MATHEMATICS AND THE PERSONAL CULTURES OF STUDENTS

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Abstract

One important aspect of teaching mathematics is to stress the harmony of mathematics with other intellectual and cultural pursuits. The history of mathematics reflects its origins as a human activity, as people sought to make sense of their world. In more recent times, with the spread of universal schooling, in more developed countries at least, formal education processes for children and adolescents have generally followed the artificial separation of disciplines which originated with the medieval universities. It is a commonly recognised phenomenon today that school students — and vocational students in my experience — carry this arbitrary separation of disciplines into their thinking processes and are unable, even unwilling, to (re)make the connections that might be logically present. This is in contrast to the social sciences, for example.

Another unintended outcome of the arbitrary separation of disciplines is that people may fail to appreciate fully the aesthetics of the natural environment or their cultural environment (e.g., music and other performing arts, visual and literary arts, history, architecture, etc.), because they have never been encouraged to connect different ways of knowing or to reconcile different forms of meaning in mathematics classes.

In this paper I share and explore topics, appropriate to the different developmental levels of learners of all ages, which might encourage boundary crossing. These involve focusing on economic, social, cultural, natural, and historical themes. My concern is that, wherever possible, mathematics should be seen by students to be immediately relevant to their lives, and as supporting them to make decisions that affect them personally.

INTRODUCTION

One important aspect of teaching mathematics is to stress the harmony of mathematics with other intellectual and cultural pursuits. The history of mathematics reflects its origins as a human activity, as people sought to make sense of their world and utilise primitive symbolic systems to overcome the limitations of human memory. With the rise of universal schooling in more developed countries, formal education processes for children and adolescents followed the artificial separation of disciplines which originated with the medieval universities in what Bernstein (2000), following Durkheim, describes as founded on two major discourses: Greek and Christian thought.

It is a well known phenomenon today that school students — and vocational students in my experience — carry the arbitrary separation of disciplines found in most schools into their thinking processes and are unable, even unwilling, to (re)make the connections that might be logically present. So, although they can successfully complete assigned tasks in the mathematics or statistics classroom, when they are confronted with textual or practical applications in their other studies or even outside of school they are unable to competently draw upon mathematical knowledges and skills to creatively solve problems in these different contexts. It is also commonplace that, in English-speaking countries at least, many adults from all walks of life claim both not to have been good at mathematics and that they never use anything they learned in school. These two aspects are very sad reflections on a near-universal education system that encourages, even enforces, separation of subjects. Following Bernstein's (2000) analysis, I have also argued elsewhere (FitzSimons, 2005) that the vertical discourse of mathematics is strongly contrasted with the horizontal discourse of (adult) numeracy, and that pedagogy which is only concerned with the former will not guarantee numerate behaviour in practice. The school subject of mathematics is strongly classified — that is, there are very strong boundaries around what is considered mathematics and what is not. This is in contrast to the social sciences, for example. Another unintended outcome of this arbitrary division is that people may fail to appreciate fully the aesthetics of the natural environment as well as those of music, visual and literary arts, architecture, and so forth, because they fail to connect different ways of knowing or to reconcile different forms of meaning.

This paper is composed of two major sections: (a) one which reflects on my practice over 25 years, and (b) a suggested framework and ideas for others who wish to pursue a theoretically well-founded approach to curriculum planning.

Reflecting on my Practice

In this section I reflect on various activities that support adult learners of mathematics in their quest for meaning through involvement in their personal cultures. Firstly, I discuss working with two groups of women returning to study mathematics after many years away from formal education. One was more focused on the compulsory years of mathematics study, while the other was more focused on preparation for entrance to tertiary studies. I then discuss institutional teaching high level statistics to people intending to be, or currently working as, laboratory technicians. Moving out of the institution into the workplace I discuss how the personal culture of experience in the workplace can be integrated into mathematics curricula for operators working in the pharmaceutical manufacturing industry. Finally, I return to institutional teaching and discuss aspects of a mathematics program for future primary/elementary school teachers.

For women returning to study, each semester classes would begin with the sharing of goals for the program, and reaching agreement on content and classroom norms for learning in an adult environment. I would also request that the women prepare a mathematics-learning history so that I could be aware of the cognitive areas where difficulties occurred in former schooling, as well as affective domain considerations. These histories evoked feelings about both the mathematics content and the pedagogies employed by school teachers as well as some very cruel classroom management strategies, especially in mathematics. The histories enabled the women to reflect on how they did and did not learn mathematics and how they were positioned by parents as well as education systems and by particular teachers in ways that were detrimental to their self-confidence as learners (see, e.g., FitzSimons, 2003). Throughout the program there were many discussions on the history of mathematics, different cultural approaches to learning and doing mathematics — many women were not born in Australia —, and individual research into mathematics topics such as Venn diagrams or the discoveries and struggles of famous women mathematicians, for example. Aesthetic aspects of mathematics were portrayed by colourful posters which were pinned to the walls of the room, as well as through activities such as curve stitching (or by drawing with coloured pens). One activity which was focused on issues important to the women was for them to design and conduct a survey of other users of the centre where the classes were run, in order to make recommendations for change. This activity highlighted the importance of communication in mathematics, and provided practice in data collection, summarisation, interpretation, and presentation, thereby giving practice in number and graphical skills in a meaningful context. One group collected data from the daily news media on sunrise/sunset times and temperature data. Plotting these week-by-week evidenced certain patterns as well as randomness. Plotting the hours of daylight eventually led to what turned out to be a trigonometric function, complete with maxima and minima (i.e., the solstices) as well as low and high rates of change (at the solstices & equinoxes, respectively). These are very big conceptual ideas, normally taught in the calculus years at school, but ones which were experienced and understood by women enrolled at the so-called 'basic' levels. This understanding was deepened by the students relating the plotted graphs to their personal experiences. Clearly, this activity works best in locations distant from the equator!

Another, more advanced, group who were preparing for tertiary entry had the challenge of finding the height of a light tower at a nearby football ground. We discussed a range of methods, and some chose the ancient method of using a shadow stick. Others chose to use an inclinometer and trigonometric methods. One creative person even made a telephone call to the local council! Not surprisingly, each method achieved a slightly different result and these had to be reconciled through discussion.

Higher level vocational students, whose mathematical backgrounds are notoriously weak, yet who wish to qualify for scientific paraprofessional work, have to make meaning of histograms, stem-and-leaf plots, boxplots; binomial, Poisson, normal, t-, and chi square distributions; regression and correlation; as well as quality control and quality assurance work. These skills are critical in scientific and medical laboratory work, for example. In order to make these abstract concepts relate to the life and work experience of the students, I adopted a variety of strategies in the classroom. Almost every lesson started with a video from the series, *Against All Odds* (COMAP, 1989, which, even though it is now almost 20 years old, sets statistical topics firmly in the everyday world of adult students. Illustrations include sickle-cell anaemia and its relationship to the Binomial theorem; the Challenger Space Shuttle disaster which was caused by faulty assumptions about the rules of probability; and quality control in a potato chip manufacturing company. There were also regular practical components, designed to actively involve the students in measuring something, whether it is themselves or objects such as different varieties of dried beans which model natural variation beautifully. These practical components help the students to ground the abstract nature of the subject firmly in reality before they turn to technology-supported calculations. One outcome of these sessions is that students naturally talk with one another about the work they are doing, posing and answering their own questions. A project component of assessment required students to apply techniques they had learned to something happening in their workplace, experimental activities science subjects, or at home. Over the years, students have shown a sophisticated grasp of techniques, and have been excited to link their learning with their workplace; workplaces have also benefited from this activity.

Working in industrial teaching setting in the pharmaceutical manufacturing sector offered many challenges as well as opportunities for linking the teaching program to the everyday work of the students. Faced with an impoverished curriculum of number work that is normally taught in elementary school — albeit with so-called industrial applications — and yet recognising that the workers were already carrying out important and responsible work and felt threatened by the prospect of being subjected once more to the demeaning practices of many mathematics classrooms in the past, I decided to adopt a new approach. This was to make an ethnographic study of the workplace activity and to fit the curriculum to workers' existing practices and skills. I observed all of the explicit and implicit mathematical practices of the workers, starting from 'inwards goods', through the different warehouses, production and packaging, and on to dispatch. From these, I was able to tailor a program which covered the set curriculum, and more, in a way that had immediate relevance to the workers, even to the point of using the actual names of the workers and the products they made. The activities included tours of the workplace in order to see how mathematics and information technologies were used in practice, and this had the benefit of enabling to workers to see the processes upstream and downstream, thus giving them a better idea of how their own efforts fitted in to the total production process. As a result of their lift in self-confidence, the workers were more willing to question work processes and to suggest improvements. See FitzSimons (2000, 2001) for more detail.

My final example is drawn from my experience of teaching mathematics to people intending to become primary (elementary) school teachers (FitzSimons, 2002). As is well recognised, many of these people are also anxious about mathematics, yet would like to teach in ways that children will find interesting and exciting. One year's assessment activity was for them to design and model an adventure playground suitable for primary-aged children. Another year, they were asked to design a 'mathematics trail' for primary children, utilising a real or hypothetical site, including activities relating to each of Bishop's (1988) six 'universals' — of counting, measuring, locating, designing, explaining, and playing[1] — with questions of varying sophistication. In both years there were many outstanding projects as the students showed creativity and a willingness to become deeply immersed in the process. Many also developed activities that they could use in their future teaching profession. At the same time, they were using and further developing the mathematical and other skills identified in the course program.

Frameworks for Planning

The unit *Program Design and Delivery*, designed for workplace educators and trainers who have not yet acquired their first degree, the major assessment task is to make a study of their own or someone else's program, analysing the various constituents, and then to make recommendations for change and/or justify retaining the current program wholly or in part. To help them prepare for this task, I draw on activity theory as espoused by Yrjö Engeström (2001). Although it is a complex theory to understand and work with, I find that the time and effort involved are justified by the high quality of student work. Once again, the outcome is directly related to their personal worlds — even though these are not usually mathematics education. I believe that this framework may be adapted by mathematics educators conducting undergraduate or post-graduate courses, or even continuing professional development.

At the ESU-5 presentation I offered a modified version of Engeström's framework as a basis for planning mathematics programs for learners of all ages:

	Cultural backgrounds	**Historical aspects**	**Tensions & contradictions**	**Moving on**
Who are learning?				
Why do they learn?				
What do they learn?				
How do they learn?				

[1]See Bishop (1988, pp. 100–103) for operationalisation of these pan-cultural mathematical activities in terms of the school mathematics curriculum.

Question arising could include:

- Who are *your* particular learners?
- Who else is learning?
- Why are they learning mathematics?
- What are their longer term goals?
- What are your objectives for them (e.g., workplace, citizenship, qualifications)?
- Why is the history of mathematics important? (note the importance of workplace historical artefacts).

Herrington et al. (2001) proposed a framework for evaluating online program which could also apply to regular classroom activities, especially projects. These could be framed as questions:

- Authentic tasks: Do the learning activities involve tasks that reflect the way in which the knowledge will be used in real life settings?
- Opportunities for collaboration: Do students collaborate to create products that could not be produced individually?
- Learner-centred environments: Is there is a focus on student learning rather than teaching?
- Engaging activities: Do the learning environments and tasks challenge and motivate learners?
- Meaningful assessments: Are authentic and integrated assessment is used to evaluate students' achievement?

Discussing mathematical awareness, Tzanakis, Arcavi, et al. (2000) identify two major categories of awareness that students might develop.

- Awareness of intrinsic nature of mathematical activity:
  - The role of general conceptual frameworks and associated motivations, questions, and problems which have led to developments in various domains of mathematics.
  - The evolving nature of mathematics in both content and form.
  - The role of doubts, paradoxes, contradictions, intuition, heuristics and difficulties while learning and producing new mathematics.
- Awareness of extrinsic nature of mathematical activity:
  - Aspects of mathematics may be seen as closely related to the arts, sciences, and other humanities.
  - The social and cultural milieu may influence or even delay the development of certain mathematical domains.
  - Mathematics is recognisably an integral part of the cultural heritage and practices of different civilisations, nations, or ethnic groups.
  - Currents in mathematics education throughout history reflect trends and concerns in culture and society. (pp. 211–212)

Although this paper has generally focused on the extrinsic aspects of mathematics in relation to the personal cultures of students, teachers and teacher educators may find appropriate moments in which to discuss some of the intrinsic aspects of mathematical activity in a natural way.

My concern is that mathematics should be seen by our students to be immediately relevant to their lives, and help them to make decisions that affect them personally. This means that teachers need to:

- keep in mind the mandated curriculum and assessment requirements
- to juxtapose these with activities which hold rich connections for the for the learners beyond the mathematics classroom
- ensure that activities take place at a variety of cognitive levels (operations — embedding & reinforcing facts and rules, actions — developing understanding of concepts and tools, and activities — creating and communicating)
- keep in mind the range of generic competencies (e.g., communicating, planning, working in teams, problem solving, using technology) which accompany workplace and other civic activities.

Topics which might encourage boundary crossing, appropriate to the different developmental levels of learners of all ages. They involve focusing on economic, social, cultural, natural, and historical themes. For example:

- mathematics trails to study local architecture and/or history
- the linking of mathematics and history at school
- the role played by mathematics in children's and adolescent literature and films
- the mathematics of a major issue in the local environment
- the mathematics of analysis and composition of art
- the mathematics of analysis and composition of music
- the mathematics of analysis and composition of dance choreography
- the comparative costs of various mobile/cellular phone schemes
- the comparative costs of various credit card schemes
- statistical investigations of events occurring in everyday life at home, in the local community, or on television (e.g., sports)

It is essential that learners of mathematics at any level from the early years to university graduates are actively involved in their learning and are able to communicate what they know to a range of other people who are at different levels of mathematical understanding.

Conclusion

As a former school and vocational mathematics teacher I am only too aware of the pressures on very busy teachers. I also remember the practicalities of focussing on the topics for the classes immediately on the horizon, and the importance of each and every assessment, leading up to the end of the semester, with the ultimate goal of supporting students to achieve their goals through gaining the required qualifications, ideally at the highest possible level. Mostly, this meant working under the constraints of curriculum and major, high-stakes assessment tasks set by external authorities.

I also remember very clearly my disappointment when students, even in vocational education, were unable to bring the mathematical skills and knowledges developed in my class to bear in their laboratory or other classes. It seemed that there were invisible walls. Similarly, it is probably a universal phenomenon that employers often complain that new graduates from school or university are unable to 'apply' what they have supposedly learnt according to their qualifications. Beyond the workplace, it is essential that learners, young and old, are able to make meaningful connections between their educational experiences and the world beyond the classroom (real or virtual). Clearly this needs more than wrapping so-called realistic settings in textual form around the mandated mathematics algorithms for the particular group of learners. It needs the learners to be involved in both cognitive and affective domains — to have a real interest in the outcome of their work and to be able to communicate the problems and the results to other interested people.

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Further reading

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