Differential Calculus in Military Schools in Late Eighteenth-Century France and Germany

A comparative textbook analysis

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Abstract

In the historiography of mathematics it has not been unusual to approach the algebraization of calculus from an internalist perspective. However, since this process can be regarded as being subject to social variables, its analysis in detailed contexts deserves more attention than it has been granted as yet, as well as the communication between such contexts. This paper explores how some aspects concerning the algebraization of differential calculus were communicated in eighteenth century. To this purpose I carry out a comparative analysis of educational books on differential calculus that were used in the French and German systems of military education in this period.

1 Introduction

The historiography of mathematics abounds with studies approaching the history of calculus from an internalist point of view. Focussing on the intrinsic development of concepts, these studies are mainly based on the history of great men and their ideas. G. Schubring, however, points to the need for revision of the perspectives hitherto developed in the historiography of mathematics and favours a hermeneutical reconstruction of conceptual development.[1]

The exploration of the algebraization of calculus — a fundamental aspect in its development — in detailed contexts has been neglected so far, as well as the interaction between such contexts. In this sense, Schubring regards algebraization as a category of conceptual development, subject to social variables, culturally and epistemologically shaped.[2] As a socially molded category, we must understand the process of algebraization as evolving within a system of national education, which in turn belongs to a specific cultural and epistemological context; this guarantees a common system of communication. Textbooks emerge from a specific national educational system to communicate a subject matter to a particular community of practitioners. Consequently, Schubring proposes comparative analysis of textbooks for examining national trends with regard to style, meaning and epistemology, and for comparing how differing concepts from other communities and contexts were received.[3]

[1] Schubring 2005, p. 7.
[2] Schubring 2005, pp. 8–9.
[3] Schubring 1996, pp. 363–364.

Taking Schubring's approach as starting point, in my paper I carry out a comparative analysis of educational books[4] centered on the algebraization of differential calculus. To this purpose I am exploring how some aspects concerning the algebraization of differential calculus were communicated in the French and German systems of military education with the help of certain educational books. In contrast to the traditional history of great men and their ideas, my preference here is to focus on some "forgotten books", as J. Topham puts it.[5]

Topham's work can be framed into a recent programmatic proposal in the study of the history of science, opened up by J. Secord. The main point of Secord's program is the conceptualization of knowledge as communication. Taking this direction, Secord suggests that "what" is being communicated can only be answered through the understanding of "how", "where", "when" and "for whom".[6] I believe the comparative analysis object of this paper fits perfectly in the frame of Secord's questions.

This paper opens with a general outline of the institutional framework involved, namely, the French and German systems of military education. After the introduction of some educational books used in both contexts, the paper proceeds with the comparative analysis of these books, which will lead to some final remarks.

2 French and German systems of military education

We may start reviewing broadly the institutional framework concerning mathematical education in eighteenth-century France and Germany, with particular emphasis on their respective systems of military education.

In eighteenth-century pre-Revolutionary France university education was mainly restricted to the *collèges*, run by religious orders, where mathematics was taught at a rather elementary level. But this system coexisted with some others. By the 1750s a well-developed network of *écoles militaires* had been established, some of which were actually formerly religious *collèges.* The state control exerted on these schools guaranteed the homogenity in the education herein. In addition, the fact that professors and examiners were often connected with the Académie des Sciences favoured this trend. Another feature of this system was its stress on applications, thus preceding the so-called *école physico-mathématique.* The syllabi of these schools usually covered arithmetics, algebra, geometry, and trigonometry. Certain *écoles*, however, showed an inclination towards the introduction of new topics. In the 1780s, for instance, public exercices on differential and integral calculus were held at the *écoles* of Brienne and Sorèze — both formerly Benedictine *collèges.* Even the latter offered a course on differential and integral calculus as early as in 1772. Such an origin provided the military schools with an adequate institutional support for mathematics during the eighteenth century.[7]

In this period German national structure differed greatly from that of France, in that there was no national or cultural unity at all. In fact the German territory was divided up into hundreds of states under either the Catholic or the Protestant faith, each with its own educational system. With regard to higher education, mathematical studies achieved a notable position within the university context of the Protestant German states. The universities of Halle and Göttingen were paradigmatic examples in this context. It is worth mentioning here that, besides teaching, professors were required to publish their research, the emphasis being laid on reflections on the foundations of science. Unlike France, German states

[4]Since the word "textbooks" was not defined in the eighteenth century yet, my preference here is to consider them as educational books or "books employed for educational purposes", as they are referred to in Bertomeu Sánchez, J. R.; García Belmar, A.; Lundgren, A.; Patiniotis, M. (eds.), 2006, "Textbooks in the Scientific Periphery", *Science and Education* **15** (7–8), p. 658.

[5]Topham 2000, pp. 566–567.

[6]Secord 2004, pp. 663–664.

[7]See Schubring 1996 and Taton 1986.

had developed no significant system of military schools. However, as a consequence of the awful losses undergone by the Prussian army in the Seven Years' War (1756–1763) Frederick II felt the need to improve the officers' education. For this purpose he ordered the establishment of institutions for military instruction, to attract young noblemen to become officers. As a part of his project in 1765 the "Académie militaire" was founded in Berlin, which in 1791 became an artillery academy. Here, the non-commissioned officers could acquire knowledge on topography, cartography and geology in order to get promoted.

3 Books on differential calculus used within the French and German systems of military education

This paper aims to analyze the transition in the process of the algebraization of differential calculus in eighteength century by examining and comparing some educational books on the subject used within the French and the German systems of military education. According to the audience they were originally written for, these works can be grouped into two categories. The first category consists of those works addressed to a larger, non-specific audience, written in the first half of the century. On the other hand, the second category gathers those books intended for a more specific audience, namely, the students of military schools.

In the first group I include the following educational books: the *Analyse des infiniment petits* (1696) by Guillaume François Antoine de L'Hospital, Marquis de Sainte-Mesme (1661–1704), the *Analyse démontrée* (1708) by the father Charles R. Reyneau (1656–1728), and the *Instituzioni analitiche* (1748) by Maria Gaetana Agnesi (1718–1799). The idea of this group occurred to me when I was examining the different practices of communication involved in the circulation of Johann Bernoulli's lessons on differential calculus in eighteenth-century France and northern Italy.[8] In 1696 L'Hospital published what was considered by contemporaries and subsequent historians as the first educational book on differential calculus, the *Analyse des infiniment petits pour l'intelligence des lignes courbes*.[9] This work originated clearly from the lectures that Johann Bernoulli (1667–1748) gave L'Hospital between 1691 and 1692.[10] L'Hospital was introduced to Johann Bernoulli by Nicolas Malebranche (1638–1715), a member of the congregation of the Oratoire. Through the group he built up in Paris, Malebranche exerted a large influence on the development and spread of mathematics, in general, and Leibnizian calculus, in particular. It was also Malebranche who in 1698 encouraged his friend the Oratorian Charles René Reyneau to write a work on the new calculus intended for beginners. To accomplish Malebranche's request, Reyneau managed to get a copy of Johann Bernoulli's manuscript, worked it out and finally published his *Analyse démontrée* in 1708. This work proved to be the most important source of Maria Gaetana Agnesi's book, *Instituzioni analitiche*, envisaged as a systematic, educationally oriented, introduction to algebra, Cartesian analysis and calculus addressed to the learned community in northern Italy. That Agnesi's book relied so much on Reyneau's is hardly surprising since her philosophical background was shared by Reyneau.

Not only did Reyneau's book travel to Italy, where it was appropriated by Agnesi. But also Agnesi's book was later translated into French and introduced before the Académie des Sciences in Paris. In 1775, a comission of the Academy of Sciences advocated the translation of Agnesi's second volume — on differential calculus — into French. The reading of this version was recommended at the royal military schools of Brienne and Sorèze in 1782 and 1784, respectively. The works of L'Hospital and Reyneau were also to be found in the library

[8] See Blanco 2007.

[9] On the publication of L'Hospital's book see for instance Bossut, C., 1802, *Essai sur l'histoire générale des mathématiques*. Paris : Chez Louis. Vol II, p. 138.

[10] There is a comparative analysis of L'Hospital's *Analyse* and Johann Bernoulli's lectures in Blanco 2001.

of several *écoles militaires*.[11] At this point I became aware of the fact that, having emerged within the context of academies and societies in the first half of the eighteenth century, the works of L'Hospital, Reyneau and Agnesi ended up being used in French military schools, most likely as inherited from their original *collège* structure.

The boundaries of the second group are more clearly defined, since I consider here educational books explicitly intended for students of military schools. Within the French system of military education the figure of Étienne Bézout (1730–1783) stands out as a popular textbook writer on mathematics, his audience being mainly the students of the various military institutions where he taught. Together with Charles-Étienne Camus (1699–1768) and Charles Bossut (1730–1814), Bézout has a place in the group of the renowned examiners and educational authors for the French military schools. Bézout originally wrote his popular *Cours de mathématiques* for navy engineers (1764–1767), followed by a book reduced as to content for artillery students (1770–1772). To carry out the comparative analysis I selected one of the many subsequent editions of Bézout's work, *Cours de mathématiques à l'usage du corps de l'artillerie* (1799–1800), because it is one of the latest in the century. It is revealing that, in clear contrast with the works of Camus (1749–1752) and Bossut (1781), Bézout's *cours* included differential and integral calculus.[12] As it is stated in the first page of the third volume, the principles of calculus came in useful for the introduction of the physico-mathematical sciences.

As we have seen above, there was no well-developed system of military schools in Germany in the eighteenth century. In spite of this apparent lack, I deemed it worth including in this second group a volume addressed to the cadets of the Royal Prussian Artillery, the *Anfangsgründe der Analysis des Unendlichen* (1770) by Georg F. Tempelhoff (1737–1807). The differential calculus is the main topic of the *Anfangsgründe*'s first volume. Although Tempelhoff studied mathematics at the universities of Frankfurt an der Oder and Halle, when the Seven Years' War started he entered the Prussian infantry and, soon afterwards, was transferred to the artillery force. His military career was marked by distinctions, to the point of being promoted to Lieutenant General in 1802. In fact Tempelhoff was appointed first director of the Artillery Academy in Berlin (1791). Beside his *Anfangsgründe der Analysis des Unendlichen* he published several mathematical works, among others, the *Anfangsgründe der Analysis der endlichen Grössen* (1768), the *Vollständige Anleitung zur Algebra* (1773) and the *Geometrie für Soldaten* (1790). Some of his works, even that on ballistics, were indeed said to be more relevant on a theoretical level than on a practical one.

4 Comparative textbook analysis

In this section I will discuss mainly the works of Bézout and Tempelhoff, with occasional references to the earlier works mentioned above. The comparative analysis focusses on the authors' views regarding the use of functions, the characterization of the limit, the concept of curve, the application of series expansions and the choice of coordinates.

Bézout opened his work with some reflections on the nature of the infinite quantities and the infinitely small ones.[13] Apart from this fact, the way Bézout introduced the basic definitions and rules of the differential calculus, and even the content of his work, resembles that of L'Hospital's. To begin with, the definitions of variable quantity and difference provided by L'Hospital in the first section of the *Analyse* did not differ substantially from the definition gathered in Bézout's book:

[11] As it is stated in Taton 1986, there were exemplars of their works in the École de Valence (1785) and the École Royale d'Artillerie de Strasbourg (1789).

[12] Schubring 2005, pp. 217–220.

[13] See Bézout 1799–1800, §§ 1–5.

> A *variable* quantity increases by infinitely small steps, the *difference* between the values of a variable in two subsequent instants being the corresponding increment (or decrement) of the variable (Bézout 1799–1800, § 6).

The rule for the differentiation of the product illustrates another coincident foundational aspect in the expositions of L'Hospital and Bézout. L'Hospital performed the differentiation of the product of xy as follows: if the quantities x and y were to increase in $\mathrm{d}x$ and $\mathrm{d}y$, respectively, then the difference of xy would yield $x\,dy + y\,dx + dxdy$. Assuming $\mathrm{d}x$ to be constant, the term $\mathrm{d}x\mathrm{d}y$ could be neglected since it was an infinitely small quantity with regard to $y\,\mathrm{d}x$ and $x\,\mathrm{d}y$. A century later Bézout proved the rule exactly the same way in his *Cours de mathématiques à l'usage du corps de l'artillerie*.[14]

Another illustrative example concerns the concept of curve. An essential point in Leibnizian calculus was that a curve could be considered to be identical with an infinitangular polygon, that is, a polygon of infinitely many infinitely small sides. This logically implied that the tangent could be taken for the extension of a side of the infinitangular polygon. We find this approach in L'Hospital's book, as well as in Bézout's.[15]

In the *Institutiones calculi differentialis* (1755) Leonhard Euler (1707–1783) considered the sequences of values as not induced by the infinitangular polygon, but by a function of an independent variable. It is known that Euler's *Introductio in analysin infinitorum* (1748) contributed essentially to the elaboration of the concept of function. Seven years later his differential calculus text provided a complete treatment of functional derivation. Hence it is worthy of mention that Bézout introduced the concept of function only in the section on integral calculus, but not in the one concerning differential calculus. This parallels the absence of the concept of function within the university context in France.[16]

By contrast, in his *Anfangsgründe* Tempelhoff introduced the use of functions. Like Euler in the *Institutiones calculi differentialis*, Tempelhoff started off with the consideration that the *difference* of a function between two consecutive values was a finite quantity. Then he extrapolated from finite differences to *differentials*, or infinitely small differences.[17] Tempelhoff's approach resembles again Euler's in that a line can be regarded as generated kinetically. Moreover, Tempelhoff referred on several occasions to Colin Maclaurin's *Treatise of Fluxions* (1742). This seems to imply that he bore an intuitive conception of the limit of ratio of differences,[18] the treatment being exclusively verbal, and not yet operational. There is a hint of this intuitive conception in Tempelhoff's definition of the tangent line as the limit of secant lines. On the operation of finding the limit, Schubring points out that Tempelhoff was the first to introduce the algebraization of the fundamental concepts of calculus in educational books for engineers and for students of military schools.[19]

Despite not providing an explicit definition, Tempelhoff grouped functions into algebraic and transcendental, and his classification proceeded as in Euler's *Introductio in analysin infinitorum* (1748).[20] In connection with the treatment of functions, chapter 6 of Euler's *Institutiones calculi differentialis* is devoted to the differentiation of transcendental functions, as derived from their series expansion. Insofar as Euler described there the rules for the differentiation of the trigonometric functions, one might expect the sine and the cosine to be treated as functions hereafter. That any function could be developed into series is actually stated in § 561 of Tempelhoff's book. Tempelhoff inferred nonetheless the differentials of

[14] Bézout 1799–1800, § 9; L'Hospital 1696, § 5.
[15] Bézout 1799–1800, § 30; L'Hospital 1696, § 3 and *Définition*, in Section II.
[16] See Schubring 2005, pp. 217–219.
[17] See for instance Tempelhoff 1770, § 255.
[18] Tempelhoff 1770, § 254.
[19] Schubring 2005, p. 251.
[20] Tempelhoff 1770, § 256.

the trigonometric lines from proportions of the segments that characterize these functions.[21] With regard to the differential of the sine, for example, Tempelhoff proved the formula from the comparison between the differential of an arc of the circle and the sine itself. In his book Tempelhoff referred to the sine and cosine as "lines", but also as "functions".[22] Surprisingly, this time it is Bézout who derived the differentials of the sine and cosine as Euler did, from the development of the formulas for the sum of two angles.[23]

Before closing this section I would like to draw attention to the fact that both Tempelhoff and Bézout chose orthogonal coordinates for the curves involved in the problems they wanted to solve, independently of the geometrical nature of the curve. Their preference gives a glimpse of the emergence of the independent variable, so crucial in consolidating the fundamental role of the function.

5 Some final remarks

Underlying the epistemological features of these works, some national trends can be made out, which uphold some of the views outlined at the beginning of this paper. When it comes to the algebraization of calculus, the books used in the French military system with educational purposes did not include the concept of function, let alone that of the limit of ratio of differences. We have seen that Tempelhoff conferred great value to the consideration of new approaches in his book. While the *Anfangsgründe* can be said to have played an essential part concerning an early reception of Euler, Bézout offered a rather elementary exposition of the differential calculus, with much in common with L'Hospital's *Analyse*. That his section on calculus introduced the sections on mechanics and hydrostatics conveys the idea of calculus as an auxiliary tool, the stress being on its applications.

We can therefore speak of two tendencies with regard to the relationship between education and research in these contexts. As a brand new system, the German military education might have been influenced by the dominant university context, wherein research tasks were encouraged. On the contrary, in the French military system teaching and research followed different paths. Not unlikely this was due to the institutional framework inherited from the religious *collèges*. Given the relevant role of the connection between teaching and research in shaping a discipline, we can conclude that the emergence of differential calculus as a discipline evolved at a different pace in the contexts object of this paper. Therefore the emergence of a discipline turns out not to be independent from the national educational system in a specific period.

In short, this different perception of teaching and research might have prevented the differential calculus from becoming a "boundary object" between the analyzed contexts. That is to say, the diverging meanings that calculus had in these two different social worlds granted no recognizable means of translation.[24] This fact confirms Schubring's statement on the rarely mutual exchange between France and the German states before the 1790s, in particular between their corresponding systems of military education.[25]

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[21] See Tempelhoff 1770, § 332–349.

[22] Tempelhoff 1770, § 332 and § 565.

[23] Bézout 1799–1800, §§ 22–ff; Euler 1755, § 195.

[24] I am borrowing here the definition of "boundary object" as quoted from Susan Leigh Star and James Griesemer in Roberts 2005, p. 3: "both plastic enough to adapt to local needs and constraints of the several parties employing [it], yet robust enough to maintain a common identity across sites... [it has] different meanings in different social worlds but [its] structure is common enough to more than one... [making it a] recognizable means of translation."

[25] Schubring 1996, p. 367.

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