The History of Mathematics Education and Its Contexts in 20^{th} Century France and Germany

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Abstract

In this plenary session, some key moments in the development of the teaching of mathematics in two countries will be presented which decisively influenced the overall history in Europe: in France and Germany. Since the respective developments in France and in Germany mutually influenced one another, the presentation will be given jointly and in a dialogue mode.

Among these key moments in history, the period from 1902 to 1914 will highlight their interaction, since it not only comprises the beginning of international cooperation in mathematics education, but also decisive exchanges about goals and directions of reform and about the modernization of teaching mathematics. Another key moment will be the "modern math" movement. The mutual relation will in particular emphasize the imbedding of mathematics education, its contents and objectives into the cultural, economic and social contexts in these periods and countries.

1 Period of reforms and cooperation 1900–1914

Germany

The situation of mathematics education in the German states by 1900 was an evident outcome of its development during the 19th century. I should like to emphasize two of its major characteristics, which are essential for the further evolution of the history:

1. In all German states, the key structural problem of secondary schools had been how to opt for classical, literary studies — which would typically lead to university studies — on the one hand, and for preparing for "civil", not-learned professions and careers on the other hand. Separate school-types providing such more practical, or more "modern", teaching lasted less long than the *Gymnasien*, and they did not provide the *Abitur* — the university entrance degree. By the end of the 19th century, these — originally complementary — schools had been expanded in duration and qualifications offered. And in 1900, it were three different types of secondary schools which had been granted the right to held the *Abitur* exam, and thus to give access to higher education: the three types being defined by the kind of classical learning they provided:

- *humanistisches Gymnasium* — with Greek and Latin,
- *Realgymnasium, with Latin*,
- *Oberrealschule*, with none of these languages.

One has to know that mathematics constituted a major teaching subject in either of these three types, but according to different views of mathematics. And one has to know that this split which corresponded to different social strata in German society, persisted for a very long time — until 1972 at least.

Moreover, in a manner parallel to the differentiation of the secondary school of 1810 into three competing types, there had also been established two competing types of higher education: the universities and — as newcomers having the same academic status by the same year 1900. And there had been no free choice of one of these two types for the graduates of the three school types. Originally, the *Oberrealschulen* graduates were restricted to the technical colleges and a few disciplines of the university. Hence, there was the danger of a culturally segregated guidance cementing barriers between classicality and modernity, between the humanities and the sciences, between *Bildungsbürgertum* and the economically active social strata. And it was in particular mathematics which was affected by this split.

What was at stake for mathematics, hence, was a problem of transition from secondary schooling to higher education. The problem was all the more acute as the technical colleges, due to their origin as polytechnical schools, provided a large portion of basically elementary mathematics and which — while young mathematics professors formed in the spirit of the new Weierstrassian rigour in analysis used them for presenting rigorous foundations of mathematics — not only annoyed their students, but even provoked the emergence of an anti-mathematical movement among engineers.

The second pivotal feature of mathematics teaching in secondary schools was its outdated nature: despite the needs of the by now industrialized country for adapted modern education, mathematics instruction everywhere was dominated by just elementary teaching goals, focussing on classical, Euclidean geometry and enhancing as key function the formation of logical thinking. The teaching of variables was banned — as not being elementary in that sense — and therefore that of functions, too. Consequently, conic sections were to be taught only via the synthetic method, i.e. as geometrical loci, but not by means of the analytical method.

In 1891, an association of mathematics and science teachers of its own had been founded — the "Förderverein": association for the promotion of the teaching of mathematics and science. It did not initiate actions for modernizing teaching mathematics and changing the structural problems yet. As a matter of fact, it was a university mathematician who became active from 1900 on and who initiated reforms: Felix Klein. Actually, his original concern was the first issue, the transition from secondary to higher learning. But upon reflecting how the problem might be solved, he became aware of the fact that an enormously more extensive and more complicated problem had to be tackled: the second issue, the reform of the syllabi for the secondary schools.

A decisive support in order to realize such reforms came from France. Thanks to the good services of *L'Enseignement Mathématique*, the first international journal mathematics education, Klein learned of the 1902 reforms in France where elements of the calculus were introduced into the syllabus of the last grades. Such an introduction in Germany would resolve the problem of the curriculum at the technical colleges, but lest an alien, novel element be presented in the upper grades, it should be systematically prepared and appear to be just the logical closing of a consequently organized new syllabus.

Having familiarized himself with some of the main problems facing mathematics teachers in the schools, Klein proceeded to coin the key phrase that would hereinafter serve as the slogan for his reform programme. This was the famous notion of *functional reasoning*, or the idea that the function concept should pervade all parts of the mathematics curriculum. This slogan of functional reasoning in hand, Klein began in 1902 to gather support for this reform movement from below. He succeeded in forging an extraordinarily broad and powerful

alliance of teachers, scientists and engineers that was to advocate a series of reforms for mathematics and sciences curricula.

A committee established in 1904 in Breslau, reflecting in its composition this broad movement, the so-called *Breslauer Unterrichtskommission*, was able to present one year later, in 1905 at Meran, to the annual meeting of the association of German mathematicians, a profoundly revised syllabus, which presented a modernized course, based in fact on that idea of functional reasoning and ending with the elements of the calculus. This was the later so famous Meran programme. The Meran text contained but one shadow: due to the resistance of some functionaries, the calculus was recommended for both realist school types, but for the *humanistisches Gymnasium* it was just optional. For Klein's conception of free transition, it should apply likewise to the realist and to the classical school types — and, hence, contribute to overcoming, at least for mathematics, the split along contrasting views of culture or cultures.

In fact, at the basis, in the schools, mathematics teachers were enormously active towards realizing the programme of functional reasoning, including the elements of the calculus, at all three school types. And there was a "modern" textbook, published by two teachers from Göttingen, which corresponded well to Klein's programme: O. Behrendsen and E. Götting, *Lehrbuch der Mathematik nach modernen Grundsätzen* (Teubner, Leipzig).

Additional impetus for the reform movement in Germany came from outside: by the establishment of the first International Commission on Mathematics Instruction (IMUK/ CIEM), in 1908. Felix Klein was elected president and he used this position not only to extend international cooperation beyond the limits envisaged by the ICM, but also to complement the compilatory official task by a reformist agenda disseminating the ideas of curricular change. An important means for that were international trend reports on some key problems of mathematics teaching. The cooperation between France and Germany signified one of the essential reasons for the success of IMUK work until 1914, until the onset of World War I.

France

Our task, here, in this plenary *"à deux voix"*, is to try to show how mathematical education is dependant on the time and the place where and when it is given. As for time, it will not be difficult to show its link with the social and political context. For place, however, either for Germany or for France, the challenge will be a little more difficult concerning some of the periods we have defined. For this very first period, a common reflection and cooperation on reforms in mathematical curricula were developed all over Europe, and between our two countries. More general institutional educational features and mathematical issues at stake were also largely common. Nevertheless, I shall try to show how these common issues were specifically managed in the French context.

Regarding the situation during the 19^{th} century, the key structural problem was altogether identical and different from that of Germany. We have three different types of school, three different schoolings, referring to different social strata and to different status of mathematics. A first type, for the learned élite — even the scientific one — were the *lycées classiques*, which provided, first and foremost, classical and humanist education. Mathematics education was on the fringe of this secondary training, postponed to the very final year of the *lycée*. And that, even for the scientific élite, as I have said, who continues their studies in *Grandes écoles* like the *Ecole* polytechnique where mathematics was so essential.

The second and third types of school trained technological and industrial staff. So, both of them, *Ecoles primaires supérieures*, for lower classes, and *modern secondary colleges* for upper classes, gave a key role to mathematics and to science education which were taught according to practical aims, and did focus on applications.

This dichotomy in the goals of education, and this monopoly of classical humanities in the *lycées* became more and more untenable for the economic and political élites of the

Third Republic. In 1899, the French parliament initiated a comprehensive inquiry all over the country to discuss *the* educational question of this time: Which training, for which elite in a modern country? What modernity, what humanities does the country need?

As for mathematics, and science, different positions were maintained, and sometimes complementary values were argued for: cultural values as part of new modern humanities — the "scientific humanities", together with other sciences, living languages and French modern literature — versus utilitarian values, mathematics seen as an applied and practical subject, its applications being another part of the modernity.

As a consequence of this enquiry, a deep reorganisation of the structures and of the contents of secondary instruction was undertaken in 1902, taking into account new goal and new audiences.[1]

The 1902 reform had a considerable impact:

- the unification, in a unique secondary level structure, of the modern and classical secondary cursus, considered - at least in principle, if not symbolically — as equal;

- the establishment of two stages in the curriculum: a first corresponding to the first four forms of the *lycée* for boys from 12 to 15 years, after which students might leave secondary instruction; and that prospect is absolutely new; a second stage corresponding to the last three forms which ended with the *baccalauréat*;

- the end of the monopoly of classical humanities and the development of "modern" subjects as languages, science and mathematics.

Regarding the structure of mathematical curricula, we have to note several convergent factors: firstly, the growing place of mathematical education, in particular of geometry, in the first forms of the *lycée*; secondly, the effects of the diversification of the goals of secondary education; and lastly, the effects of a third factor, from the outside of the educational world, the new conceptions mathematicians had then about geometry. All these factors led to new contents and new methods for all the mathematical curricula.

Regarding geometry teaching, it was emphasized, for example by the syllabus in 1905, that it should "be essentially concrete". Even more innovative were the introduction of the concepts of function, of continuity, derivative, graphical representation, and links to physics and to applications, since the beginning of the second stage

A quote by Emile Borel, given in a pedagogical conference for mathematics teachers still in 1904, is characteristic for the mathematical trends at stake in this 1902 reform:

> "We have to introduce more life and more sense of reality in our mathematics education,"
>
> "That is the only way to prevent that mathematics be one day suppressed because of budgetary economy."
>
> "Don't we risk diminishing this great educative value [of secondary instruction] when making mathematics education more practical and less theoretical?"[2]

[1] See the speech given by G. Leygues, minister of Public Instruction, in: Georges Leygues: Séance des débats à la Chambre, 12 et 14 février 1902, *Le Journal Officiel*, 666.

[2] Emile Borel, "Les exercices pratiques de mathématiques dans l'enseignement secondaire", *Revue générale des sciences pures et appliquées* 14 (1904), 431–440.

2 Between the two World Wars. Germany 1920–1933/France 1920–1940

Germany 1920–1933

After the end of World War I, the entire political, social and economic situation had changed. For Germany, France now was an enemy, even the "hereditary enemy" **("Erbfeind")**. German scientists were being internationally isolated and boycotted. And the precondition for Klein's activities to have mathematics acknowledged as a key moment of culture was no longer fulfilled: due to the horrors of the War, the sciences had lost their legitimacy to a considerable extent and had to act from a defensive position. There was a cultural crisis of mathematics and the sciences. Subjects now valued in the school context were of a quite different, nationalist character: "Kulturkundliche" subjects, i.e., German language and literature, geography, and history were favoured, at the disadvantage of mathematics and the sciences. The weekly hours for these subjects were reduced in all types of secondary schools. A fourth, new type of secondary school now established characterizes the political trend: *Deutsche Oberschule* — German secondary school.

A few positive instances can be named, however. Firstly, the applications of mathematics were more valued in the syllabi and had to be taught more extensively. And secondly, the new Prussian syllabus of 1925 enacted now officially what had for a long time been practiced by mathematics teachers: the Klein programme with the elements of the calculus in all types of secondary schools.

And there were profound changes in the school system — thanks to the Revolution of 1918 — the only true revolution in German history: the social separation between a primary school system for the lower classes and a system of secondary schools with separate preparatory schools was abolished, and replaced by an obligatory consecutive system where children from all social classes had to attend the primary schools. And the formation of teachers for these new primary schools became attributed to institutions belonging to higher education: the Pedagogical Academies, admitting only students provided with an *Abitur*. The professorships established there for the methodology of teaching reckoning and geometry constitute the origin of didactics of mathematics in Germany.

The restructuration of the school system was accompanied by a reform of pedagogical methods with a deep impact, best exemplified by the method of so-called *Arbeitsunterricht*: i.e., replacing old formalist teaching addressing only memory and the head by active methods, claiming proper activities by the students themselves, and emphasizing in fact manual occupations.

In a number of textbooks, one finds, in the Weimar period, examples of nationalistic contents in exercises given to the students.

3 Fascism — World WAR II

Germany 1933–1945

It is remarkable and characteristic that these nationalist overtones were directly transformed in Nazi times into militaristic, anti-Semitic and eugenic indoctrination.

Immediately after the seizure of power by the Nazi Party, the two organizations for mathematics teaching — the *Förderverein* and the *Reichsverband deutscher mathematischer Gesellschaften* — decided themselves their "Gleichschaltung", i.e. adoption of key principles of the Nazi system:

- replacement of elections for the presidency by the "Führerprinzip",
- change of their statutes by adopting the so-called Aryan paragraph (i.e. excluding so-called Jews from membership).

And it was this *Reichsverband* who had decided to compose a handbook for mathematics teachers which was to help or guide them to accommodate their teaching to the Nazi system:

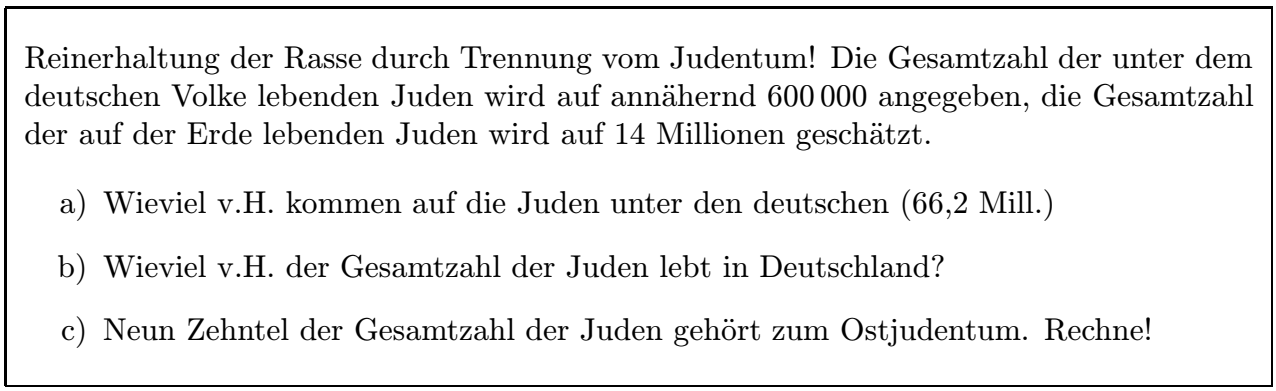
Adolf Dorner (ed.), *Mathematik im Dienste nationalpolitischer Erziehung mit Anwendungsbeispielen aus Volkswissenschaft, Geländekunde und Naturwissenschaften.*

The handbook, published in 1935, was recommended for use in schools by the ministries and reedited several times.

It contained a collection of ideologizing, indoctrinating and discriminating exercises. It did not meet refusal. How was the rapid adaptation possible? On the one hand, the methodology of *Arbeitsunterricht* allowed for number of textbook writers and didacticians to be already near to romantic and irrationalist tendencies so that they would easily become adherents of "Blut und Boden" ideology. On the other hand, it is clear that no instance in the state or in the National Socialist Party had given orders to write textbooks in this sense. What one observes can be characterized as — according to the terms used in history of science — **Self-Mobilization**. Instead to await orders or the elaboration of a respective policy, the functionaries and activists in the respective field — here: the textbook writers — engage themselves in elaborating a policy in their field which they judge to please the *Führer* and to contribute to Nazi policy.

The schoolbooks even for primary grades are full of examples of such self-mobilization: the illustrations are featuring militarist context for playing youngsters; exercises for multiplication are visualized by showing SA troops marching — in groups of four, six, etc.

Already the title pages serve as indoctrination for air battle war (see fig. 1). Most horrifying is how word problems on percentage calculations was used for propagating elimination of Jews:



Reinerhaltung der Rasse durch Trennung vom Judentum! Die Gesamtzahl der unter dem deutschen Volke lebenden Juden wird auf annähernd 600 000 angegeben, die Gesamtzahl der auf der Erde lebenden Juden wird auf 14 Millionen geschätzt.

a) Wieviel v.H. kommen auf die Juden unter den deutschen (66,2 Mill.)

b) Wieviel v.H. der Gesamtzahl der Juden lebt in Deutschland?

c) Neun Zehntel der Gesamtzahl der Juden gehört zum Ostjudentum. Rechne!

Figure 1 – Büttners Rechenbuch. Ergänzungen. Ausg. E, Heft 4

World War II was led by the German state in particular again against France. Germany's goals included not only occupation of large parts of the French territory, but also collaboration of the Vichy Regime allegedly governing the remaining territory.

France 1920–1940

Here, political events dramatically influence the subdivision into periods for the two countries. The convenient period of study for France will be the two decades between the World Wars.

The years, just after the war, were marked by a strong nationalism, a manifest consequence of World War I. French politicians, French elites, and among them some mathematicians, desired then to promote classical humanities, a tradition which they called to be specific to "Latin" nations as France, as opposed to German practical culture. In that period, the 1902 reform was accused of having greatly weakened classical humanities — the

pretended French identity — by imitating German approaches. And, in 1923, a conservative parliament voted a new reform.

This reform excluded modern secondary instruction from the *lycée* — Latin became again compulsory in the first grades; it cancelled the organisation in two stages; prescribed the monopoly of classical humanities values — the only goal of the *lycée* being to educate minds and hearts of an intellectual and social elite; and lastly imposed the *"égalité scientifique"*, that is imposed the same curricula in science and mathematics for all students till the very last grade. The consequences were that there was less instruction in science and in mathematics than after 1902, and that treating nearly all mathematical notions was postponed to the last scientific grade (as before 1902).

This reign of *"égalité scientifique"* and classical humanities as *the* model for the training of the elite persisted during the entire inter-war period, even when the compulsory Latin was abolished as early as 1925 (by a left-majority parliament) and modern secondary instruction reinserted into the *lycées*. Thus, the predominance of humanities and the reduction of mathematics and science teaching was maintained even under the *Front populaire* (1936) when a soft re-organisation of the *"enseignement moyen"* (for children from 12 to 15 years old) was undertaken. These features remained the distinctive sign of the specificity of this secondary instruction — cultural, liberal and disinterested — which excluded all practical and concrete aims.

We should mention, since it became important in the aftermath, the growing success of the altenative model proposed by the primary structures in charge of a part of the *"enseignement moyen"* which attributed great importance to science and mathematics, and to their applications, in their curricula.

France and the régime de Vichy (1940–1944)

If nothing very specific took place for mathematics education during these years, this period is nevertheless important, since the Vichy *régime* took some structural measures, which affected the evolution of the French educational system even after the war.

For political reasons, Vichy tried to destroy the very independent, homogeneous and strong world of the "primary schooling" (that is primary school, higher primary school and *"école normale"*: the institutes for future primary teacher training, for students of an age of 15 to 18), a "primary world", which was very much attached to republican ideas and against the collaboration of the Vichy regime with the Nazi occupation authorities. Firstly, Vichy abolished the "primaire supérieur" (higher primary level) in order to integrate it into secondary level instruction, creating the "modern college", less valuable than the *lycée* and where, once again, Latin became compulsory. Secondly, Vichy abolished the teacher training institutes, since future teachers had now to attend the *collèges modernes*.

4 After World War II, 1945–about 1965

France: Economical stakes in society

The period after World War II was characterized by an enormous increase of the importance of mathematics and the role of mathematicians in contemporary time. This was documented, as G. Kurepa put it, not only by the now high number of mathematicians, including applied mathematicians being engineers, but foremost by the unprecedented fact of mathematical laboratories being established in big industrial and commercial enterprises. The fundamental new achievements of mathematics in fields as diverse as structures, logic, optimisation, calculators and numerical analysis, statistics, computer science, caused him to assert a key role of mathematics in the industrialized society.[3]

[3]See Kurepa's report for the ICME study: Georg Kurepa: "Le rôle des mathématiques et du mathématicien à l'époque contemporaine. Rapport général", *L'Enseignement mathématique* (2), 1 (1955), 93–111.

The consequence that reforms of mathematics education were needed to meet these new demands were shared not only by mathematicians, but in particular by agencies for economic development. In fact, the initiatives undertaken by the OEEC (organisation for European economic cooperation) later renamed as OECD, since the late 1950s and the beginning of the 60s, were to become the motor for the second international movement of curricular reforms. In1958, the OEEC opened an office in Paris in order to "make more efficient science and mathematics education" and to promote a reform of the contents and the methods of mathematics instruction for 12 to 19 years old students. The expert meetings organized by OEEC/OECD initiated the "new math" movement: in 1959 in Royaumont, in 1960 in Dubrovnik, in 1963 in Athens.

A very specific epistemological context

The new math movement was nurtured in particular by an epistemological context, which was specific for France. It was the impact of the mathematical achievements of Bourbaki, the innovative group of essentially French mathematicians, who familiarized the new central role of the notion of structure in mathematics, which should become the core of what was called "new math".

Mentioning the French anthropologist Claude Levi-Strauss will remind of the huge importance of structuralism, which constituted the philosophical trend dominating in France at that time in all sciences - including human and social sciences. "New math" and its structure were generally understood as the essential scientific tool and language to access any knowledge.

In the field of education, one of the consequences was the convergence between mathematicians in the current of Bourbaki, and psychologists and philosophers like Piaget and Gonseth. Meetings were organised from the beginning of the 1950s by a newly created international organisation, the *CIEAEM* where French mathematicians played an important role.

French mathematicians and French mathematic teachers were quite mobilised, individually and collectively in their association, *APMEP*, since the beginnings of 1950s, to think, experiment and promote a reform of contents and methodology in mathematics education. More, quite a lot of French mathematicians were requested as experts in the OEEC and OECD meetings.

Institutional educational context

Two important institutional reforms took place in these years in France, establishing for all children from 12 to 16 compulsory instruction in a more or less complicated system of various "middle schools" belonging all to secondary instruction. That meant two essential things: firstly, primary instruction became for all children the first stage of an extended school attendance in secondary system; this stage can be considered to present the dynamic of the necessary math reform; secondly, "middle school" had then new aims and new publics which differed from precedent periods, providing education to children whose educational and social future was as different as long and general studies, practical studies or apprenticeship.

At the same time, the baby boom which followed the years of war, provoked an enormous growth of the number of students in this secondary level and, related to that, a decisive lack of qualified mathematics teachers. The term of "qualified", however, is too much ambiguous and appeals to different dimensions of the situation: it either meant that teachers were former upper-primary teachers, trained in a "primary tradition", or it meant undergraduate mathematics teachers.

These institutional reforms were understood either as a factor of democratisation of the educational system or as a factor of its *"massification"*, that is of quantitative growth without any strong qualitative social change.

The essential event for this 1945–1965 period in France was the creation, in December 1966, of a *Commission ministérielle d'étude pour l'enseignement des mathématiques*, whose president became André Lichnérowicz.

Germany

While France constituted, as we have just seen, the centre for the elaboration of key mathematical and didactical conceptions for what was to become "modern mathematics", Germany lagged behind and played no active role during this period.

Concerning the East German Democratic Republic, I should just mention that primary and secondary schooling there constituted a consecutive and unitary system and that mathematics and the sciences occupied there a highly valued position. I have to concentrate on Western Germany, the FRG, however.

After World War II, a conservative stabilization was effected by a return to the pre-Nazi period; in particular, the segregated school structure was reinforced. Ideologically, a a backward-oriented conservatism ruled and emphasized the values of an allegedly "Christian West", thus establishing a cultural distance to the barbaric East. In fact, this ideological orientation expressed militant anti-Communism. On the other hand, this policy intended to integrate the FRG into Western Europe, and therefore not only to the first structures of integrated European institutions began to emerge, but also the Franco-German Youth Exchange Program. Paradoxically, Anti-Sovietism, thus, helped to overcome the traditional hate of the French arch-enemy, and to enhance a new friendship between the two nations.

The conservatism of West-German society directly affected the teaching of mathematics and the sciences. This is illustrated by a fact unique for the Western countries. In 1960, while other Western countries had already been profoundly affected by the Sputnik-shock and had reinforced mathematics and sciences teaching, and while the OECE was strongly active in modernizing mathematics teaching, the KMK — *Kultusministerkonferenz*, the body of the federal education ministers — decided to reduce the weekly hours for mathematics and the sciences in the secondary schools, in favour of the humanities, convinced to thus be able to save the *Abendland*, the West, — the so-called *Saarbrücker Rahmenvereinbarung*.

One will not be surprised to hear that in such a conservative situation the separated education of boys and girls in secondary schools was maintained, but you might be astonished to see that there were separate mathematics schoolbooks for girls in the 1950s and 1960s: *Mathematik für Mittelschulen. Für Mädchen. Geometrie und Stereometrie* — Verlag Ernst Klett Stuttgart.

Regarding curricular change, there was nothing comparable to France. Only a few, relatively isolated discussions were led, since 1955, and these concerned exclusively the *Gymnasium*. One of the exponents of this group was Hermann Athen, director of a *Gymnasium* and an influential schoolbook writer. When the group presented, in 1965, its proposals for a rather moderate reform within the Gymnasium, to the *Förderverein* annual meeting, it met flat refusal by the mathematics teachers. Regarding the primary schools, there were no reform discussions at all: neither among the teachers, nor among the teacher educators, at the Pedagogical Academies.

When external agencies like the OECE began to look for supporters for curricular changes, they met difficulties in finding active and willing personalities. In 1959, for the decisive first international meeting, at Royaumont, the OECE — which had looked for two to three representatives from each of its member countries – had invited that Hermann Athen and Heinz Schoene, a functionary of the education ministry of Rheinland-Pfalz who was later to become one of the most active personalities among the German *Länder* governments.

For the next international meeting, at Dubrovnik in 1960, there were no active German promoters of reform: the famous mathematician Emil Artin (Hamburg), and two today unknown persons: O. Botsch and B. Schöneberg.

Also in 1963, at the important international conference in Athens, with a great number of participants, there were only two Germans: still Hermann Athen and now Hans-Georg Steiner who was later to become so important for the national and international development of mathematics education.

5 "Modern mathematics" — ca. 1965 to ca. 1985

Germany

Due to the refusal of an internal reform in 1965 by the teachers, at least for parts of the Gymnasium, all reform initiatives came to be imported from abroad. Thus, the decisive document became a text voted in 1968 from above, by the KMK, decreeing a profound reform, which was to be be enacted from 1972 on. For the first time, primary and secondary education were seen as a unity, subject to a common curriculum developing the key thematic issues of mathematics. These issues, organized in thematic areas, should range from sets, magnitudes, positional systems, to congruences, real numbers and trigonometry — hence less revolutionary than accused later.

These reform decisions fell on teachers and educators entirely unprepared. There existed didacticians (teacher educators) for primary teaching, but they had in no way been involved in the preparations by the KMK. And for the secondary domain, there barely existed didacticians but just practitioners of teacher training. The execution of the reform decision was thus taken over by the textbook industry, which produced quickly numerous, but poor textbooks for school which grossly exaggerated the importance of the set language.

Soon, public resistance became organized concentrating on the alleged set theoretical nonsense. The public uproar led in 1975 to a backlash in which the syllabi were replaced by new ones free of sets. This was then understood as a return to basics. In the long run, this was not confirmed. Rather, the main effect of a common curricular structure of school mathematics developing the fundamental concepts of mathematics was maintained.

And a consensus emerged in all syllabi of the federal states stating a few conceptual fields as constituting school mathematics, like, say:

- number
- figure and form,
- magnitudes,
- functions,
- data.

The growing consensus was also due to the eventual constitution of a discipline *Didaktik der Mathematik* common to all school grades, enhanced by the international work of the IDM at Bielefeld, founded in 1973, and a growing international cooperation in mathematics education.

France

This last period was in France the time of the official reform led by the ministerial committee. The reform was first desired and supported nearly unanimously in France. The agenda of the committee was clear. Firstly, it had to work on new options for primary and secondary curricula, making them experimented and tested. Secondly, it had also to work on in-service training for teachers and on the establishment of new institutes devoted to it — later named the IREM.

I cannot discuss here the mathematical characteristics of these new curricula. I should just mention the importance given to modern algebra and set theoretical concepts in the whole curriculum, from elementary level to baccalaureat, to classical Euclidean geometry and to classical calculus. I should like, however, to stress two key points of the reform, which turned out to be two major difficulties. The first was that this reform had to be for all students whatever their future, at school or in society. The second was that the reform had to comprise the entire range from pre-elementary school to university.

Two quotations illustrate those difficulties. The first shows the consequence of the reform for primary level curricula whose goal aim was no longer to prepare children for vocational or everyday life.

> This teaching being only a prelude to various middle school teachings, we have to make lighter knowledge that is required today, in particular concerning practical applications, and to privilege instead a better comprehension of basic notions and a better learning of mathematics techniques.

The second quotation shows the objective difficulty and, all together, goodwill, and inability and unpreparedness of the Lichnerowicz committee to deal with the "democratisation" issue and to think of anything but secondary-long training necessary followed by universities studies. Evoking, in a meeting of the *commission*, the question of the curricula reform for the "filières courtes", this part of middle school which trains to vocational life, one of the member resumed the matter, saying: "Do we have to teach obsolete mathematics to less clever children?".

Because of the coincidence of *massification* reforms and new mathematics reform, it was the first time that identical mathematical curricula for the middle school had to be thought of all together for pupils entering vocational life and for pupils continuing with higher studies. And, ideas on democratisation of education, inherited from the inter-war period, supposed, as an evidence which was not even disputed that the model for the elite was the best and had to be adhered to for the education of all. And thus it was also for mathematics, the mathematical and pedagogical traditions of the primary system being cancelled for the benefit of upper school ones.

At the beginning of the 1970s, dissension among the *commission* exploded, and the unanimity of the beginning collapsed. First, some of the mathematicians and some physicist inside the committee, then outside, criticised the formal and abstract dominating side of the mathematical programs. It was not fit for the greater part of teachers and pupils, too poorly prepared for it. It was fit, they said, neither for the training of future physicists or scientific researchers, nor for that of future engineers. These criticisms came just as heatedly from the mathematics education community, like APMEP or even the IREM, from the academic community, and from professional societies or the *Académie des sciences* itself, and from economic and industrial circles.

The story ends quite sadly. At first, the *commission's* work ceased, since in June 1973 Lichnérowicz resigned, and the *commission* never carried through the second stage of the reform. Then, the entire reform was abandoned in the 1980s, disputed even by its supporters who thought that it did not really correspond to their recommendations.

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