The Emergence of Mathematics as a Major Teaching Subject in Secondary Schools

Gert SCHUBRING, Hélène GISPERT,
Nikos KASTANIS, Livia GIACARDI

Bielefeld, Germany; org., Paris, France, Thessaloniki, Greece, Torino, Italy

gert.schubring@uni-bielefeld.de, helene.gispert@u-psud.fr,
nioka@math.auth.gr, livia.giacardi@unito.it

Abstract

One of the most decisive characteristics in establishing public systems of education in the European countries was the introduction of mathematics — hitherto a marginal subject at the existing secondary schools — as a major teaching subject, as a constitutive dimension of general education. This introduction did not take place in a homogeneous manner. Rather, the forms, contents, and methodologies depended upon different cultural, social, and political contexts prevalent within the respective countries. Moreover, after the introduction, no steadfast evolution was assured — in several countries, mathematics teaching suffered backlashes, jeopardizing, or in fact reducing, its function as a major subject. The panel will confront these processes and experiences had in four of the European countries:

- *Mathematics for the first time being established as a major school subject in public education with the French Revolution, declining under French Restoration, and eventually being resurrected in the course of the 19th century (Hélène Gispert, University of Orsay, France),*
- *Mathematics becoming a constitutive element of general education in Prussia and Bavaria around 1810 (Gert Schubring, University of Bielefeld, Germany),*
- *The transition from private, church-organized teaching to public instruction in the newly established Greek national state (Nikos Kastanis, University of Thessaloniki, Greece),*[1]
- *Mathematics in the educational reforms of unified Italy since 1861, and ensuing conflicts caused by the predominance of humanistic culture (Livia Giacardi, University of Turin, Italy).*

Introduction

As shown in the preceding Plenary, mathematics teaching was firmly established in France, and in Germany, during the 20th century. Its status as a major teaching subject was not challenged — there were merely problems with regard to reducing the number of lessons resp. teaching hours per week.

This unchallenged status is all the more remarkable as mathematics now enjoyed a relatively new preference. At least until the early 19th century, mathematics had had, and

[1] N. Kastanis had not been able to assist the Congress. See the paper: Iason and Nikos Kastanis, "The Transmission of Mathematics into Greek Education, 1800–1840: From Individual Initiatives to Institutionalization", *Paedagogica Historica. International Journal of the History of Education*, **XVII**, 515–534.

continued to have, in some other countries even until the end of the 19^{th} century, a minor, and even marginal role as a teaching subject at secondary schools.

What can be observed thus is a revolutionary rise of mathematics, from being marginally taught in just a few grades of the *Gymnasium*, *college*, *colegio*, *collegio*, to attaining a key function within reformed systems of public instruction. Trying to understand how such a radical change came about is quite a challenge. This is why it is rewarding to undertake a comparison between various European countries as to how the process evolved within these. Such a comparison has been attempted for the first time. It is based on recent research.

A few instances characterizing the respective context of such revolutionary change can be listed from the outset: A major drive follows from the fact that the rise of mathematics coincided with the establishment of public school systems, and with a process of secularization of society which implied a separation between religion and state.

This was conducive to a novel role of the state, which took on the responsibility of organizing a national system of education providing general education for the young generation, or for some socially selected members of the latter.

The issue thus is to study how mathematics came to be accepted as a constitutive part of general education — while before it had only been regarded a a marginal subject apt to provide some auxiliary, or other less essential aptitudes.

It is also evident, therefore, that any investigation confined to mathematics instruction alone will not yield significant answers. As the issue of why mathematics rose in importance is intimately related to the social, cultural, and political histories of the countries concerned, it must take these into account as well.

These interplays are all the more illustrative as the initial rise of mathematics did not guarantee any continuous expansion of mathematics teaching. The contributions will show that a host of interventions from the environment led in all countries to ups and downs, some of the downs even again reducing mathematics to a minor subject.

We will proceed chronologically: beginning with France, the first national state to establish a public school system; continuing with states in Germany, which followed suit shortly afterwards, sparking developments in Greece, and closing with Italy, which became unified into a national state after 1861.

France

The first decisive French measure was to establish the *lycée*, in 1802, during the First Empire, the "lycées napoléoniens" that Napoleon Bonaparte conceived to be interbred by two separate traditions, so-to-say a political "entre-deux", a political compromise of these post-revolutionary times.[2] The first and more recent tradition was that of Enlightenment which had inspired the French revolutionary educational projects conveying a major role to mathematics and science; mathematics being highly valued both as a theoretical subject and for its applications. The other was the classical and humanist tradition in which Latin and Greek constituted the major teaching subject. Thus, the *lycée* was erected on two pillars, on Latin and on mathematics, both to be taught from the very beginning to the final grade, the one preparing for the *baccalauréat.*

This structure did not last long, that is it did not outlast the Empire period. In fact, when French Restauration set in after 1815, when the *Ancien Régime* had returned from exile — staunchly upholding values dating from before the French Revolution — there was no more mathematics in the *lycée*, except in its final grades, and the *lycées* had been renamed *"collèges royaux"*. The major teaching subjects after that were classical, concentrating on humanities, the unique goal of the *collèges royaux* being to train an intellectual and social

[2]Bruno Belhoste, "Introduction", in *Les sciences dans l'enseignement secondaire français. Textes officiels . 1789–1914*. Paris: INRP & Economica, 1995. pp. 1–62 (quote 27–29).

elite for administration. The values to be taught to this elite concerned a feeling — taken literally from the contemporary comments for teachers in the syllabi — for "le beau, le bien, le vrai", the beautiful, the correct, the true.[3].

Mathematics, after that, was again considered merely a speciality, not a general educational discipline — and it was the same for science. Young men — I can speak only of the training of boys (secondary schools for girls were not established before the last third of the 19$^{\text{th}}$ century)[4] — were supposed first to be educated in humanities, opting for their special subjects afterwards. And only at this late point in their school career, those wanting to become natural scientists, engineers, or the like, were permitted to choose mathematics as a major subject to be taught to them. Even the designation of the various grades indicates that, the lower being called "grammar grades" and the two final ones "rhetoric" and "philosophy".

For the *lycées*, this state of affairs held until the middle of the 19$^{\text{th}}$ century. In 1833, however, when a new *régime* succeeded to that of Restauration, still under a monarchy, albeit a more liberal one, a new kind of intermediate school was established to train the boys of the middle classes who were barred from, or did not desire to attend, the *collèges royaux* and to submit to their classical schooling. These were the "écoles primaires supérieures" (upper primary schools).[5] Within these, their educational objectives being practical and concrete, mathematics became a major teaching subject. These schools, however, were never intended for the elite, not even for that belonging to the bourgeoisie.

In the middle of the century, with the Second Empire and after the Revolution of 1848, we find a decade favourable to mathematics and to science at the *collèges*. This was a period of strident and short-lived reform, of change refused by almost all teachers of secondary education, even by mathematics teachers, of the the so-called "réforme de la bifurcation".

The problem had been festering since the 1830s. Since mathematics was taught only in the final year of the *collèges*, candidates for the military, technical or engineering *grandes écoles* could not be properly prepared. Thus, a parallel system of auxiliary courses and private institutions had been growing for decades, rivalling the *collèges royaux*, a parallel world where mathematics lessons dominated training.

In 1852, Louis Napoleon — much more enamoured of science, technology and "progress" than his royal predecessors, and in line with the positivist current of his period — decreed a reform of secondary education.[6] I just spoke of the 1848 Revolution, the first in which the working class assumed an important political role, throwing a terrible scare into the governing classes and into the bourgeoisie. Napoleon III intended to kill two birds with one stone, to train both a scientific and technological elite and to contain the drive of the dangerous upstarts from the middle and working classes by education. Secondary instruction was no longer confined to the intellectual and administrative elite. Its task now became to train and educate managers for industry and business as well.

Latin ceased to be compulsory in colleges after the third year, and new contents and new methods were defined for all subjects. Mathematics acquired another importance, the problem being, however, that it was not taught but with practical purposes in mind: no more Euclid, and no more proofs in geometry; rather, the focus now was on applications, and we are able to note more advanced topics than beforehand .

[3]Martine Jey, *La littérature au lycée, l'invention d'une discipline (1880–1925)*, Metz: CELTED, Paris : Klincksiek, 1998.

[4]Nicole Hulin, *Les femmes et l'enseignement scientifique.* Paris : PUF, 2002.

[5]Jean-Pierre Briand & Jean-Michel Chapoulie, *Les collčges du peuple.* Paris: INRP, CNRS, ENS Fontenay-Saint Cloud, 1992. Renaud d'Enfert, "Introduction" in *L'enseignement mathématique à l'école primaire. Textes officiels 1791–1914.* Paris : INRP, 2003. pp. 13–44 (quote 25–27 & 40–43).

[6]Nicole Hulin, *L'Organisation de l'enseignement des sciences: la voie ouverte par le Second Empire.* Paris : Editions du CTHS, 1989. Bruno Belhoste, *op. cit.* pp. 41–47.

This abrupt change, this deterioration of goals, was felt to be unbearable by the intellectual elite and by teachers; the reform did not outlast ten years. The solution to this conflict was to separate the two ways of schooling by creating a novel secondary institution — another beside the *lycée* which returned to its classic role and confinement to intellectual and purpose-free cultural studies. The new separate institution was named "secondaire spécial", because its goal was to teach what was opportune and not to provide any general cultural education. Mathematics, like science, had a major role, and advanced topics were taught referring to applications, with practical goals in mind. In the *lycées*, Euclid and geometrical proofs returned, but with a very minor role because mathematics as a whole was marginal as well.

It is evident that less educational value was attributed to this "secondaire spécial" than to the classical *lycées*. Perhaps not so evident is the overwhelming success of this *secondaire spécial* over the classical *lycées*.

It must be stated, however, that these two institutions together did not receive more than 5 % of young boys in France until the end of the 19th century.

At the close of this century, one of the results of the competition between the two schools was first the growing success of "secondaire spécial", the increasing attendance of boys from the bourgeoisie, which compelled it to assign a higher symbolic value to this school type. Thus, it shed its name of "special secondary school", instead assuming that of "modern secondary school". It did so at a cost, however: simultaneously, its links to applications were severed, and some advanced mathematical topics were struck from the syllabi In the 1890s, graduation from this school was eventually honoured by the title of *baccalauréat* — a modernized *baccalauréat* — increasingly conveying the image of classical schooling, but somehow in a watered-down mode.

A second result was that the classical *lycée*, confronted by this menace, relied more and more on classical options, and mathematics was once more confined to the final year/term.

Finally, the situation — and here I come back to my former lecture — had become untenable at the close of the 19th century. I shall not repeat my mention of the 1899 inquiry[7] and the 1902 reform.[8] I should only like to stress one point, and this will be my conclusion. In 1902 with the beginning of the new century, two historically opposed or even contradictory issues of mathematical training were for the first time reconciled in another novel institution, the 1902 *lycée* both modern and classic. For the first time, Borel's question: "Will we not risk diminishing its great educative value when we make mathematical education more practical and less theoretical?",[9] was answered in the negative. No, we will not risk diminishing its great educative value.

Alas, as we have seen just before, however, the latter reconciliation of the goals classical and modern, which raised mathematics to the status of a major subject in secondary education, was not to last.

Germany

The difficulty in analysing aspects of German history is given by the multitude of coexisting independent states. For the time between 1815 and 1866, there were 39 separate German sovereign states, each with a educational system of its own. I can present here no more but

[7] Renaud d'Enfert, "La question des disciplines scientifiques dans l'enquŕte Ribot (1899)", in H. Gispert, N. Hulin, M.-C. Robic (dir) *Science et enseignement. L'exemple de la grande réforme des programmes du lycée au début du XXe siècle.* Paris : Vuibert & INRP, 2007. pp. 65–80.

[8] Bruno Belhoste, *op. cit.* pp. 55–60. Hélčne Gispert, "Quelles lectures pour les conférences de mathématiques: savante, pédagogique, politique?" in H. Gispert, N. Hulin, M.-C. Robic (dir) *op. cit.* pp. 203–222.

[9] Emile Borel, "Les exercices pratiques de mathématiques dans l'enseignement secondaire", *Revue générale des sciences pures et appliquées* 14 (1904), 431–440.

some characteristic cases. Since the Protestant Reform, these educational systems had been split, according to the religious affiliation of the respective sovereign, into Protestant systems and Catholic systems. Major differences concerned their higher education, but, regarding secondary education, mathematics constituted a marginal subject in both systems:

- In Catholic territories, only in the final grade, the class of philosophy, there were a few months of mathematics instruction, directed towards an interest in astronomy — but not in the preceding grades;

- In Protestant territories, there used to be some arithmetic teaching in lower grades. This became complemented later on — during the 18$^{\text{th}}$ century — by some mathematics in upper grades.

One can observe the beginnings of change during the second half of the 18$^{\text{th}}$ century: mainly in Catholic territories, due to the dissolution of the Jesuit order. Some states now introduced mathematics as a subject to be taught in all grades, but these were only rather regional practices.

Profound changes occurred, however, in the wake of the French Revolution.[10]

It was Bavaria, which became the first model for fundamental social and political reforms in a German state. These reforms included education, too, so that a system of public schools was established. Reorganizing secondary education occurred in 1808. Two parallel types of general education were institutionalized: the *Gymnasial-Institute* with a classical profile, and the *Real-Institute*, with a modern profile. Mathematics was a major teaching subject in both types.

The next state, which followed was Prussia. In 1810, Wilhelm von Humboldt and Friedrich Daniel Schleiermacher cooperated in establishing the famous neohumanist conception of education. These educational reforms were part of the fundamental reforms both political and social of these years — implementing an "intellectual" revolution from above, instead of a political one from below. Even the educational reforms themselves proved to be systemic: The neohumanist conception of education envisaged an intellectual formation by several major teaching subjects — not just two, as Latin and mathematics in Napoléonic France, but three constitutive elements of general education in the reformed *Gymnasien*: classical languages, mathematics and the sciences, and history and geography. Since the syllabus provided six weekly lessons[11] for mathematics in all grades, there was a considerable demand for mathematics teachers. In fact, the simultaneously reformed Philosophical Faculties were endowed for the first time with proper courses of study, for future teachers of these three major disciplines. The demand for teacher training led to the emergence of research and teaching in specialized, pure mathematics. Despite all problems of implementation of such a profound educational reform, the ministry succeeded in maintaining the basic dimensions during the first half of the 19$^{\text{th}}$ century.[12]

In Bavaria, however, the political backlash after Restoration in 1815 effected a turnabout in the educational system, too. In 1816, the *Real-Institute* were dissolved, and their teachers dismissed. The *Gymnasial-Institute* became the only type of general secondary schools, but now with a lopsidedly classical profile. Mathematics teaching was reduced to just one weekly hour, entrusted to the now generalist teacher for all subjects of a given grade, since the

[10] Gert Schubring, "Essais sur l'histoire de l'enseignement des mathématiques, particuličrement en France et en Prusse", *Recherches en Didactique des Mathématiques*, 1984, 5, 343–385.

[11] Gert Schubring, "Die Geschichte des Mathematiklehrerberufs in mathematik-didaktischer Perspektive", *Zentralblatt für Didaktik der Mathematik*, 1985, 17, 20–26.

[12] Gert Schubring, *Die Entstehung des Mathematiklehrerberufs im 19. Jahrhundert. Studien und Materialien zum Prozeß der Professionalisierung in Preußen (1810–1870).* Zweite, korrigierte und ergänzte Auflage (Weinheim: Deutscher Studien Verlag 1991).

mathematics teachers here were dismissed as well. Bavaria, now, fell into social and political backwardness.[13]

A next telling case is presented by the kingdom of Wurttemberg, where secularization occurred particularly late. This situation found its characteristic expression in the fact that instruction in the lower and middle grades of the secondary schools was determined by an exam which was external to the school system: it was the so-called "Landexamen", the entrance exam for the Protestant seminaries, i.e., the obligatory propaedeutics for theological studies. Since becoming a Protestant pastor constituted still the dominant professional career at Württembergian secondary schools, what was taught there essentially was just that what would be examined in that *Landexamen*. And for mathematics, that was just a bit of arithmetics. This marginal and moreover elementarist position became changed only after 1891, and that slowly.[14]

A last characteristic example is provided by Kurhessen, a rather small and agrarian state with Kassel as its capital. Since there existed only six *Gymnasien* in the country during the first half of the 19th century, the government had no ministry of education of its own. Educational matters were handled by the ministry for the interior. And for *Gymnasium* questions, this ministry relied on its being counselled by the board of the Gymnasium directors — all being philologists. Mathematics held, formally seen, the position of a major teaching subject since mathematics was examined in the final *Abitur* exam. The directors became, however, increasingly concerned about this status, since poor achievement in mathematics was apt to lower the predicate of students excelling in classical languages. Eventually, the directors succeeded in having the ministry issue a decree in 1843, which drastically reduced the contents of mathematics teaching in that state. The decree was based on the notion of limit — evidently not limit in the sense of calculus, but as limit of school mathematics. Regarding arithmetic and algebra, it defined that equations of second degree already transcended the limits of school mathematics and belonged instead to university mathematics! Thus, without excluding mathematics from the *Abitur* exam, and without challenging its formal status as a major subject, it became in fact so reduced that exams on such elementary topics could no longer influence the final outcome.[15]

These four cases illustrate the enormous scope of variation in the real status of mathematics education in Germany, which was supposed to have a common culture, but where marked differences in political and economic development also shaped different school structures and views on general education.

Italy: Mathematics and Scientific humanitas in secondary teaching in Italy

1 Italian Schools Post-Unification

After the unification of Italy, young nation's difficult and important task of forging Italian citizenship was entrusted to the schools — in particular, to secondary schools — and among those who took up the gauntlet are some of the greatest Italian mathematicians.[16] The

[13] Gert Schubring, "Die Mathematik — ein Hauptfach in der Auseinandersetzung zwischen Gymnasien und Realschulen in den deutschen Staaten des 19. Jahrhunderts", *Bildung, Staat und Gesellschaft im 19. Jahrhundert. Mobilisierung und Disziplinierung. Hrsg. K.-E. Jeismann.* (Stuttgart: F. Steiner 1989), 276–289.

[14] Gert Schubring, "Der Aufbruch zum 'funktionalen Denken': Geschichte des Mathematikunterrichts im Kaiserreich. 100 Jahre Meraner Reform", *N.T.M.*, 2007, 15, 1–17.

[15] Gert Schubring, as note 13.

[16] For further details on the subject of this paper cf. Giacardi, L., 2006, "From Euclid as Textbook to the Giovanni Gentile Reform (1867–1923). Problems, Methods and Debates in Mathematics Teaching in Italy", *Paedagogica Historica. International Journal of the History of Education*, **XVII**, 587–613 and Giacardi, L. (ed.), 2006, *Da Casati a Gentile. Momenti di storia dell'insegnamento secondario della matematica in Italia*, La Spezia: Pubblicazioni del Centro Studi Enriques, Agorŕ Edizioni. The most important legislative

earliest legislation aimed at giving a comprehensive organisation to the Italian education system was the Casati law, from the name of the Minister for Education, Gabrio Casati, who drafted it. The new law of 1859 was designed to reorganise the school system in Piedmont and Lombardy, and was gradually and with difficulty extended to the other Italian regions. All legislation regarding education in Italy was based on this law until 1923, when Giovanni Gentile, a prominent figure among Italian Neo-Idealist philosophers, introduced the reform that brought important changes to the school system, while maintaining various of its key features.

Its distinguishing characteristics are the dominant role of university studies in the overall scheme, the bureaucratic centralisation, and the concern for forming a ruling class rooted in the values of humanistic culture. In conformity with this aim, the Casati law divided secondary education into two branches: classical (consisting of 5 years of *ginnasio* and 3 years of *liceo*) leading on to university studies and intended to form the elite — both scientific and technical — of the future; and technical (lower 3 years and upper 3 years), intended as training for trades, and not leading to university admission. However, it was the *ginnasio-liceo* that formed the core of the secondary school system in Italy. In the technical institutes, only the physics-mathematics stream, created in 1860, gave access to university (science faculties). Despite ups and downs, for about sixty years it remained the branch of secondary education where mathematics was of prime importance. Mathematicians of scientific standing such as Vito Volterra, Corrado Segre, and Francesco Severi attended it.

In order to appreciate and evaluate the legislative measures adopted after the Casati Law, the choices made and their consequences for mathematics teaching, it is essential to know the situation of Italian schools post-unification.

First of all it is necessary to bear in mind the very high rate of illiteracy that was present, which, according to the census of 1861, reached almost 87 % in Palermo and almost 92 % in Cagliari. Secondly, the number of students who attended secondary school was extremely low, equal to 0,7 per 1 000 inhabitants.[17] The problems which afflicted the secondary schools emerge clearly from a Higher Council for Public Instruction report of 1864: the inadequate recruitment of teachers, poor-quality textbooks, the "premature bifurcation" in classical and technical courses which excluded from the *ginnasio* all disciplines useful to everyday life, and low standards regarding the final exams for the diploma.[18]

The greatest Italian mathematicians of the time were well aware of the situation and sought at first to make up for the lack of Italian treatises with numerous translations of French and German elementary textbooks. Among these Luigi Cremona and Enrico Betti stand out.[19]

2 Cremona and mathematics as "a means to develop general knowledge, a kind of mental gymnastics"

Important changes for the teaching of mathematics resulted from the Act of Parliament issued in 1867 by the Minister for Education, Michele Coppino. The mathematics curricula and instructions on teaching methods were actually the brainchild of the geometry scholar Luigi Cremona, who re-introduced Euclid's *Elements*, "the most perfect model of rigorous

measures concerning the teaching of mathematics in Italy from 1859 to 1923, can be found on the web-site http://www.dm.unito.it/mathesis/documents.html.

[17]Talamo, G., 1960, *La scuola dalla Legge Casati alla inchiesta del 1864*, Milan : A. Giuffré, 61–62.

[18]Bertini, G., 1889, *Relazione e proposte sull'istruzione secondaria*, 1865, in *Per la riforma delle scuole medie. Scritti vari*, Torino : G. Scioldo, 81–114.

[19]I only mention the Italian translations of the treatises on geometry by Legendre (Rubini 1855; Panunzio 1858; Poli 1877; ...) and by Amiot (Novi 1858); on trigonometry by Serret (Ferrucci 1856); on algebra and arithmetic by Bertrand (Betti 1859, Novi 1862); on the elements of mathematics by Baltzer (Cremona 1865–1868).

reasoning", as the textbook to be used in the classical secondary schools. Indeed he was convinced that:

> ...it [mathematics] is principally a means to develop general knowledge, a kind of mental gymnastics aimed at exercising the faculty of reason.[20]

Just one year after the Coppino Act, an Italian translation of Euclid's *Elements* with supplementary notes and exercises, *Gli Elementi di Euclide con note aggiunte ed esercizi ad uso de' ginnasi e de' licei*, was published by Enrico Betti and Francesco Brioschi, but the real author was Cremona, as can be gathered from his letters to Betti. Cremona's aim was threefold: to do away with the myriad of worthless books, compiled merely to make profit; to foster the publishing of good Italian text-books; to oppose the A. M. Legendre approach to geometry:

> "Above all", he says, "the teacher must not pollute the purity of the geometry of ancient times, by transforming geometrical theorems into algebraic formulae, thus replacing the concrete magnitudes with their measures".[21]

His final aim was to educate the future ruling class.

3 The flourishing of mathematics textbooks for secondary schools

The reintroduction of Euclid's text and the publishing of the Betti-Brioschi textbook provoked a heated debate among teachers and mathematicians, as can be inferred from the correspondence of the Italian mathematicians and from articles published in *Giornale di Matematiche* soon after the Italian translation of a paper by J. M. Wilson, who criticized Euclid's *Elements* from both the scientific and the didactic point of view. The most significant consequence of this debate was the publication of high quality textbooks written by the foremost Italian mathematicians, which was exactly what Cremona hoped for.

This phenomenon did not go unobserved abroad; in particular, Felix Klein noted it several times, but he also observes:

> ...great mathematicians have been involved in this enterprise and have produced texts of great scientific value while of modest pedagogical quality.[22]

Indeed, many of these manuals were translated or reviewed in international journals. I will mention only those that had a marked influence on the debate on teaching geometry. The *Elementi di Geometria* by Achille Sannia and Enrico D'Ovidio (1869)[23], the *Elementi di geometria ad uso dei licei* by Aureliano Faifofer (1880)[24] follow the Euclidean method, while improving it where it shows weaknesses, and adding supplementary topics. Riccardo De Paolis' textbook *Elementi di geometria* (1884) marks the beginning in Italy of *fusionism*, the name given to a teaching method where the related subjects of plane and solid geometry are studied together, properties of the latter being applied to the former in order to gain the

[20] Cf. "Istruzioni e programmi per l'insegnamento della matematica nei ginnasi e nei licei." *Supplemento alla Gazzetta Ufficiale del Regno d'Italia*, Florence, 24 October 1867.

[21] Cf. Ibid.

[22] Klein, F., 1925, "Der Unterricht in Italien", in *Elementarmathematik vom höheren Standpunkte aus*, Berlin : Springer, 1925–1933, II, 246.

[23] This textbook had editions in 1869, 1876, 1895, and an eleventh edition at the end of the century. It was reviewed by J. Hoüel and T. A. Hirst, and partially translated into English.

[24] This textbook had editions in 1880, 1882, 1890, and a seventeenth edition in 1909. It was reviewed by P. Mansion, G. Teixeira and A. Buhl, among others, and translated into French, Spanish, and Japanese.

maximum benefit. However *fusionism* spread in Italy thanks only to the *Elementi di geometria* by Giulio Lazzeri and Anselmo Bassani (1891)[25] which were more careful of didactic demands.

There are some manuals that explicitly show the influence of the studies on the foundations of geometry. Among them I mention only the *Elementi di geometria* by Giuseppe Veronese, (1895), which were criticised by Klein for the scant attention given to didactic aspects,[26] and the textbook by Michele De Franchis (1901), which is notable and innovative for the rigorous approach to the theory of congruence (the "group of motions" is introduced), but was considered too difficult by teachers.

Instead, attention to the teaching method and to didactic needs characterises the textbook written by an eminent figure in the Italian school of algebraic geometry, Federigo Enriques, together with Ugo Amaldi, *Elementi di geometria, ad uso delle scuole secondarie superiori* (1903)[27]. Here the subject is approached through the rational-inductive method, in an attempt to overcome the defect typical of Euclidean exposition. The scientific and methodological bases for this acclaimed textbook, as Enriques himself states, derive from the *Questioni riguardanti la geometria elementare* (1900), a collection of papers on problems of elementary mathematics seen from a higher point of view, written with the contribution of Enriques's friends and of the members of his school, and clearly influenced by Klein.

The publication of these manuals served to stimulate the debate that was reflected in a series of legislative measures concerning the teaching of geometry: a Circular (1870) limited the obligation to follow Euclid to plane geometry only; the Baccelli Decree (1881) introduced the teaching of intuitive geometry into the lower *ginnasio* in order to attenuate the impact with rational geometry; the Coppino Decree (1884) established (E. Beltrami) that the study of rational geometry be reinstituted in the fourth year of *ginnasio*; the Gallo Decree (24. 10. 1900) no longer referred to Euclid's *Elements* for the teaching of geometry, left the teacher at liberty to follow either separation or fusion, and reinstated the study of intuitive geometry in the first classes of the *ginnasio* excluding the disquisitions on the foundations of science from the schools.

Textbooks for geometry, above all else, influenced the debate on methodology. There were, however, two algebra textbooks with different methodological approaches — one by Cesare Arzelà, the other by Giuseppe Peano —, which influenced subsequent mathematical literature. Moreover, it is in the algebra texts written for the physics-mathematics stream of the technical institutes that the concept of function and the first elements of infinitesimal calculus were introduced for the first time. The *Trattato di algebra elementare* (1880) by Arzelà was one of the most widely adopted textbooks in secondary schools. Written for the physics-mathematics section of the technical institutes, it featured a new methodological approach: actually the core concept behind the presentation of the material was not the equation, but rather the function. Peano's *Aritmetica generale e algebra elementare* (1902) featured the systematic use of logical symbols which, according to the author, contribute not only brevity, but also precision and clarity. For this reason it was generally greeted with puzzlement by teachers.

4 Light and shadow in secondary teaching of mathematics at the end of the 19th century

The years from the Unification of Italy up to the early twentieth century were a period of great political and social ferment. Italians were also making advances of considerable

[25] This textbook had editions in 1891 and 1898 and was reviewed by L. Ripert, and translated into German by P. Treutlein in 1911.

[26] Klein, F., 1925, "Der Unterricht in Italien", cit., 247–248.

[27] This textbook had numerous editions up to 1992 and was reviewed by F. Palatini 1903, G. Vailati 1904, etc.

importance in scientific research, achieving international recognition at the highest levels with the successes of the Italian school of Algebraic Geometry and Peano's studies on Logic.

Towards the end of the nineteenth century the studies on the foundations of mathematics created a common area of interest between elementary mathematics and advanced research. As a result, certain mathematicians who were deeply committed to pure research were also personally involved not only in preparing school textbooks, but also, on the politico-cultural side, in developing an improved framework of laws on education and in teacher training. The mutual interchange between universities and secondary schools was a further source of enrichment: university teachers had often begun their careers as secondary school teachers (Cremona, Betti, D'Ovidio, De Paolis, ...), while the most distinguished secondary school teachers (Lazzeri, Faifofer, Bettazzi, Vailati, ...) taught courses at university. This enabled them to incorporate the experience of teaching on two different levels into their daily work.

Teacher Training Schools (*Scuole di Magistero*) were established, and the first teachers' associations were founded. The most important of them was the *Associazione Mathesis*, founded by Rodolfo Bettazzi in Turin. Its specific aim was "improvement of the school system and the training of teachers in both scientific and methodological aspects of mathematics". Under the leadership of its presidents, including the prominent mathematicians Severi, Castelnuovo and Enriques, this association was often to make its voice heard on issues regarding legislation for secondary schools.

Strangely enough, this commitment on the part of mathematicians did not correspond to a significant improvement in the quality of mathematics teaching during the last twenty years of the nineteenth century. We need only consider the series of legislative measures enacted between 1881 and 1904 to see how the role of mathematics was progressively weakened both in the curriculum contents and in the number of teaching hours allocated.

Some of the causes of this situation become evident from the Ministry of Education report in 1887[28], which presents a comparative analysis of the curricula and timetabling of classical secondary schools (*ginnasio-liceo*) in Italy and in the rest of Europe. This report clearly shows the defects of the Italian *ginnasio-liceo*, particularly when compared to schools in Germany: the excessive number of hours devoted to the native language and the lack of foreign languages teaching; the poor coordination between mathematics and physics teaching, and, finally, the adoption of a teaching method which was purely rational, allowing very little room for practical application. (see Table 1)

Moreover in 1893 the Baccelli decrees suppressed the written examination in mathematics in the diploma exams for the *ginnasio* and *liceo* and in 1904 the Orlando decree gave second-year *liceo* students the option of choosing between Greek and Mathematics, "releasing congenitally incapable students from a useless burden".[29] This decision, which was severely criticised by the various teachers' organisations, was abolished only in 1911.

The discussions in the milieu of the *Associazione Mathesis* — Turin 1898, Livorno 1901, Naples 1903, Milan 1905 — and debates within the National Federation of Middle School Teachers — Milan 1905 — not only provided evidence of a increasingly numerous participation of teachers in scholastic politics, but also focused on the weaknesses and defects of secondary teaching.

5 A good reform, which was not realized

Due to the evident deficiencies in secondary school teaching, in 1907 the minister of education Leonardo Bianchi appointed, a Royal Committee to prepare a radical reform of the secondary school system. After comprehensive inquiries, in 1908 it presented, a draft for a law, that

[28] Cf. "Esame comparativo dei programmi nelle scuole secondarie classiche." *Bollettino Ufficiale dell'Istruzione* XIII (Ottober 1887), 193–241.

[29] "Programmi di matematica per i ginnasi ed i licei." *Bollettino Ufficiale del Ministero dell'Istruzione Pubblica* XXXI, II, n. 52, Rome, 29 December 1904, 2851.

proposed a drastically changed school structure and innovative curricula: a three-year course for the lower secondary school, common to all types (*scuola media unica*) should be followed by three different branches of the *liceo*: *classico*, *scientifico*, and *moderno*. The reform proposed was based on the acknowledgement of the educational importance of scientific culture and was inspired by a positivist and liberal-democratic school of thought.

The syllabi for mathematics and the instructions on teaching method were written by Giovanni Vailati (1863–1909) and expressed his own vision of mathematics, where positivist principles, epistemological propositions from Peano's school, and the need to make culture democratic, blend harmoniously with pragmatism, as well as with his deep-rooted belief in the unity of knowledge and in the educational importance of mathematics.

Critizing the teaching approach based on passive learning, he proposed active modes of learning: students should show that they know *how to do things*, not merely *how to repeat things*. Other methodological aspects were stressed by Vailati: first of all, the importance of showing the applications of algebra to geometry, and vice versa, in order to make students appreciate immediately the underlying unity of the mathematical disciplines, and to train them to approach any one problem with a variety of methods, choosing, as the situation requires, the best possible approach. He also considered it important to find a balance between intuition and rigour in mathematics teaching. Moreover, in view of the aims of the different courses of study, the concepts of function and of derivative were introduced in all three branches of *liceo*, the concept of integral was introduced in the *scientifico*, while probability theory and its applications were taught in the *moderno* to students intending to enter the world of work, or to continue with technical studies. In the *liceo classico* the emphasis was on Euclidean geometry, accompanied by readings from the original writings of the great geometers of the ancient world, thus offering the students a more complete picture of classical civilisation, not limited to the fields of art and literature.[30]

The structural reform, and especially the unification of the lower secondary schools, was considered to be too radical. The mathematics curricula prepared by Vailati also attracted criticism. They were discussed during the congress of the *Associazione Mathesis* held in Florence in 1908. The *Mathesis* committee appointed to present a report on Vailati's proposals criticized the absence of any treatment of the theory of proportions, or of a rational treatment of arithmetic, the excessive fragmentation of some parts of the programme, and the abolition of descriptive geometry.[31]

In any case, due to the manifold resistances, the proposed reforms were never carried through. However a part of Vailati's proposals was implemented in 1911 when the minister Luigi Credaro established the *liceo moderno*, which diverged from the *classico* after the second year of *liceo*, and where Greek was replaced by a modern language and greater scope was given to scientific subjects. Castelnuovo, then president of the *Associazione Mathesis*, was given the task of preparing the curricula and the instructions on teaching method for the new courses. He gave great importance to numerical approximations and introduced the concepts of function, derivative and integral illustrating them by applications to the experimental sciences. He also highlighted the importance of coordinating mathematics teaching with that of physics and of avoiding the over-refinement of modern criticism, and, at the same time, the trap of simplistic empiricism. This syllabus for the *liceo moderno* began to be introduced in the schools from 1914–1915, despite the difficulties caused by the lack of trained teachers, by the hostility of the teachers in the *liceo classico*, who sent the

[30] Vailati, G., 1910, "L'insegnamento della Matematica nel nuovo ginnasio riformato e nei tre tipi di licei." *Il Bollettino di Matematica*, **IX,** 57; cf. Giacardi, L., 1999, "Matematica e humanitas scientifica. Il progetto di rinnovamento della scuola di Giovanni Vailati." *Bollettino della Unione Matematica Italiana*, **3-A**, 339–341.

[31] Berzolari, L., Bortolotti, E., Bonola, R., Veneroni, E, . "Relazione sul tema: I programmi di matematica per la Scuola Media riformata." In *Atti del I Congresso della Mathesis Società Italiana di Matematica, Firenze 16–23 Ottobre 1908*. Padua: Premiata Societŕ Cooperativa Tipografica, 26–33.

less able pupils to the *liceo moderno*, and by the absence of funds, which made it difficult to provide science laboratories.

6 Towards the predominance of Humanistic Culture

In those same years, the *Associazione Mathesis* also invoked a reform of the curricula of mathematics in the technical institutes, "curricula that are dated and defective, in terms both of the gaps that they present and of the plethora of arguments of scant educational and scientific value".[32] In particular, they proposed introducing, as was done in the *liceo moderno*, differential and integral calculus. They also asked that the mathematics curriculum of the physics-mathematics section be differentiated from that of other sections starting from the second year of the course. The *Mathesis* suggestions were in large part absorbed into the syllabi of the secondary schools developed in 1917. In the instructions there, it was underlined that the aim of the teaching of mathematics in the physics-mathematics section was "not only to provide the students with a valuable instrument for collateral studies, for higher studies, and for life, but also, and more importantly, to educate them to rigorous reasoning".[33] Further, teachers were invited to give importance to physical applications and not to tire the students with "worries of overwhelming rigour"; and they were also advised to introduce the concepts of limit, derivative and integral according to their historical development. These new curricula never became effective because of the particular historical period Italy was going through.

In autumn 1923, following the March on Rome, Mussolini became head of the government and the Fascist dictatorship began. Gentile, then minister for public instruction, taking advantage of the full powers given to him by the first Mussolini government, realized in one single year a complete and organic reform of the Italian scholastic system according to pedagogical and philosophical lines that he himself had developed from the early years of the 20$^{\text{th}}$ century. The decree relating to the secondary school was issued in May 1923[34], and the curricula and timetables were approved in October. Fascist principles and the ideologies of neo-idealism were opposed to a wide spread of scientific culture, and above all, to its interaction with other cultural sectors: the humanistic culture had to constitute the cultural axis of national life, and in particular, of education. This vision drastically conflicted with the scientific *humanitas* that mathematicians such as Cremona, Vailati, Castelnuovo had sought to introduce into Italian schools, and negated the formative role of mathematics. None of the protests by mathematicians were given a hearing.

[32] "Proposta di programmi di matematica per gli Istituti Tecnici", *Bollettino della Mathesis*, **VI**, 1914, 178–181.

[33] "Riforma dei programmi delle Scuole Medie", *Il Bollettino di matematica*, **XVI**, 1919, 84.

[34] *Orari e programmi per le regie scuole medie*, Bollettino Ufficiale del Ministero dell'istruzione pubblica, 17 Novembre 1923, 50, II, 4 413–4 510.