also dealing with probability theory. Recall that Masaryk studied philosophy and philology at the university in Vienna. In 1878 he was there appointed associate professor on the basis of the treatise *Suicide as the Social Phenomenon of Present Time*. Four years later, Masaryk became professor at Charles University in Prague; for his inaugural lecture he chose the topic *David Hume's Scepsis and Probability Calculus* that was later published in Czech and English (1883 and

[6] The domain of true of the evidence E is tacitly but naturally supposed to be non-empty.

[7] We shall not omit the work of Pierre-Simon Laplace; nevertheless, Bolzano's treatise was more exact, clear and brief.

1884, respectively). The aim of this contribution was to disprove Hume's scepsis consisting in the following: mathematics alone deserves our confidence, sciences based on experience are unsafe since the understanding of causal connections evades us. On one hand, we must agree: indeed, we are not able to predict anything on the basis of our experience; a new premise may appear and everything changes. On the other hand, we need predictions, we need hypothesis about our surroundings, we need sciences based on experience. Thus it is not satisfactory to say they are unsure and logically groundless, so that we should stop to develop them.

To accomplish his aim, Masaryk provides a detailed historical overview of attempts to disprove Hume's scepsis. He starts with the Scottisch school (T. Reid, J. Beattie, J. Oswald), I. Kant, F. E. Beneke and J. G. Sulzer, then he discusses the attempts to disprove the scepsis with the help of probability theory, namely the contributions of J. G. Sulzer, M. Mendelssohn, J. M. Degérando, S. F. Lacroix, S. D. Poisson. Finally he deals with inductive logic and probability theory in general; here he cites G. W. Leibniz, J. Bernoulli, P. S. Laplace, A. Quetelet and R. Herschel. Masaryk concludes: *All these newer treatises miss an explicit reference to Hume; they miss therefore, I would like to say, a true point* [...] *Hume himself spoke much about probability, but it seems that he did not know the mathematical rules of probability calculus, since he was not able to distinguish subjective and objective probability, and it is therefore understandable how he came to his sceptical theory of induction...* (Masaryk, 1883, pp. 14–15). At the time of writing his treatise, Masaryk seems not to be aware of the work of Bernard Bolzano who explicitly cited Hume (Bolzano, 1834) and who gave the foundations of inductive logic (Bolzano, 1837).[8]

Four years after his arrival to Prague, Masaryk became widely known in the connection with his fight for the truth about supposititious old Czech manuscripts that were found in 1817 in Dvůr Králové nad Labem (Königinhof an der Elbe) and Zelená Hora (Grünberg). The former was originally placed to the end of the 13$^{\text{th}}$ century, the later to the 9$^{\text{th}}$–10$^{\text{th}}$ century.[9] Soon after their discovery, doubts about the authenticity appeared. First mainly in the connection with the older one, later also in the case of the Königinhof manuscript. Nevertheless, the defenders were very vehement, both manuscripts significantly influenced Czech literature and national renaissance. A new discussion arose in 1886 when Masaryk provided space to opponents of the authenticity in the journal *Athaeneum* of which he was the editor. He invited the philologist and literary historian Jan Gebauer to publish his reasons for falsification. This analysis was followed by many other contributions disproving the authenticity for other reasons, e.g. historical, sociological, aesthetical and paleographical. Although the response of the defenders was passionate, the falsification was finally proved.[10] It is interesting that it was also the probability theory that contributed to this proof.

Briefly, Gebauer (1886) gave two main philological grounds for the falsification hypothesis: grammatical "oddities", i.e., deviations from the Czech grammar of that time determined from other, provably authentic manuscripts, and concurrent occurrence of "suspicious" forms in Grünberg and Königinhof manuscripts and in the works from the 19$^{\text{th}}$ century written before 1817. Historian Josef Kalousek and other defenders of the authenticity claimed that these oddities and suspicious forms were only accidental. August Seydler, physicist and Masaryk's friend, therefore decided to calculate the probability that all those forms were really accidental. He did so in the couple of papers published in 1886 and the result was clear: probability that all deviations from the old Czech grammar and all coincidences were

[8] However, when a Bolzano Committee was established after the First World War with the aim to organize and publish all Bolzano's manuscripts, Masaryk supported its activities both as the state president as well as a private person.

[9] A continuous series of provably authentic Czech manuscripts starts in the 13th century.

[10] In the scientific circle the opinion soon prevailed that both manuscripts were really falsificated. In 1967 it was once more and definitively proved.

accidental, was

$$P < \frac{1}{3 \cdot 10^9} \cdot \frac{1}{10^{14}}.$$

The oddities and coincidences require therefore an explanation, it is not satisfactory to blame the mere chance.

EMANUEL CZUBER (1851–1925)

One more name cannot be missing in this section: Emanuel Czuber, professor of mathematics at the technical secondary school in Prague, later at the Technical University in Brno (1886–1891) and at the Technical University in Vienna (1891–1921). Eight years after arriving to Vienna he published an extended study on the probability theory (Czuber, 1899). Its first chapter is devoted to the foundations of probability theory from the historical as well as philosophical point of view. Czuber emphasizes the logical interpretation of probability and besides the well-known names, he cites e.g. J. von Kries and C. Stumpf. Further parts of the treatise deal with various applications of probability theory; each topic contains the outline of its history, the greatest stress is laid on the concept formation and its philosophical aspects. In 1923 Czuber published the book solely devoted to the philosophical foundations of probability theory. Again, Czuber promoted the logical interpretation of probability, put stress on its significance for epistemology and natural philosophy, and among the predecessors he cited Bernard Bolzano.

3.2 SUBJECTIVE INTERPRETATION

Let us recall that the subjective interpretation regards probability as the degree of belief of a particular individual. That is, in the formula (1) the aposterior probability $P(H \mid E)$ expresses the degree of belief in a hypothesis H based on the evidence E (situation, circumstances, witnesses). As before, an important role is played by conditional probabilities. Note that this approach corresponds to our everyday considerations ("this street is probably more dangerous", etc.), it deals with real concepts, with subjective acceptance or rejection of hypothesis. Nevertheless, numerical expression is not at all trivial. Let us remark that one of possible solutions is to use an analogy to a betting system.

As the founders and main representatives of the subjective interpretation of probability are usually considered Frank Plumpton Ramsey (1931) and Bruno de Finetti (1937), later Leonard Jimmie Savage (1954).

VÁCLAV ŠIMERKA (1818–1887)

But almost half a century sooner, the Czech priest Václav Šimerka published a remarkable treatise *Power of Conviction* (Šimerka, 1882 and 1883), which can also be included into this direction of thoughts. Šimerka asks: how can the conviction be expressed by numbers? He states: *For this purpose the probability calculus is exceptionally convenient, since our conviction about the possibility of an event increases in the same rate as does the mathematical probability, that is, everything is more believable, the more it seems to be probable. The terms in the sequence [...] empty mind,*[11] *feeling, ..., up to knowledge and certainty can therefore be expressed by numbers between* 0 *and* 1*, where* 0 *corresponds to none,* 1 *to the highest conviction.* (Šimerka, 1883, p. 517)

Causes or sources of the conviction are called *grounds*, their power v is expressed by *probability.* To assemble more convictions together, Šimerka introduces the concept of an *imperfection of a conviction* as a difference $\varepsilon = 1 - v$ between the complete knowledge and the given conviction v. Consider convictions v, v', v'', ... and the corresponding imperfections. The

[11] This term denotes either a complete ignorance or a state in which the grounds supporting and disproving a hypothesis are in equilibrium.

resulting power of conviction V is given by the formula $1 - V = (1 - v)\,(1 - v')\,(1 - v'')\cdots$, which can be expressed as follows: the imperfection of a human conviction is a product of imperfections of its grounds. For $v = v' = v'' = \ldots = 0$ we have $V = 0$; according to Šimerka's words: *empty grounds provide no belief.* For $v' = v'' = \ldots = 0$ we obtain $V = v$ and the characterization: *in an empty mind every ground enroots with its full power.* Šimerka continues:

This is attested not only by the experience from schools and common men, many of which believe even very shaky novels and stories, but also the experiences of missionaries who give evidence that Christianity enroots the best in the nations with disordered minds, when their original superstitions were rebuttet, without being substituted by anything else; otherwise is it much more difficult. [...] *The empty mind can therefore be deceived by false grounds, what would be otherwise not so simple. It is clear that this is the basis of the old immoral principle: slander, something will stick in the memory.* (Šimerka, 1883, p. 517)

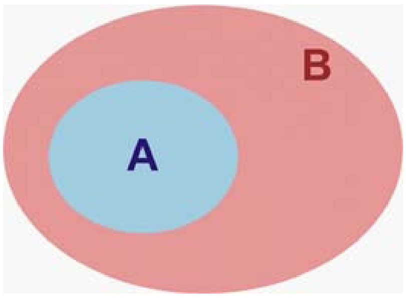
Šimerka's extensive and interesting treatise was appreciated by Masaryk (1885). Otherwise, although it was published also in German, it remained without any substantial influence on the later development of the subjective interpretation of probability.

4 Contributions to the development of probability theory

Let us finally mention some of the Czech contributions to the development of probability theory as a mathematical discipline. Czech mathematicians of the 19$^{\text{th}}$ and first half of the 20$^{\text{th}}$ century gained the greatest respect in two directions, namely in the domain of geometrical probability and in the field of Markov chains.

4.1 Geometrical probability

Recall that the geometrical probability concept originated as an extension of the classical definition of probability to situations with uncountable sets of elementary events. Then it is necessary to replace the numbers of favourable and all cases by convenient measures. For example, we can look for the probability that a point randomly chosen in a set B belongs to a subset $A \subseteq B$, too:



$$P\,(X \uparrow A \mid X \uparrow B) = \frac{\text{measure of the set } A}{\text{measure of the set } B}$$

Intuitively, it is reasonable to use length, area or volume as a measure of line segments, plane areas or space areas, respectively (and Lebesgue measure in general). Instead of points we can also consider randomly chosen lines or planes and appropriate multiple integrals for corresponding measures. Then, if we replace geometrical points, lines or planes by *probes* or *cuts*, we come to great many applications in medicine, biology, material engineering, geology, etc. As we could see it in other contexts, also geometrical probability is necessarily conditional: for example, it is meaningless to ask after an "absolute" probability that a point hits a bounded set in a plane, since the measure of the whole plane is infinite and the probability would always be zero. It is therefore necessary to condition the probability by hitting another specific bounded set.

Recall that the roots of geometrical probability begin in 1733 when Louis Leclerc, Comte de Buffon, presented the solution of today famous needle problem and several other examples. Buffon's ideas were further developed throughout the 19$^{\text{th}}$ century by P. S. de Laplace and I. Todhunter. In 1865 various problems concerning geometrical probability started to be published in the British journal *Mathematical Questions with Their Solutions. From the "Educational Times"*. Among the most important authors we can find J. J. Sylvester, M. W. Crofton, T. A. Hirst and A. Cayley. In the following years, these and several more British mathematicians continued in the investigation of various specific problems concerning geometric probability in the plane. Approximately at the same time but almost independently was geometrical probability studied by French mathematicians G. Lamé, J. Bertrand and J. É. Barbier.

EMANUEL CZUBER (1851–1925)

Emanuel Czuber started to work in the field of geometrical probability already as the secondary school teacher in Prague. In 1884 he published a treatise where he extended Crofton's results concerning lines in plane to lines and planes in space, and showed possible applications of the proven general theorems (Czuber, 1884). In the same year he published the first monograph summarizing the state of the art of geometrical probability of that time and containing also new results and generalizations (Czuber, 1884a). In the introduction Czuber briefly recalled the history of this theory from Buffon over Laplace up to a more intensive development in the second half of the 19^{th} century. Among the names he cited we can find British mathematicians A. R. Clarke, H. Mc'Coll, E. B. Seitz, J. J. Sylvester, S. Watson, J. Wolstenholm and W. S. B. Woolhouse, and French mathematicians J. É. Barbier, C. Jordan, E. Lemoine a L. Lalanne. A special recognition is attributed to M. W. Crofton. Seneta, Parshall and Jongmans (2001) expressed a conjecture that only Czubers monograph draw Crofton's attention to the contributions of French mathematicians and thus created a bridge between England and France.

Czuber returned to geometrical probability also in later treatises and incorporated it also into his probability textbook. In all cases he started from the latest state of the theory and enriched it with original ideas.

BOHUSLAV HOSTINSKÝ (1884–1951)

The first contribution of Bohuslav Hostinský, professor of theoretical physics at Masaryk University in Brno, in the field of geometrical probability concerned Buffon's needle problem. Hostinský (1917, 1920) criticized the traditional solution for being based on an unrealistic assumption that parallel lines are drawn on an unbounded board and the probability that the mid point of the needle hits a region of a given area is proportional to this area and independent of the position of the region. Hostinský argued that no real experiment could satisfy such an assumption, and replaced it by a more realistic one: parallel lines are drawn on a square table board and the experiment requires the needle to fall on it; now the probability that the mid point of the needle hits a square of a given area nearby the edge of the table is lower than the probability that it hits a square of the same area nearby the middle. To solve this problem, Hostinský generalized Poincaré's method of arbitrary functions, and came to the solution that contained the classical one as a limit case. In 1920 Hostinský sent the French variant of his paper to *Bulletin des Sciences*. Subsequently he discussed it in the correspondence with M. Fréchet, which could have awoke Fréchet's interest in probability theory.[12] Six years later Hostinský published the first (and for a long time the unique) Czech book on geometrical probability (Hostinský, 1926).

4.2 MARKOV CHAINS

The second domain in which Hostinský played a significant role was the theory of Markov chains, that is, stochastic discrete-state and discrete-time processes in which the probability

[12] For more details see the paper of Havlová, Maziljak, Šišma (2005).

of a transition from state x_t to state x_{t+1} depends only on x_t and is independent of the way how the system has attained it.

A detailed analysis of Hostinský's contributions exceeds the scope of this paper. Let us only remark that at the international congress of mathematicians in Bologna in 1928 both Hostinský and Hadamard presented contributions (based on their previous publications) dealing with the cards problem. Still at the congress, G. Pólya draw their attention to a 20 years older work of A. A. Markov containing similar ideas. Thus the concept of Markov chain emerged and then spread immediately. Nevertheless, a similar method was used already by L. Bachelier in his thesis from 1900. And according to A. P. Juškevič,[13] such method appeared at first in the treatise of Francise Galton from 1889. Let us finally point out that while Markov applied "Markov chains" to an analysis of part of the text of Evzen Onegin, Hostinský emphasized physical applications concerning Brownian motion and ergodic principle. It is perhaps not necessary to recall that today Markov chains play a fundamental role in physics, queuing theory, railway safety systems, internet applications, mathematical biology and many other domains.

5 Conclusion

In this paper we were discussing the contributions of Czech mathematicians to probability theory. A golden thread of all sections was the attempt to stress that probability is everywhere around us — only it does not seem to be properly at schools. Let us therefore conclude with the question to us, mathematics teachers: what shall we do with it?

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