MATHEMATICAL EDUCATION AT CAMBRIDGE UNIVERSITY IN THE NINETEENTH CENTURY

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Abstract

Students who studied mathematics at Cambridge university in the 19th century faced a challenging course. Mathematics was viewed by the authorities as an integral part of a liberal education and it was extremely competitive. While the rigour instilled by mathematics was seen as the key to all further study, the idea that the Cambridge course provided a technical education for future mathematicians and scientists was not an objective in the 1840s. This brief article investigates some important moments in the evolution of the Cambridge mathematics course from the 1840s until the 1900s.

1 INTRODUCTION

The market town of Cambridge with its ancient university was the most important place for mathematics in Great Britain in the nineteenth century. There were two reasons for this. The first was that Cambridge University housed the famed Mathematical Tripos as the mainstream course of study for its students, and the second was the position of Cambridge as the institutional centre for mathematical research in Great Britain.

2 The 1840s

Mathematics at Cambridge was the basis for a 'liberal education'. It was a sort of preknowledge for the learning of all other knowledge and it was important to teach it to the young. Taught too late, it would be as useless as trying to teach 'the violin to a grown man', it was said. The knowledge of mathematics was not claimed to be useful in itself (except for the future tutor or schoolmaster), but it was believed that the study of mathematics would develop and strengthen the faculties of the mind. After the completion of this study, it was held, one could go on to other fields and be more effective in them. Mathematics gave the 'art of acquiring all arts', and like physical games which prepared the body, mathematics toned the intellectual muscle.

The Mathematical Tripos and its examination evolved over the nineteenth century. The word 'tripos' is believed to have been derived from the three-legged stool sat upon by undergraduates while they were being examined. These oral 'tripos' examinations were discontinued by the beginning of the 19th century but the name associated with them became

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ingrained. By the 1840s written examinations were the norm. This evolution continued. The Tripos examination sat by the mathematician Arthur Cayley in the 1840s was not the Tripos sat by the mathematical physicist James Clerk Maxwell in the 1850s. In turn this differed from the one sat by the statistician Karl Pearson in the 1870s, still less the Tripos of the logician and philosopher Bertrand Russell in the 1890s, a course that was barely recognized as one which existed in the 1840s. In the 1840s the Mathematical Tripos was a wide-ranging course covering most aspects of mathematics in some depth, but, by the time Russell sat the examination in 1894 it had become specialized.

Some aspects of the Mathematical Tripos course were resistant to change. These included:

- the once-and-for-all nature of the Mathematical Tripos final examination
- the order of merit the final and unchangeable listing of students according to their examination marks obtained in the final examination
- the position of the 'Senior Wrangler' the top student in the order of merit

The order of merit did not measure a student's real knowledge of mathematics. In the 1837 list, for example, there was a highly creative mathematician like J. J. Sylvester 'beaten' by someone unknown to the mathematical community. The order of merit — the grandfather of all academic league tables — did not indicate any research potential, and it was not supposed to. The Mathematical Tripos examination was primarily a mathematical contest designed for clever schoolboys who could jump through hoops at speed.

At the pinnacle of the order of merit was the champion student, the Senior Wrangler.

He signified all that was good about the Mathematical Tripos and acted as a focus for the whole system. One contemporary remarked:

In my opinion it is this continuance of solving problems, this general course of not only acquiring principles but applying them, that at last makes the senior wrangler, who perhaps at the time is one of the most expert mathematicians in existence.¹

The serious students coming to Cambridge for the first time were rapidly moved into examination mode. There were examinations at every turn — on arrival, at the annual College examinations, and the whole procession culminating in the university Mathematical Tripos examination at the end of ten terms. The skills of solving problems and working quickly under pressure were all part of the Cambridge package for its students. If they succeeded, there would be week long examinations for the two Smith's Prizes. Then there were the College fellowships to strive for. If they were at Trinity College, the high flyers with a fellowship in prospect would face another batch of examinations nine months after the exertions of the Mathematical Tripos and the competition for the Smith's Prizes.

The Tripos examinations of the 1840s spread over six days of $5\frac{1}{2}$ hours each day covered equal proportions 'pure' questions and 'applied' questions (applied mathematics was then called 'mixed mathematics'). The examinations then contained questions on such subjects as astronomy, algebra, elliptic functions, differential equations, mechanics, and the application of mathematics to such questions as the shape of the rotating earth. A question might ask for the reproduction of known facts (a 'bookwork' question) and be accompanied by a following 'rider' which required mathematical technique. A different type was the 'problem' question which required students to solve previously unseen problems but perhaps similar to

¹Much has been written on the educational system at Cambridge University. Further references to the Mathematical Tripos and sources of quotations can be found in recent books: A Warwick, *Masters of Theory*, (2003), Chicago Univ. Press; T. Crilly, *Arthur Cayley*, (2006), Johns Hopkins Univ. Press.

ones they had already done. A quarter of the questions were allocated to 'problems'. It is customary to criticize the Mathematical Tripos of the 19th century as a 'great writing race', as described by Augustus De Morgan but it was remarkable in its coverage of both ancient and contemporary mathematics.

To make success in the examination a reality, private mathematical coaches came into their own and supplied extra tuition. One student described their teaching methods:

not a day, not an hour was wasted; the perfect candidate should be able to write the bookwork automatically while his thoughts were busy with the rider, and the fingers could be trained even when the brain was weary; above all, curiosity about unscheduled mathematics was depravity.

A leading Cambridge mathematician George Peacock criticized the 'unhappy system' of private tuition and the notion that mathematics was good medicine for all students. In 1848, the Board of Mathematical Studies was set up and reforms put in place but could do nothing about the issue of private coaching. It was recommended that the Mathematical Tripos examinations be in two parts, thus reducing the pressure brought about by a battery of examinations one after the other. Peacock saw all too plainly that the Mathematical Tripos was crammed full of subjects resulting in an indigestible course of study, and a reduction in the wide coverage was proposed and accepted. The master of Trinity College William Whewell argued strongly for student attendance at the lectures given by the professors who, he observed, had little input to the education of Cambridge undergraduates.

One outcome of the centrality of mathematics to Cambridge education was the founding of the *Cambridge mathematical journal* in 1838 and its successor, the *Cambridge and Dublin mathematical journal* in 1845. While the Cambridge journals had an international dimension and enjoyed the support of a few continental mathematicians they also brought together students and fellows from the different colleges of Cambridge. In the 1840s teaching was college based and a man had no necessity to mix in with students from other colleges. The Cambridge journals performed the useful function of removing this insularity and when *Dublin* was added to the title, of enlarging the research base in Britain.

3 The 1850s and 1860s

In 1850, a Royal Commission was appointed to look into the workings of both Oxford and Cambridge universities. The resulting Cambridge University Act (1856) gave a new impetus to the creation of the University as something more that a collection of autonomous colleges. A new form of governance was given to the university and the powers of the individual colleges reduced.

The road to mathematical research proved bumpy. Through financial problems, the *Cambridge and Dublin mathematical journal* which Thomson had launched with such brio in 1846 collapsed in 1854. It was re-branded in the following year as the *Quarterly journal of pure and applied mathematics*. After a rocky start when it was doubtful if it would continue, it made a long run until 1927 for many of its years under the editorship of the Cambridge don J. W. L. Glaisher.²

The Mathematical Tripos mattered most at Cambridge. It was a Cambridge affair, which in hindsight now seems somewhat parochial. Some admired the stability of the system, as did the theologian F. J. A. Hort in the 1850s, bemoaning no Trinity College Senior Wrangler in 11 years, wrote: 'I feel a proper pride in the mathematical tripos and senior wranglership as great existing institutions'. In the Cambridge *Student's guide* of 1863, J. R. Seeley said:

 $^{^{2}\}mathrm{A}$ 'don' is a traditional term used for staff attached to an Oxford or Cambridge college.

The Mathematical Examination of Cambridge is widely celebrated, and has given to this University its character of the Mathematical University *par excellence*.

William Everett, an American student who spent three years at Cambridge and who graduated in 1863, noted the characteristics of the mathematical education he received and its continued reliance on Newton and Euclid:

Englishmen hate going back to first principles, and mathematics allows them to accept a few axiomatic statements laid down by their two gods, Euclid and Newton, and then go on and on, very seldom reverting to them. This system of mathematics developed in England, is exceedingly different from that either of the Germans or the French, and though at different times it has borrowed much from both these countries, it has redistilled it through its own alembic, till it is all English of the English.

When reform of the Mathematical Tripos was considered in the 1860s the newly installed Sadleirian professor Arthur Cayley engaged in debate with George Biddell Airy, the Astronomer Royal and a former Lucasian Professor of mathematics at Cambridge. Cayley thought of his subject independently of any students, while Airy's thinking was shaped by the ideals of the university as a teaching institution. It was Cayley's ill advised sentence: 'I do not think everything should be subordinated to the educational element,', which caused Airy the greatest consternation, and he replied:

I cannot conceal my surprise at this sentiment, assuredly the founders of the Colleges intended them for education (so far as they apply to persons in *statu pupillari*), the statutes of the University and the Colleges are framed for education, and fathers send their sons to the University for education. If I had not your words before me, I should have said that it is impossible to doubt this.

There was clearly a wide chasm between the idea of mathematics as a living subject that constantly expanded its domain and the subject set in stone which passed as the basis for a mathematical education.

4 The period after 1870

Major reforms of the Mathematical Tripos came into operation in 1873. The syllabus now included the introduction (and reintroduction) of such topics as the mathematical theory of elasticity, heat, electricity, waves and tides, these new specialisms arranged in divisions which students could select for their study. Karl Pearson praised the Mathematical Tripos examination of the 1870s for it being 'not specialised, but [it] gave a general review of the principia of many branches of mathematical science' and he valued the challenge of 'problems' thus forcing the private coaches to deal with them in their classes. He observed that this essence of mathematical research was missing in the much-heralded German system which he saw as laying the emphasis on the teaching of theory.

But overall the reforms of the Mathematical Tripos brought into play in 1873 were not a success and even in the first year of operation their failure was apparent. Drilled in examination technique by their coaches, students quickly learned that the art of cherrypicking across the subject divisions was an efficient method for amassing marks. This led to a superficial knowledge of a wide range of subjects rather than knowledge to any depth. Drastic action was required, and in May 1877 a large and influential University committee was appointed. High on the agenda were

• whether the order of merit should be retained

- the status of the Senior Wrangler
- how to cope with the increase in mathematical knowledge, and whether the Mathematical Tripos could or should cover the whole of mathematics
- whether the honours students should be allowed to sit the Mathematical Tripos examinations in June or keep to the traditional January examinations

Reaching an agreed radical solution was impossible. Syndicate members were successful products of the very system they were investigating, and there would inevitably be a strong tendency to preserve their own 'golden age'. The private coaches had a powerful incentive for maintaining a system, which benefited them financially.

But change was in the air. The first shoots of progress towards the higher education of women began in the 1870s, and a decade later a woman was recognized as the equivalent of wrangler though the formal admittance to a degree was still a long way off. The Devonshire Commission on Scientific Instruction and the Advancement of Science which sat 1872–1875 and produced a voluminous report. The Oxford and Cambridge Commission of 1877 resulted in a University of Oxford and Cambridge Act which enforced further changes in the governance of the university.

Attention was turning towards research being part of the university's mission. Five university lectureships in mathematics were created in 1883. In theoretical physics Cambridge was led by G. G. Stokes while on the experimental side the Cavendish Laboratory was created in the early 1870s. James Clerk Maxwell was the first Director and he led an active school. He was followed by such luminaries as Lord Rayleigh and J. J. Thomson. Applied mathematics enjoyed a high reputation.

But what of pure mathematics? It fell to Cayley to gather a nucleus of researchers around him. Cayley did have a handful of protégés (J. W. L. Glaisher, W. K. Clifford, A. R. Forsyth, and H. F. Baker) and he gave assistance to a number of promising students including women students who were beginning to arrive on the scene in the 1880s. But this was nothing like the research school underway in Germany under the direction of Felix Klein. Cayley was in the end a 'General without Armies'.

G. H. Hardy, a later Sadleirian professor identified the period between 1880–1890, as the time the Mathematical Tripos was at the 'zenith' of its reputation in the public eye, but one which coincided with research mathematics in England being at its lowest ebb. The lone star in pure mathematics was the ageing Cayley but Hardy did not value his work highly. At school level Cayley opposed the proposed reforms in the teaching of mathematics, and he emerged as the leader of the conservatives who insisted on the retention of Euclid's *Elements* for the teaching of geometry.

From 1886 a newer two-part Mathematical Tripos was created. It was a very different Mathematical Tripos from the one of the 1840s when mathematics had no other competing subjects and students had little choice of subject to study. Towards the end of the century the number of students opting for the Mathematical Tripos course fell rapidly. In the period 1840–1850, there were on average 124 Mathematical Tripos students graduating each year with an honours degree, but in 1890–1900 there were only 92.

In 1890 G. T. Bennett graduated Senior Wrangler and was winner of the First Smith's prize for a paper on number theory. While Bennett was the *official* male Senior Wrangler, it was Phillipa Fawcett's performance that which electrified the student population when she graduated 'above the Senior wrangler':

Hail the triumph of the corset,

Hail the fair Philippa Fawcett

The procession of males in the order of merit had been topped by this Newnham College scholar. And, it was as if the newly liberated female students at Cambridge saw the Emperor without clothes. In George Bernard Shaw's play *Mrs Warren's profession*, Vivie gave voice to the curiosity that the Mathematical Tripos had become:

do you know what the mathematical tripos means? It means grind, grind, grind for six to eight hours a day at mathematics, and nothing but mathematics. I'm supposed to know something about science; but I know nothing except the mathematics it involves. I can make calculations for engineers, electricians, insurance companies, and so on; but I know next to nothing about engineering or electricity or insurance. I don't even know arithmetic well.

Tension existed between the teachers of mathematics and the 'active' mathematicians who researched the subject and passed this on. They could not believe in a teaching a system which was dominated by an examination consisting of artificial questions which could only be justified by their being good Mathematical Tripos examination questions.

5 DENOUEMENT

'Victorian mathematics' at Cambridge continued a little longer. The big fight in the cause of Mathematical Tripos reform took place in 1907. The majority of active mathematicians at Cambridge were in favour of the change — the abolition of the order of merit and the coveted title of the Senior Wrangler. There was a minority who opposed the reforms and one private coach thought that the proposed reforms would mean the end of mathematics at Cambridge.

The voting took place in February 1907 and about 55 % were in favour of reform. It was a close call, but in a first past the post voting system 'one is enough'. The last examination conducted under the old regulations was held in 1909. It was truly the end of an era. The institution of the private coach melted away, and in the tumultuous events of 1914 the veritable old Tripos became a distant memory.