Manuscripts and Teachers of Commercial Arithmetic in Catalonia (1400–1521)

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Abstract

During the last 40 years, an increasing number of commercial arithmetic and algebra texts from the period 1300–1600 has been studied. Considering the great economic development of many Italian cities during this period, it is hardly surprising that most of these texts were composed in Italy. Francesc Santcliment's Summa de l'art d'Aritmètica is a book on commercial arithmetic that was written in Catalan and published in 1482 in Barcelona. It was the first mathematics book printed in the Iberian Peninsula and the second printed commercial arithmetic in Europe. However, even when printed treatises like this started to be published and to be widely used, manuscripts continued to play an essential role in the teaching of commercial arithmetic and algebra. In fact, some of these manuscripts give us a closer view on the teacher's daily work, showing how contents of the printed treatises were adapted to each educational context.

This talk is based on my research on Catalan manuscript sources for commercial arithmetic and algebra during the late Medieval and early Renaissance periods. I will resort to some of these manuscripts to show which contents were studied and which teaching methods were used, among other topics.

Keywords: Commercial arithmetic, Catalonia, Fifteenth-century

1 The context

The growing complexity of commercial practice in late Medieval Europe made knowledge of arithmetic more necessary for many people, particularly for merchants. The teaching of the Hindu-Arabic numeration system, its methods of calculation and applications to commerce was made mainly through vernacular treatises that started to appear towards the end of the thirteenth century.[1] The fact that most of them were written in Italian is an obvious consequence of the economical development of the Tuscan cities. These treatises were usually known as *trattati d'abaco* in Italy. When using expressions like "abbacus teacher" or "abbacus school", we will be referring, respectively, to a teacher of practical mathematics in a vernacular language and a school where commercial arithmetic was taught. Thus we will use this terminology not only in the Italian but also in the broader European context.

[1] To get introduced in Italian commercial arithmetic between the 14th and the 16th centuries, see (Franci, Toti Rigatelli, 1982), (Swetz, 1987) and (Van Egmond 1980, 3–33). For the French-Provençal case, see (Benoit, 1982), (Beaujouan, 1988) and (Spiesser, 2003). For the Catalan case, see (Malet, 1998), (Docampo, 2004a) and (Salavert, 1990).

It seems that abbacus schools (or *botteghe d'abaco*) started to appear in the second half of the thirteenth century. Some of them were public schools, but most of those that were established in the biggest trading centres such as Florence and Venice were mostly private institutions. Florence was a very important centre for the teaching of commercial arithmetic: in 1338 there were 6 abbacus schools that were attended by more than 1 000 students in total. Boys entered them when they were 10 or 11 and stayed there for 2 years, before leaving them in order to complete their education in bank offices, trading centres, etc.[2]

As far as we know, two programs for teaching in abbacus schools have been found so far. One of them was used in Pisa in the first half of the 15th century, and the other was followed in 1519 in a Florentine school.[3] Even when the level of the knowledge and the skills that were acquired could be quite diverse, we know that the basic contents that were taught were the numeration system, the four elementary operations with integers and fractions and the rule of three. Then, the pupils would learn how to apply these essential tools to face real-life trading situations: mostly problems of partnership, barter, exchange and alligation. These are also the most usual topics that appear in abbacus treatises, together with some recreational problems. In some of the texts, we also find a section on practical geometry, mainly consisting of calculations of areas and volumes and simple applications of the right-angled theorem.[4] Algebra appears in several texts of the abbacus tradition, although false position methods were often applied to what we would call first-degree problems.

Most of the main features that we have quoted for the Italian abbacus tradition can also be found in Catalan commercial arithmetic. On the other hand, there are clear connections between Catalan and French-Provencal treatises.[5]

It is significant that the first mathematics book that was printed in the Iberian Peninsula was a commercial arithmetic. The *Summa de l'art d'Aritmètica*, by Francesc Santcliment, was written in Catalan and published in 1482 in Barcelona.[6] This early printing attests for the wide audience of this kind of books. The *Summa* of Santcliment was also, as far as we know, the second printed commercial arithmetic in Europe. In the Iberian Peninsula, it was followed by its version in Castilian (Zaragoza, ca. 1487),[7] and the treatises of Juan de Ortega (Lyon, 1512), Juan Andrés (Valencia, 1515), Gaspar Nicolas (Lisbon, 1519) and Joan Ventallol (Lyon, 1521).[8] Ventallol's arithmetic, which was titled *Pratica mercantívol* was the last important work on commercial arithmetic printed in Catalan until 1596, when Bernat Vila's *Reglas breus de arithmètica* appeared.

2 Teaching commercial arithmetic in 15th-c. Barcelona

Those who prepared themselves to be merchants had a long and hard way ahead. It is well known that they received their professional education in vernacular language, and that it was based on four main fields: writing of commercial letters, practical arithmetic, book-keeping and maritime and commercial laws.[9]

As we have seen, the abbacus schools played an essential role in the education of many Italian merchants. We have not found evidence for the existence of similar schools in Cat-

[2](Goldwaithe, 1972, 420). See (Ulivi, 2002) to get introduced to Italian abbacus schools.

[3]See (Arrighi, 1965–1967) and (Goldthwaite, 1972) respectively.

[4]See (Rankin, 1992, 28).

[5]See, for instance, (Malet, 1998, 33–40, 63–72), (Docampo, 2004a, 343–344, 539–540).

[6]See (Malet, 1998) for a critical edition of this text.

[7]*Ibid.*, 40–43.

[8]Ortega and Andrés' arithmetics were composed in Castilian, although the former was translated into French (1515) and also into Italian (1515 and 1522). Gaspar Nicolas' was the first printed Portuguese arithmetic. For a detailed analysis of the problems appearing in the arithmetics of Santcliment, Ortega and Ventallol, see (Labarthe, 2004).

[9]For the apprenticeship of medieval Catalan merchants, see (Docampo, 2004b).

alonia, and there are even some contemporary sources that point at the opposite direction.[10] Similarly to what probably happened in the French-Provençal area, everything seems to suggest that Catalan students of commercial arithmetic did not attend centres that can be compared with well established Italian abbacus schools.

Up to now, as far as Catalonia is concerned, we just have found explicit references to commercial arithmetic teachers in the city of Barcelona. This is hardly surprising if we consider that this city was the main political and economical centre of the Crown of Aragon. In the following table we include the name of those commercial arithmetic teachers that have been identified so far and that were active in the fifteenth century:[11]

Name of the teacher	**Profession**	**Name for the subject**	**Year**
Christoforo Grillo	*magister abbaque*	[*abbaco*]	1442
Jaume Verdaguer	*canviador de menuts*	*comptar d'abba i compte pla*	1459
Galceran Altimir	*scriptore littere rotunde*	*còmputs en 4 espècies*	1460
Joan de Tremp	merchant	*còmpot d'abba*	1479
Francesc Santcliment	arithmetic teacher[12]	*art d'arismètica*	1482

As it can be seen, the professional profile of these teachers is quite diverse, and we can find a money exchanger, a specialist in round script and a merchant among them. In each case, we have included the year in which they are mentioned in a contract or in any other contemporary reference that shows that they were active then.

Christoforo Grillo was an abbacus teacher of Pisan origin, who had the Barcelonese citizenship and died in Barcelona (ca. 1474). Galceran Altimir was put in charge of Ferran II's books during a visit of the king to Barcelona in 1481.[13] Francesc Santcliment is the author of the *Summa de l'art d'Aritmètica*, and according to his own words, he taught arithmetic in Barcelona (ca. 1482) and also in Zaragoza (ca. 1487).[14]

Jaume Verdaguer, Galceran Altimir and Joan de Tremp respectively appear in three contracts where the teaching of arithmetic is mentioned. In the first case, Bernat Alemany, a tailor from Barcelona, agrees to leave his son Miquel (13) in Jaume Verdaguer's place for three years to work as a servant and learn the profession of money exchanger as well as arithmetic. The second contract is dated in 1460. In it, it is stated that Pere de Mont-real (20), will live and work as a servant in Altimir's place for 3 years, while Galceran Altimir will instruct him in good manners, and taught him how to write commercial letters and to perform the four basic arithmetical operations, as well as other questions related to them. In the third contract, Pere Cicart, a wool worker from Northern Catalonia, leaves his son Francesc (13) with Joan de Tremp, to work as a servant and to learn the profession of merchant. Francesc will learn to read and write, bookkeeping and practical arithmetic. Joan de Tremp will be paid 7 pounds per year.

In the first two cases, the apprentice will be provided food and clothing. In fact, these contracts were not too different from those used for the learning of other professions. It is possible that a professional had a few apprentices at the same time and thus he gave lectures on commercial arithmetic to a little group of pupils.

[10] See (Docampo, 2004b, 700–702).

[11] Otherwise anything else is indicated, our sources, as far as these teachers are concerned, are (Del Treppo, 1976, 486), (De la Torre, 1971, 252–253), (Hernando, 2002, 426), (Hernando, 2005, 953, 956, 974–975, 979) and (Malet, 1998, 28, 352).

[12] In this case, we have not seen him explicitly referred to as an "arithmetic teacher" anywhere, but he certainly worked as such.

[13] See (Docampo, 2004a, 194).

[14] See (Malet, 1998, 352), (Santcliment, ca. 1487, f. 47v).

It has been observed that abbacus teachers did not have professional associations, as happened with artisans, to supervise their professional practice. Then they had to build their reputations just from their practice and not in exams to prove their skills.[15] Having this in mind, it seems reasonable to assume that each one of them often had to promote his teaching in order to overcome the competence and get as many pupils as possible. Perhaps we can feel the flavour of this need at the beginning of a commercial arithmetic treatise in which Galceran Altimir is invoked:

> Quam vis aresmetica in septem partes fuerit divisa secundum Algorismi tamen secundum praticam et doctrinam magistri Galcerandi Altimir, yllusstrísimi Ferdinandi Yspaniarum Reges librarii quatuor specibus videtur e s·escritura quarum prima espes decitur addiçió, secunda mulltiplicaçió, terça substracctió, quarta deviçió.[16]

As we see, it is stated that arithmetic was divided in 7 parts according to Algorismi, but in 4 parts according to the practice and doctrine of Master Galceran Altimir, librarian of Ferdinand, king of the "Spains" (sic, "yspaniarum" being the genitive plural of "hispania-ae"). These four parts are addition, multiplication, subtraction and division. This is the only Latin quote of the manuscript apart from the sentence that introduces the Hindu-Arabic numerals. Even when commercial arithmetic was basically cultivated using vernacular languages, some headings and titles could help to make treatises more "respectable".[17]

3 Some manuscript sources

Two anonymous Catalan arithmetic texts that can be dated around 1440–1450 are preserved in the Biblioteca degl'Intronati di Siena (Italy).[18] One of them seems to be a fragment of a commercial arithmetic treatise and consists of a few pages with some solved problems. The other one is a long list of more than 200 exercises that are systematically ordered.[19] The solution is provided only in 5 of them and no solving process is explained. We do not know whether this collection was part of a larger work or a separate exercise book. However, it is clear that it is an ideal tool for a systematic practice of the most usual proceedings in elementary commercial arithmetic.

Exercises are ordered with an increasing degree of difficulty into each section. We can differentiate the following sections (we indicate the number of exercises in brackets): multiplication (72), calculation of the value, in pounds[20], of different amounts of money (14), subtraction (5), division (82) rule of three (53),[21] and partnership (5). Virtually all the exercises reproduce situations that the future merchant would have to face in his daily practice. For instance, in one of the examples in the section on subtraction, a merchant should get an amount of money from another one in *florins*, *sous* and *diners*, and he has received part of it in *ducats*, *sous* and *diners*. The question is how much is still lacking. As happens in the

[15] See (Radford, 2003, 130).

[16] *Llibre que esplica lo que ha de ser un bon mercader*, f. 76v. See next section.

[17] See (Rankin, 1992, 11).

[18] Both manuscripts are preserved together in Ms. 102 (A.III 27) Biblioteca degl'Intronati di Siena. They were edited and briefly commented in (Arrighi, 1982). For a detailed analysis, see (Docampo, 2004a, 161–176).

[19] Ms. 102 (A.III 27), ff. 158r–169v.

[20] Money of account.

[21] The rule of three is called *regla de si tant vall tant*. Francesc Santcliment states that this is the way in which it is called in "our vulgar language": "E comença la dita spècia en nostre [parlar] vulgar: si tant val tant, ¿què valrà tant?" (Malet, 1998, 163). We can find a very similar expression for the rule of three in a Castilian manuscript of ca. 1400 (see below, note 43): "(. . .) sy tanto fase tanto ¿qué sería tanto?" (Caunedo and Córdoba, 2000, 147). This statement is typical of the Ibero-Provençal area, and the same phrase is used by the Arabic author Ibn al-Khid?r al-Qurašī (11th-c.) to refer to the kind of problems that have to be solved by the rule of three. See (Høyrup, 2007b, 4).

rest of the manuscript, spaces are left to be filled in with the answer.[22] In another example, now in the section on the rule of three, the price of a certain amount of cloth must be found and, as happens in most of the exercises, calculations involve quantities in complex form: "the *peça* of cloth is worth 15 *lliures* 7 *sous* 3 *diners*, how much will 5 *canes* and 3 *palms* be worth?"[23]

It must be noted that these exercises are not only aimed to practise the main operation that roughly classifies the different kinds of exercise (subtraction, rule of three, ...), but also to remember the different unit equivalences and constantly practise the main changes of units. In these changes, divisions were often performed using "short rules" (*regles breus*) and not by applying the usual algorithms.[24] Fractions appear in several exercises, especially in the section on division.

It seems reasonable to think that this collection was used as a reserve of exercises by the teacher. Pupils would copy these lists in their notebooks in order to practise exhaustively on the different kinds of exercises, and this practice would enable them to perform operations quickly and precisely. It is clear that their learning was based on repetition of the procedures, as can be seen in the Pisan teaching program mentioned above. It must be noted that we have not found such a large collection of unsolved exercises like this in any other medieval treatise.

For the period we are dealing with, the only known manuscript on commercial arithmetic in Catalan that can be considered a complete treatise is contained in a merchants' handbook from around 1490 titled *Llibre que esplica lo que ha de ser un bon mercader*.[25] The distribution of its arithmetical contents can be seen in Appendix A. Having into account the invocation of the "practice and doctrine" of Galceran Altimir at the beginning of these contents (see above), it is reasonable to think that this arithmetic was related, at least partially, to his teaching.[26] On the other hand, this treatise has important coincidences with Santcliment's *Summa* of 1482 in some of its chapters.

The style is direct and simple, a characteristic of commercial arithmetics. A clear didactic aim can be seen all over the text, and many examples and unsolved exercises are included in order to practise the four elementary operations and the rule of three with its main applications. However, the chapters on the elementary operations do not contain any explanation on how these operations are performed, but consist of long series of solved examples. On the other hand, the rule of three and its applications, operations with fractions and alternative calculation techniques, among other contents, are explained in detail. In several occasions, after a long series of similar exercises is given, the author includes one or two recreational problems, which are not connected with the previous series, but were surely aimed to challenge and motivate the students.[27]

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[22] Un mercader ha aver de hun altre 1623 florins, 2 sous 1 a raó de 11 sous. E an rabut 1237 ducats, 4 sous 7 diners a raó de 14 sous 5 per ducat. Quant serà la reste ne quall aurà a cobrar? ....
Lo mercader dels florins ha aver del dels ducats ....
(Ms. 102 (A.III 27), f. 162v). It is indicated that 1 *florí* = 11 *sous* and 1 *ducat* = 14 *sous* 5 *diners*. It is well known that 1 *sou* = 20 *diners*.

[23] "La peça del drap vall 15 lliures 7 sous 3, què valran 5 canes 3 palms? ...." (Ms. 102 (A.III 27), 167v). It is known that 1 *peça* = 12 *canes*, 1 *cana* = 8 *palms*. On the other hand, 1 *lliura* = 20 *sous*; 1 *sou* = 20 *diners*.

[24] These rules often exploited the systems of sub multiples of the main units in order to avoid performing a general algorithm of division. See (Docampo, 2004a, 68–70).

[25] "Book that explains what should the good merchant be". Diversos 37 B/2 Arxiu del Regne de Mallorca, Palma de Mallorca. This codex is already presented in (Sevillano, 1974–1979). For a detailed analysis of the commercial arithmetic part of this book, see (Docampo, 2004a, 190–306).

[26] In some abbacus works, a well-known teacher was cited at the beginning, not as its author but as an authority. See (Van Egmond, 1980, 27). Even when this could happen in our case, and even when we cannot know if this treatise was directly based on a work by Altimir, some sort of connection is very likely.

[27] Many of them can be found in similar versions in contemporary arithmetics. There is also a complete section of the manuscript that is mostly devoted to this kind of problems (ff. 143r–152r).

Arithmetic operations often are represented inside rectangles, as happened in other arithmetics like Leonardo Pisano's famous *Liber abaci* (1202)[28]. It seems clear that they were (at least originally) related to a calculation board in which operations were performed and numbers could be easily rubbed out. Leonardo Pisano (also known as Fibonacci), mentions a "whitened table in which numbers are easily erased"[29] where numbers should be written down. He does it when he describes the multiplication of 12 by itself. In order to illustrate the explanation in the main text, he includes the following figure:[30]

descriptio	
prima	4
	12
	12
Secunda	44
	12
	12
Vltima	144
	12
	12

Rectangles containing operations and appearing in the margins next to the main text can be found all over the *Liber abaci*.

In the Catalan manuscript, ninety squares like the following one (f. 87v) appear in the section on multiplication:

9 4 0 3 3 2 4 8
9 8 3 2
9 5 6 4
$\boxed{6}$

Most of them show multiplications of a number by itself, and in all of them numbers that are multiplied have the same number of digits. As we see, each square just contains both factors and the result in the upper part. The result of applying the proof by casting out of nines can be seen in the lower right corner. We can find squares that are almost identical to these in Jacopo da Firenze's *Tractatus algorismi* (Montpellier, 1307).[31]

Everything seems to suggest that these multiplications were performed by the method that was known in Italian vernacular treatises as *per crocetta*.[32] This method was often

[28]This work is edited in (Boncompagni, 1857) and translated into English in (Sigler, 2002).

[29]He writes "in tabula dealbata in qua littere leviter deleantur" (Boncompagni, 1857, 7). This expression clearly reminds us of the dust board: the board would be whitened by the sand or dust that was spread over it and operations were performed with the fingers or with an stylus. However, it is known that a wooden board with a plate of clay was used in the Maghreb before the 13th century, and that white clay was used for it, numbers being written down using an stylus with ink and rubbed out with wet clay (see (Abdeljaouad, 2002, 19–20; Lamrabet, 1994, 203)).

[30]See (Boncompagni, 1857, 7).

[31]See (Høyrup, 1999, 20–25; 2007a, 18–25). This work was one of the first vernacular texts on commercial arithmetic and contains the earliest known account on algebra in a vernacular European language.

[32]This is specially clear if we look at the crosses that appear in the first squares that can be found in Jacopo da Firenze's treatise (see (Høyrup, 1999, 20)). For a description of this method see, for instance, (Swetz, 1987, 203–204). Fibonacci explains this method when he includes the squares we have mentioned. On the other hand, the method of multiplication that we use today is used in many examples in the Catalan manuscript (for instance, in ff. 88v–94v).

performed without writing down the partial results, but keeping them in mind and also with the help of finger symbolism, specially in the more simple cases (for example, in calculating the squares of 2-digit numbers). For instance, in a Pisan program of the first half of the 15th century,[33] it is stated that pupils will have to perform all multiplications of 2-digit numbers by themselves "alle mano" (using finger symbolism). Furthermore, they will have to perform in this way at least some of the products of two different 2-digit numbers, and some products of 3 or more digit numbers in general.

It is interesting to note that the order in which the squares appear in the Catalan manuscript and some instructions in this Pisan program fit fairly well: first of all, calculations of the squares of 2-digit numbers. Secondly, products of two different 2-digit numbers. Then, (after some other contents in the case of the Italian document) squares of 3-digit numbers and products of different 3-digit numbers in this order. After this, the same scheme must be followed for 4-digit numbers. Finally, products of numbers with a different number of digits are dealt with, and we must remark that this kind of products do not appear in the squares of the Catalan manuscript nor in those of the *Tractatus algorismi*.

On the other hand, the section on division in the *Llibre que esplica...* starts with eight divisions that are performed by the method that is known in Italian abbacus treatises as *partire a regola*. It was mainly used with 1-digit or 2-digit divisors lesser than 20. This method can be quickly performed and allows the student to easily generate a lot of exercises for a continuous practice.[34] This method also opens the section on division in the Pisan program.[35] According to Luca Pacioli, it was used by Florentine teachers to prepare their pupils for other methods of division.[36] In the Catalan manuscript it is only used for 1-digit divisors. In the first example (f. 95r), 97483027894 is divided by 2:

2	97483027894
0	48741513947

Larger divisions are performed by the method that was known in contemporary Italian abbacus treatises as *partire a galera*.[37] These divisions also appear inside rectangles. Numbers are arranged in a quite unusual manner, with the divisor in the upper part and the "casting out of nines" checking performed between the divisor and the rest of the operation. For instance, in the division of 3942650 by 19 (f. 97v) we find:[38]

[33] Codice 2186 of the Biblioteca Riccardiana di Firenze. The program is described by one Cristofano di Gherardo di Dino, who starts by declaring (f. 1r): "Questo è la forma e 'l modo a insegniare l'anbaco al modo di Pisa (...)" ("this is the way to teach arithmetic in Pisa"). We have used (Arrighi, 1965–1967) as long as this program is concerned.

[34] See (Rankin, 1992, 154–156) or (Docampo, 2004a, 65–66).

[35] See (Arrighi, 1965–1967, 122).

[36] (Pacioli, 1494, 32v). A good example of long series of divisions *a regolo* can be found in Jacopo da Firenze's *Tractatus algorismi*. See (Høyrup, 1999, 26–33; 2007a, 31–34).

[37] This method was the most common one during the Middle Ages. It is explained, for instance, in (Rankin, 1992, 160–162) and (Swetz, 1987, 216–217). During the fourteenth century, however, this method was known, at least by some authors, as *partire a danda* (see note 38), a term that was used in the fifteenth century to refer to the method of long division that was quite similar to the one we use nowadays.

[38] Roughly similar boxes with divisions can be found in two of the three manuscripts containing Jacopo da Firenze's *Tractatus algorismi* (see (Høyrup, 2007a, 35–37)) and in a manuscript (ca. 1340) containing a draft autograph of Paolo dell'Abbaco's *Trattato di tutta l'arte dell'abacho* (ms II,IX.57 of the *fondo principale* in the *Biblioteca Nazionale Centrale* of Florence). However, it must be noted that in these Italian sources the placement of numbers is different, the zeros on the top do not generally appear and the divisions are checked by the "casting out of sevens" method. I am grateful to Jens Hřyrup for letting me know about the boxes in ms II,IX.57 and for providing me with a copy of a page in which they appear (f. 29r) and are referred to using the expression "partire a danda" (*addanda* in the original). This expression is also used by an anonymous Pisan author of ca. 1300 (see (Franci, 2003, 41)).

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   19
    1
2       8
   3 2
-----------
  0 0 0 0 1
0 1 7 4 1 8
1 1 4 9 1 5 7
-----------
3 9 4 2 6 5 0
-----------
  2 0 7 5 0 7
```

[the quotient is in the lower rectangle]

As we have seen, there are several coincidences between the organization of the commercial arithmetic in the *Llibre que esplica...* and the Pisan program described by Cristofano di Gherardo di Dino. We should not forget that we have few examples of the programs that were followed in Italian abbacus schools, and thus we cannot state that Pisan methods were *more* influential in Catalan teachers than those from other Italian cities. However, important contacts between both environments are clear, and the Pisan abbacus teacher Christophoro Grillo, who was in Barcelona in the mid-15th century, was surely not an isolated figure.[39]

4 Final remarks

We have seen the influence of Italian abbacus tradition in the *Llibre que esplica lo que ha de ser un bon mercader*. The Italian influence is also evident in the first manuscript in Catalan that contains an account on algebra: Joan Ventallol, the Majorcan author of the *Pràtica mercantívol*, is the most likely author of a set of notes (ca. 1520) preserved in Barcelona[40] that are mainly related to Luca Pacioli's *Summa de Arithmetica, Geometria, Proportioni et Propotionalità* (first published in Venice in 1494). These notes contain, at the present state of investigations, the first account on algebra in a vernacular Iberian language and include an interesting kind of diagrams to perform algebraic operations. Joan Ventallol could have taught mathematics in Barcelona at the beginning of the 16th century, because the commercial arithmetic part of these notes seems clearly directed to merchants from this city.[41]

As we have seen, even when the structures for the teaching of commercial arithmetic in Catalonia can not be compared to those in the largest cities of Northern Italy, many of the contents that were taught were the same in both areas, and as happens with the program described by Cristofano di Gherardo di Dino, the knowledge of Italian teaching methods is very useful to better know those used in Catalonia.

On the other hand, we believe that the research and study of more Catalan manuscripts on arithmetic and algebra should provide a better view on some very interesting points, such as the influence of those mathematics that were cultivated in Jewish circles on both sides of the Pyrenees, the role played by Catalan authors in the transmission of Arabic algebra into

[39] In this sense, it might be interesting to note that a son of Cristofano di Gherardo di Dino is known to have travelled to Barcelona in 1443: "Ricordo a me xpofano come al nome di dio e della vergine Maria a di 18 del mese di sett[en]bre 1443 una mezzedima mactina Dino mio figluolo si parti da Livorna per andare in Barsellona, insu lla ghalea di Giovanni Bandini citadino fiorentino, la qual galea fecie la volta di barbaria, li quali iddio mandi astruamento se di suo piacere (...)" These notes appear in f. 131v of the codex that contains the mentioned Pisan program. See (Van Egmond, 1980, 148).

[40] Ms. 71 de Sant Cugat, Arxiu de la Corona d'Aragó.

[41] For more information about this manuscript, see (Docampo, 2006). A research on the possible sources of the diagrams to perform algebraic operations in Ms. 71 will appear in (Docampo, Forthcoming).

Europe[42] and the connections with Castilian arithmetic treatises.[43]

The basic elements of commercial arithmetic have not changed too much since the late middle ages, and the abbacus treatises are the predecessors of modern elementary arithmetic texts. Furthermore, those treatises played an essential role in the transmission and development of Arabic algebra. These facts mark their significance in the history of mathematics education and make them worth of a deep study.

APPENDIX A

Contents of the arithmetic treatise included in the *Llibre que esplica lo que ha de ser un bon mercader*:

Presentation (f. 76r-v.); introduction of the Hindu-Arabic numeration system (ff. 76v–77r); multiplication tables (ff. 77v–78r); addition (ff. 78v–81v); subtraction (ff. 82r–85r); multiplication (ff. 85v–94v.); division (ff. 94v–107v); partial index (f. 108r–v); exercises of multiplications, subtractions and divisions with real units (ff. 108v–120v)[44]; rule of three (ff. 121r–127v); partnership (ff. 128v–135r); barter (ff. 135r–138r); exchange (ff. 138v–142r); false position [?][45] (f. 142r–142v); collection of miscellaneous problems (ff. 143r–152r); special rules to calculate prices in certain situations (ff. 152v–156r); advices and information for the merchant (ff. 156r–157r); annual, monthly and daily interests (ff. 157r–158r); *reglas de montiplicar* (ff. 158r–160r); further information for the merchant (ff. 159v–161v); operations with fractions (ff. 161v–168r); special rules to divide in certain cases (f. 168r).

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[42] This role could have been more important that what has been traditionally believed. See (Høyrup, 2006, 25, 34).

[43] Up to know, a commercial arithmetic in Castilian (ca. 1400) has been edited and commented in (Caunedo and Córdoba, 2000). Jens Høyrup has observed important similarities of some parts of this text with "the various extant 15th-century Provençal-Catalan algorisms" (Høyrup, 2006, 25). Some clear coincidences in problems of this treatise (Ms. 46 of the Real Colegiata de San Isidoro de León) and the arithmetic in the *Llibre que esplica...* are pointed out in (Docampo, 2004a, 252, 258–263). Another Castilian arithmetic (14th-c.) is presented in (Caunedo, 2003).

[44] In spite of the titles, the rule of three is explicitly applied in an example of this section (f. 115r–v).

[45] In spite of its title, this section consists of two problems related to money exchange which are not solved by simple or double false position.

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