

LEWIS CARROLL IN NUMBERLAND

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Abstract

If Charles Dodgson (Lewis Carroll) had not written the Alice books, Alice's Adventures in Wonderland and Through the Looking-Glass, he would be remembered as a pioneering photographer, one of the first to consider photography as an art form, rather than simply as a means for recording images. If he had not been a photographer, he might be remembered as a mathematician, the career he held for many years at Christ Church, Oxford University. But what mathematics did he do? How good a mathematician was he? How influential was his work?

1 INTRODUCTION

Letter to my child-friend, Margaret Cunnyngname, Christ Church, Oxford, 30 January 1868:

Dear Maggie,

No carte has yet been done of me
that does real justice to my smile;
and so I hardly like, you see,
to send you one. Meanwhile,
I send you a little thing
to give you an idea of what I look like when I'm lecturing.
The merest sketch, you will allow —
yet I still think there's something grand
in the expression of the brow
and in the action of the hand.

Your affectionate friend, C. L. Dodgson

P.S. My best love to yourself, to your Mother
my kindest regards, to your small,
fat, impertinent, ignorant brother
my hatred. I think that is all.

This letter to Margaret Cunnyngheame shows up two aspects of Lewis Carroll — his love of children and the fact that he was a teacher — in fact, a teacher of mathematics.

If he hadn't written the Alice books, he'd be mainly remembered as a pioneer Victorian photographer. And if he hadn't been known for that, he might have been largely forgotten — as an Oxford mathematician who seems not to have contributed very much. But is that really the case?

Certainly, mathematical ideas pervaded his life and works — even the Alice books. For example, in the Mock Turtle scene in *Alice's Adventures in Wonderland*, the Mock Turtle begins:

When we were little, we went to school in the sea. The master was an old turtle — we used to call him Tortoise.

Why did you call him Tortoise, if he wasn't one?

We called him Tortoise because he taught us. I only took the regular course. Reeling and Writhing, of course, to begin with; and then the different branches of Arithmetic — Ambition, Distraction, Uglification and Derision.

And how many hours a day did you do lessons?

Ten hours the first day, nine hours the next, and so on.

What a curious plan!

That's the reason they're called lessons — because they lessen from day to day.

And in *Through the Looking-Glass*, the White Queen and the Red Queen set Alice a test to see whether she should become a Queen.

Can you do Addition? What's one and one and one and one and one and one and one and one and one and one?

I don't know. I lost count.

She can't do Addition. Can you do Subtraction? Take nine from eight.

Nine from eight I ca'n't, you know: but —

She ca'n't do Subtraction. Can you do Division? Divide a loaf by a knife. What's the answer to that?

Bread-and-butter, of course.

She can't do sums a bit!

Another character who couldn't do sums was Humpty Dumpty. Alice was admiring his cravat:

It's a present from the White King and Queen. They gave it me — for an un-birthday present.

What's an un-birthday present?

A present given when it isn't your birthday, of course.

I like birthday presents best.

You don't know what you're talking about! How many days are there in a year?

Three hundred and sixty-five.

And how many birthdays have you?

One.

And if you take one from three hundred and sixty-five, what remains?

Three hundred and sixty-four, of course.

I'd rather see that done on paper.

Three hundred and sixty-five . . . minus one . . . is three hundred and sixty-four.

That seems to be done right — though I haven't time to look over it thoroughly right now.

Mathematical ideas also appear in his other children's books. In *The Hunting of the Snark*, the Butcher tries to convince the Beaver that 2 plus 1 is 3:

Two added to one — if that could be done,

It said, with one's fingers and thumbs!

Recollecting with tears how, in earlier years,

It had taken no pains with its sums.

Taking Three as the subject to reason about —

A convenient number to state —

We add Seven, and Ten, and then multiply out

By One Thousand diminished by Eight.

The result we proceed to divide, as you see,

By Nine Hundred and Ninety and Two:

Then subtract Seventeen, and the answer must be

Exactly and perfectly true.

And in his last major novel, *Sylvie and Bruno Concluded*, Dodgson's ability to illustrate mathematical ideas in a painless and picturesque way is used in the construction of *Fortunatus's purse* from three handkerchiefs. This purse has the form of a projective plane, with no inside or outside, and so contains all the fortune of the world. Since it cannot exist in three dimensions, he ceases just before the task becomes impossible.

Another quirky result concerned map-making.

There's another thing we've learned from your Nation — map-making. But we've carried it much further than you. What do you consider the largest map that would be really useful?

About six inches to the mile.

Only six inches! We very soon got to six yards to the mile. Then we tried a hundred yards to the mile. And then came the grandest idea of all! We actually made a map of the country, on the scale of a mile to the mile!

It has never been spread out, yet. The farmers objected: they said it would cover the whole country and shut out the sunlight! So we now use the country itself, as its own map, and I assure you it does nearly as well.

And talking of maps, here's a map-colouring game for two people that Mr Dodgson used to play with his young child-friends, related to the celebrated four-colour problem.

A is to draw a fictitious map divided into counties.

B is to colour it — or rather mark the counties with names of colours — using as few colours as possible.

Two adjacent counties must have different colours.

A's object is to force B to use as many colours as possible. How many can he force B to use?

Dodgson was an inveterate letter-writer: in the last 35 years of his life, he sent and received many thousands of letters, cataloguing them all. Although many letters were to his brothers and sisters or to distinguished figures of the time, the most interesting ones were to his child-friends, often containing poems, puzzles, and word-games. He had a deep understanding of their minds and an appreciation of their interests, qualities that stemmed from his own happy childhood experiences.

Many of his friendships were with young girls, such as with the Liddell children Alice, Edith, and Lorina. Indeed, he once wrote in joke:

I am fond of children (except boys).

In spite of that, here is a rare letter to a young lad of fourteen, Wilton Rix:

Honoured Sir,

Understanding you to be a distinguished algebraist (that is, distinguished from other algebraists by different face, different height, etc.), I beg to submit to you a difficulty which distresses me much.

If x and y are each equal to 1, it is plain that

$$2 \times (x^2 - y^2) = 0, \text{ and also that } 5 \times (x - y) = 0.$$

$$\text{Hence } 2 \times (x^2 - y^2) = 5 \times (x - y).$$

Now divide each side of this equation by $(x - y)$.

$$\text{Then } 2 \times (x + y) = 5.$$

$$\text{But } (x + y) = (1 + 1), \text{ i.e. } = 2. \text{ So that } 2 \times 2 = 5.$$

Ever since this painful fact has been forced upon me, I have not slept more than 8 hours a night, and have not been able to eat more than 3 meals a day.

I trust you will pity me and will kindly explain the difficulty to

Your obliged, Lewis Carroll.

His pen-name Lewis Carroll derived from his real name — Carroll (or Carolus) is the Latin for Charles, and Lewis is a form of Lutwidge, his middle name and mother's maiden name. He used it when writing for children, and in particular for his *Alice* books.

He was also keen on word games, and on constructing poems with hidden messages. Here's the best-known of these, with the letters of Alice's full name (Alice Pleasance Liddell) at the beginning of the lines.

A boat, beneath a sunny sky
 Lingering onward dreamily
 In an evening of July —
 Children three that nestle near,
 Eager eye and willing ear,
 Pleased a simple tale to hear —
 Long has faded that sunny sky,
 Echoes fade and memories die;
 Autumn frosts have slain July.
 Still she haunts me, phantomwise,
 Alice moving under skies
 Never seen by waking eyes.

Children yet, the tale to hear,
 Eager eye and willing ear,
 Lovingly shall nestle near.
 In a Wonderland they lie,
 Dreaming as the days go by,
 Dreaming as the summers die:
 Ever drifting down the stream —
 Lingering in the golden gleam —
 Life, what is it but a dream?

2 EARLY LIFE

Charles Dodgson was born in 1832 into a ‘good English church family’ in Daresbury in Cheshire, where his father, the Reverend Charles Dodgson, was the incumbent. In 1843 they all moved to Croft Rectory in Yorkshire, where he and his seven sisters and three brothers enjoyed a very happy childhood. When he was 14 he was sent to Rugby School, where he delighted in mathematics and the classics, but was never happy with the rough-and-tumble.

In 1850 he was accepted at Oxford, and went up in January 1851 to Christ Church, the largest college, where he was to spend the rest of his life. His University course consisted mainly of mathematics and the classics, and involved three main examinations. In his second year he gained a 1st class in Mathematics.

Whether I shall add to this any honours at Collections I cannot at present say, but I should think it very unlikely, as I have only today to get up the work in The Acts of the Apostles, 2 Greek Plays, and the Satires of Horace and I feel myself almost totally unable to read at all: I am beginning to suffer from the reaction of reading for Moderations.

I am getting quite tired of being congratulated on various subjects: there seems to be no end of it. If I had shot the Dean, I could hardly have had more said about it.

In the Summer of 1854, shortly before his Mathematics Finals examinations he went on a reading party to Yorkshire with the Professor of Natural Philosophy, Bartholomew Price — everyone called him ‘Bat’ Price, because his lectures were way above the audience. He was immortalized later in the Hatter’s song:

Twinkle, twinkle, little bat,
 How I wonder what you’re at...
 Up above the world you fly,
 Like a tea-tray in the sky,

Dodgson’s Finals examinations took place in December 1854, and ranged over all areas of mathematics. Here’s a question from an examination paper of that year:

Compare the advantages of a decimal and of a duodecimal system of notation in reference to (1) commerce; (2) pure arithmetic; and shew by duodecimals that the area of a room whose length is 29 feet 7 $\frac{1}{2}$ inches, and breadth is 33 feet 9 $\frac{1}{4}$ inches, is 704 feet 30 $\frac{3}{8}$ inches.

He was very successful in his Finals examinations, obtaining the top mathematical First Class degree in his year.

I must also add (this is a very boastful letter) that I ought to get the Senior Scholarship next term. One thing more I will add, I find I am the next 1st class Math. student to Faussett so that I stand next for the Lectureship.

3 A LECTURER IN OXFORD

Dodgson did not get the University's Senior Scholarship, but he was appointed to the Mathematics Lectureship at Christ Church. He became the College's Sub-librarian and many years later Curator of the Senior Common Room, moving into a sumptuous suite of rooms for which the eminent artist William De Morgan, son of the mathematician Augustus De Morgan, had designed the tiles around his fireplace.

In his early years as a lecturer at Christ Church, Dodgson took up the hobby of photography, using the new wet collodion process. He was one of the first to regard photography as an art, rather than just a means of recording images, and if he were not known for his Alice books, he'd be primarily remembered as a pioneering photographer who took thousands of fine pictures. The Liddell daughters used to love spending the afternoon with Dodgson, watching him mix his chemicals, dressing up in costumes, and posing quite still for many seconds until the picture was done. One picture is of Alice, dressed as a beggar girl; Alfred Tennyson described it as the most beautiful photograph he had ever seen.

From Hiawatha's photographing:

From his shoulder Hiawatha

Took the camera of rosewood

Made of sliding, folding rosewood;

Neatly put it all together.

In its case it lay compactly,

Folded into nearly nothing;

But he opened out the hinges,

Pushed and pulled the joints and hinges,

Till it looked all squares and oblongs,

Like a complicated figure

In the Second Book of Euclid. . .

4 GEOMETRY

And talking of Euclid brings us to Dodgson's enthusiasm for the writings of this great Greek author. Influenced by him, Dodgson produced for his pupils a *Syllabus of Plane Algebraic Geometry*, described as the 'algebraic analogue' of Euclid's pure geometry, and systematically arranged with formal definitions, postulates and axioms. A few years later he gave an algebraic treatment of the *Fifth Book of Euclid*, recasting the propositions in algebraic notation.

But in geometry he's best known for his celebrated book *Euclid and his Modern Rivals*, which appeared in 1879. Some years earlier, the Association for the Improvement of Geometrical Teaching had been formed, with the express purpose of replacing Euclid in schools by newly devised geometry books. Dodgson was bitterly opposed to these aims and

his book, dedicated to the memory of Euclid, is a detailed attempt to compare Euclid's *Elements*, favourably in each case, with the geometry texts of Legendre, J. M. Wilson, Benjamin Peirce, and others of the time.

It is written as a drama in four acts, with four characters: Minos and Radamanthus (two of the judges in Hades), Herr Niemand (the phantasm of a German professor), and Euclid himself. After demolishing each rival book in turn, Euclid approaches Minos to compare notes.

Dodgson's love of geometry surfaced in other places, too. His *Dynamics of a Particle* was a witty pamphlet concerning the parliamentary election for the Oxford University seat. Dodgson started with his definitions, parodying those of Euclid's *Elements*, Book I:

Plain anger is the inclination of two voters to one another, who meet together, but whose views are not in the same direction.

When a proctor, meeting another proctor, makes the votes on one side equal to those on the other, the feeling entertained by each side is called right anger.

Obtuse anger is that which is greater than right anger.

He then introduced his postulates, again based on those of Euclid:

1. Let it be granted, that a speaker may digress from any one point to any other point.
2. That a finite argument (that is, one finished and disposed with) may be produced to any extent in subsequent debates.
3. That a controversy may be raised about any question, and at any distance from that question.

And so he continued for several pages, leading to the following geometrical construction. Here, WEG represents the sitting candidate William Ewart Gladstone (too liberal for Dodgson), GH is Gathorne-Hardy (Dodgson's preferred choice), and WH is William Heathcote, the third candidate.

Let UNIV be a large circle, and take a triangle, two of whose sides WEG and WH are in contact with the circle, while GH, the base, is not in contact with it. It is required to destroy the contact of WEG and to bring GH into contact instead. . . .
When this is effected, it will be found most convenient to project WEG to infinity.

5 ALGEBRA

In 1865, Dodgson wrote his only algebra book, *An Elementary Treatise on Determinants, with their Application to Simultaneous Linear Equations and Algebraical Geometry*. In later years the story went around, which Dodgson firmly denied, that Queen Victoria had been utterly charmed by *Alice's Adventures in Wonderland*:

Send me the next book Mr Carroll produces —
— the next book being the one on determinants —
We are not amused.

Unfortunately, Dodgson's book didn't catch on, because of his cumbersome terminology and notation, but it did contain the first appearance in print of a well-known result (sometimes called the 'Kronecker-Capelli theorem') involving the solutions of simultaneous linear equations. It also included a new method of his (the 'condensation method') for evaluating large determinants in terms of small ones, a method that Bat Price presented on his behalf to the Royal Society of London, who subsequently published it in their *Proceedings*.

6 PUZZLES AND PARADOXES

Let's now turn to something a little more light-hearted. During his early years as a lecturer Dodgson started to teach a class of young children at the school across the road. He varied his lessons with stories and puzzles, and he may have been the first to use recreational mathematics as a vehicle for teaching mathematical ideas.

He enjoyed showing puzzles to his young child-friends. One that he enjoyed and may have invented was based on the well-known 1089 puzzle, but involving pounds, shillings and pence — note that there are twelve pence in a shilling and twenty shillings in a pound.

Put down any number of pounds not more than twelve, any number of shillings under twenty, and any number of pence under twelve. Under the pounds put the number of pence, under the shillings the number of shillings, and under the pence the number of pounds, thus reversing the line. Subtract — reverse the line again — then add. Answer, £12 18s. 11d., whatever numbers may have been selected.

Another problem, hotly debated in Carroll's day, was the Monkey on a Rope puzzle.

A rope goes over a pulley — on one side is a monkey, and on the other is an equal weight. The monkey starts to climb the rope — what happens to the weight?

Some contemporaries thought that it went *up*, while others said that it went *down*.

A later puzzle book of his, *A Tangled Tale*, contains ten stories each hiding a number of mathematical puzzles. Here is its preface, which conceals the name of the child-friend to whom it was dedicated:

Beloved Pupil! Tamed by thee,
Addish-, Subtrac-, Multiplica-tion,
Division, Fractions, Rule of Three,
Attest thy deft manipulation!
Then onward! Let the voice of Fame
From Age to Age repeat thy story,
Till thou hast won thyself a name
Exceeding even Euclid's glory!

The second letters of each line spell Edith Rix, the sister of Wilton Rix to whom he wrote the algebra letter earlier.

In his last years Dodgson produced a book of mathematical problems, *Pillow-Problems*, consisting of seventy-two ingenious 'problems thought out during wakeful hours'. All of these problems he thought up in bed, solving them completely in his head, and he never wrote anything down until the next morning.

7 TENNIS TOURNAMENTS AND THE MATHEMATICS OF VOTING

Another interest of Dodgson's was the analysis of tennis tournaments.

At a lawn tennis tournament where I chanced to be a spectator, the present method of assigning prizes was brought to my notice by the lamentations of one player who had been beaten early in the contest, and who had the mortification of seeing the second prize carried off by a player whom he knew to be quite inferior to himself.

To illustrate this, let us take eight players, ranked in order of merit, and let us organise a tournament with 1 playing 2, 3 playing 4, 5 playing 6, and 7 playing 8. Then the winners of the first round will be 1, 3, 5, 7, and those of the second round will be 1 and 5; the final will then be won by player 1, defeating player 5 who wins the second prize but actually started in the lower half of the ranking. To avoid this difficulty, he managed to devise a method for re-scheduling all the rounds so that the first three prizes go to the best three players.

Yet another interest of his was the study of various methods for holding elections and counting the votes. The simplest example that Dodgson gave of the failure of conventional methods is that of a simple majority.

There are eleven electors, each deciding among four candidates a, b, c, d . The first three of the electors rank them a, c, d, b ; the next four rank them b, a, c, d ; and so on. Which candidate, overall, is the best?

Candidate a is considered best by three electors and second-best by the remaining eight electors. But in spite of this, candidate b is selected as the winner, even though he is ranked worst by over half of the electors.

Some of Dodgson's recommendations were later adopted in England, such as the rule that allows no results to be announced until *all* the voting booths have closed. Others, such as his various methods of proportional representation, were not. As the philosopher Sir Michael Dummett later remarked:

It is a matter for the deepest regret that Dodgson never completed the book he planned to write on this subject. Such was the lucidity of his exposition and mastery of this topic that it seems possible that, had he published it, the political history of Britain would have been significantly different.

8 LOGIC

Throughout his life, Mr Dodgson was interested in logic. In *Through the Looking-Glass*, Tweedledum and Tweedledee are bickering as always:

I know what you're thinking about — but it isn't so, nohow.

Contrariwise — if it was so, it might be; and if it were so, it would be; but as it isn't, it ain't. That's logic.

Dodgson believed that symbolic logic could be understood and enjoyed by his many child-friends, and he published *The Game of Logic* in order to help them sort out syllogisms. This consisted of a board and nine red and grey counters which are placed on sections of the board to represent true and false statements in order to sort out such syllogisms as the following:

That story of yours, about your once meeting the sea-serpent, always sets me off yawning.

I never yawn, unless when I'm listening to something totally devoid of interest.

Conclusion: That story of yours, about your once meeting the sea-serpent, is totally devoid of interest.

As he claimed:

If, dear Reader, you will faithfully observe these Rules, and so give my book a really fair trial, I promise you most confidently that you will find Symbolic logic to be one of the most, if not the most, fascinating of mental recreations.

He followed *The Game of Logic* with *Symbolic Logic* (originally planned to be in three parts). In the first part he carefully avoided all difficulties which seemed to be beyond the grasp of an intelligent child of (say) twelve or fourteen years of age. He himself taught most of its contents to many children, and ‘found them to take a real intelligent interest in the subject’.

Some of his examples are straightforward to sort out:

Babies are illogical.

Nobody is despised who can manage a crocodile.

Illogical persons are despised.

Conclusion: Babies cannot manage crocodiles.

Others needed more thought, but could be sorted out using his counters. The following example contains five statements, but the most ingenious of his examples went up to forty or more:

No kitten that loves fish is unteachable.

No kitten without a tail will play with a gorilla.

Kittens with whiskers always love fish.

No teachable kitten has green eyes.

No kittens have tails unless they have whiskers.

Conclusion: No kitten with green eyes will play with a gorilla.

Dodgson died in January 1898, before Part 2 of his *Symbolic Logic* was completed, and parts of the manuscript did not turn up until the 1970s. If it *had* appeared, then Charles Dodgson might have been recognised as one of the greatest British logicians between the time of George Boole and Augustus De Morgan and that of Bertrand Russell.

9 CONCLUSION

But let’s leave the final word with Lewis Carroll. One night in 1857, while sitting alone in his college room listening to the music from a Christ Church ball, he composed a double acrostic, one of whose lights has often been quoted as his own whimsical self-portrait:

Yet what are all such gaities to me

Whose thoughts are full of indices and surds?

$$x^2 + 7x + 53 = \frac{11}{3}.$$

REFERENCES

The material in this article is discussed in much greater detail in

- Wilson, Robin, 2008, *Lewis Carroll in Numberland*, London : Allen Lane.
where detailed references for all the above quotations will be found.

Of the many biographies of Dodgson, one of the best known is:

- Cohen, Morton, N., 1995, *Lewis Carroll: A Biography*, London : Macmillan.

Much useful information about his life can be found in his diaries and letters:

- Wakeling, Edward (ed.), 1993–2005, *Lewis Carroll's Diaries: The Private Journals of Charles Lutwidge Dodgson*, in ten volumes, London : The Lewis Carroll Society.
- Morton, N. Cohen (ed., with the assistance of Roger Lancelyn Green), 1979, *The Selected Letters of Lewis Carroll*, London : Macmillan.

There are several collections of his writings. One of the best known is:

- 1988, *The Complete Works of Lewis Carroll*, London: Penguin Books.

Several of his works were reissued as paperback books by Dover Publications, New York:

- 1958: *The Mathematical Recreations of Lewis Carroll: Pillow Problems and A Tangled Tale*.
- 1958: *The Mathematical Recreations of Lewis Carroll: Symbolic Logic and The Game of Logic*.
- 1973: *Euclid and His Modern Rivals*, with a new Introduction by H. S. M. Coxeter.

Much of his Oxford output (both mathematical and of a more general nature) appear in the three volumes of *The Pamphlets of Lewis Carroll*, published for the Lewis Carroll Society of North America, and distributed by the University of Virginia:

- Wakeling, Edward (ed.), 1993, *The Oxford Pamphlets, Leaflets, and Circulars of Charles of Charles Lutwidge Dodgson*.
- Abeles, Francine, F. (ed.), 1994, *The Mathematical Pamphlets of Charles Lutwidge Dodgson and Related Pieces*.
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