Navigation Instruments and Teacher Training

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Abstract

The objective of this article is to present and to argue the viability and implications of teaching mathematics with the help of an historical application of trigonometrical concepts. The historical context explored here is the Portuguese navigations in the XIV–XVII centuries, especially the techniques of finding the position of boats at sea, with special emphasis on one of the instruments used at that time: the cross staff. The mathematical concepts we speak of are those related to angle and to tangent. The present study focuses attention on the initial and continuing formation of mathematics teachers and shows that the teaching of mathematics in this perspective is not only successful from the point of view of learning of mathematics but also contributes to a wider education for mathematics teachers. The article calls attention to the fact that the treatment of the mathematical concepts has to be done with extreme care so as not to risk losing the aspects related to the learning of the proper mathematical concepts. Some difficulties with this approach are also highlighted.

1 Introduction

This article discusses the viability of teaching mathematics based on an historical application of a mathematical concept. The main objective of our investigation was to verify the implications of this approach in training mathematics teachers.

Initially, we relate our previous experience in a similar course on the same theme, in which the participants (mathematics teachers and teacher trainees) showed difficulties in learning the trigonometrical concepts that we introduced. This led to the conjecture that teaching mathematics through historical applications may be unviable.

Next we describe a scheme of investigation that allowed us to verify if the conjecture was true or false. The investigation included a course for mathematics teachers that gave us an opportunity to collect data for the investigation. We also show the qualitative analysis of the data.

The results of the study refute the proposed conjecture and show that the approach that uses historical applications of math does not make the acquisition of the mathematical concepts by the learner unviable. In addition, the results also show that the historical approach contributes to the teacher's general education by covering a wider scope of knowledge and pedagogical skills.

Finally, the study discusses the difficulties we may meet if we decide to adopt a teaching approach based on historical applications.

One more remark should be made here. In the courses that we refer to in this article, the participants were in-service math teachers or undergraduates preparing to be math teachers. We call all these participants 'students'. We call those researchers that led the course 'teachers'.

2 The research problem

Trigonometry and its history have always been present in our studies. In 2004 we were working on themes about the application of trigonometry to the Portuguese navigations in the XIV–XVII centuries. We participated in a project placed on the worldwide web by some Portuguese schools, which invited other schools to participate. This, as we know until now, is the only project that links the Portuguese navigations with the teaching of school mathematics. We modified, adapted and complemented the material accessible in the site http://www.cienciaviva.com/latlong/ in order to serve as the basis of a course to be offered for mathematics teachers (MOREY, MENDES, 2005).

The theme is important to us Brazilians because the arrival of the Portuguese in 1500 started the events that determined our national identity. Also, it is a recurrent theme in social studies, specifically in the History of Brazil, but in these studies there is never any mention of mathematics. To the undergraduate math students in our courses, the information that the Portuguese navigations were enterprises that were undertaken with help of mathematics is very interesting. Thus, there is no lack of interest and motivation at the start of our courses.

While we were talking about the details of the enterprise and trying to create a living picture of what a journey across the sea to America or to India was like at that time, attention levels remained high. It was interesting to see what instruments were used and how they actually helped locate the ship in the middle of the ocean.

The difficulties began when we started to detail the mathematical knowledge that was necessary to construct and use these instruments. In the case of circular instruments like the quadrant and the astrolabe, the necessary knowledge is no more than angles, their measure and a few simple properties. For linear instruments, like the kamal, the Jacob staff or the cross staff, however, the necessary mathematics includes understanding and manipulating trigonometrical tables.

It was at this moment, therefore, as we started to stress the mathematics behind the instruments, that the students began to lose interest and had difficulty in following the course. That situation worried us because we were working with (present or future) math teachers, so we decided to investigate the situation more deeply. Our objective was thus to identify the reasons that caused the students to lose interest and to determine how to overcome the linked difficulties that arose in the course.

Our preliminary data suggested that the difficulties were mostly due to the lack of knowledge about trigonometrical concepts and of the use of trigonometrical tables. Students who were interviewed indicated that trigonometry, although a standard school subject is very poorly studied in the high schools and avoided thereafter whenever possible.

There remained the question of how to capitalize on the students' initial motivation and interest in order to overcome their rejection of trigonometry, so we began an assessment of the various aspects of the last course offered.

3 Assessing the course and making hypotheses

From our point of view, the objective of teaching trigonometrical concepts via historical applications was not reached. That led us to ask ourselves if the methodological choice (historical approach) was or was not responsible for the failure of the course's objective. In order to answer this question, we analyzed the development of our course in the light of the objections against the use of history in mathematics teaching that various authors have suggested. To give an example, we can point one of the objections that says: "many students dislike history and by implication will dislike history of mathematics, or find it no less boring than mathematics" (FAUVEL and VAN MAANEN, 2000, p. 204). We concluded that these objections weren't pertinent to our context and, thus, the cause of failure was probably due to other factors.

One of our suppositions in designing the course referred to the fact that the participants were mathematics teachers or undergraduates preparing to be mathematics teachers. This fact led us to suppose that a few explanations would be enough for the students to understand the trigonometrical concepts introduced in the course. Obviously, we were wrong in our supposition and we started to look for ways to clarify whether the difficulties were or not were linked with the initial studies of these teacher trainees.

Later we had the opportunity interview some of the participants about what they found difficult in the course. They said that trigonometry was poorly studied at school and many of them added that they considered it a difficult subject. They avoided studying or teaching the topic as much as possible. None of them mentioned history as a source of difficulties.

So we decided to offer the course again, with some improvements. The starting point now was the hypothesis that the difficulties experienced in the previous course were due to the fact that we did not pay enough attention to the trouble the students have with trigonometry due to inadequate primary and secondary studies. So, in this second edition of the course, we carefully followed two points:

- to keep the same approach (the use of historical applications of trigonometrical concepts)
- to introduce the trigonometrical concepts in that way that would allow the participants to overcome their own difficulties.

4 A COURSE FOR MATH TEACHERS, SECOND EDITION

The continuation of our investigation included preparing and offering a course to in-service math teachers and future math teachers. The course was offered to a class of 50 students and it lasted 8 hours. Similarly, the course focused on the navigational instruments used by Portuguese seamen in XIV–XVII centuries.

Our aim was for the students to understand the way the Portuguese navigators used the concepts of angle and tangent implicit in their instruments to determine the localization of ships at sea. We introduced the following topics:

- historical aspects of the age of the Portuguese navigations and of the difficulties of undertaking the enterprise of long sea voyages. We hoped that the students would feel like they were going back in time to understand the atmosphere and problems of that era. The site http://www.cienciaviva.com/latlong brings good texts about this topic of the course. However, more information can be found in (Albuquerque, 1989; Albuquerque, 1988; FONTOURA da COSTA, 1983 and PIMENTEL, 1819);
- basic notions of astronomy, enough to understand the link between of the height of celestial bodies and the localization of the ship;
- manipulation and analysis of the navigational instruments to understand their functioning and the mathematical concepts implicit in them;
- careful introduction to the notion of the tangent as a measure of inclination and its relation to the angular height of a celestial body.

The tangent notion was introduced as a measure of inclination (calculated as the ratio between the perpendicular sides of a right triangle). This was done in several contexts, each closer to the proposed application in the context of navigations. Our previous experience indicated that problems linked with understanding trigonometrical concepts could arise, so we introduced the notion of tangent gradually, from various points of view. Thus, the concept

was introduced as: ratio between the shadow of the students and the (angular) height of the sun; inclination from which we see familiar visible things such as, a bird perched on a post; inclination of the sun at a given moment and in given place; (angular) height of a celestial body (not the sun); in the analysis of the mathematical scheme of the cross staff; in the process of the construction of a cross staff.

During the course we used continuous oral dialogue, audiovisual recourses, practical manipulation of the instruments, development of individual and group tasks, both in and outside the classroom.

5 Others investigation tools

The analysis of the results of the present article is based upon data collected by the three teachers present in the classroom during the whole course. The course was prepared in this way to make it possible for us to detect the students' difficulties. Such procedures provided conditions to collect and register data, characterizing the performance of the students in the learning situations we proposed. During the course, we focused our attention on:

- the interest of the students in following the activities proposed;
- their engagement in the tasks;
- their participation in the discussions;
- their skills in performing the algorithmic procedures.

We were also able to detect those things that revealed themselves as difficult for the participants to follow the course, such as:

- lack of familiarity with an important period of our history;
- difficulties in the reading and interpretation of historical texts;
- lack of familiarity with the configuration of the sky;
- unfamiliarity with astronomical concepts;
- difficulties in the understanding and application of the trigonometrical concepts of angle and tangent;
- difficulties in the manipulation of ruler, compass and protractor;
- difficulties in making interdisciplinary relationships.

At the end of the course, we discussed, with the students, certain aspects of the course with the intention of detecting data that our observations had not captured.

The recording of the data was done by means of field notes, observations of the researchers on the actions and discussions of the students and written notes made by the students.

6 Results

In the second edition of the course, the participants followed the lessons without losing their way or their attention when we started to explore the trigonometrical concepts involved in the navigation instruments.

The following items indicate that the students learned trigonometrical concepts: the manner in which students expressed their doubts and formed questions; through the solutions in their notebooks and on the blackboard of the tasks that they were presented with; the

persistence and the success achieved in the resolution of challenges that extrapolated what they had studied during the course; the use of trigonometrical tables in the resolution of problems and in the construction of the cross staff; the comments of the students at the end of the course in which they talked about the aspects of the concepts that were new to them.

We emphasize here that the students had initial difficulties with trigonometry. The history of their failure in the comprehension of this subject was a real datum in the experience of the majority of the students in the course. However, such difficulties were gradually overcome thanks to the persistence of the students and to the care that we took in the introduction of the trigonometrical concepts. Such care demanded extra time to prepare the activities, but it contributed to the understanding and acquisition of the concepts by the students. It is true that careful introduction of concepts is necessary in all mathematics courses, not only in those that use the history of mathematics. But, we are not here affirming that recourse to history is the only way to teach and to learn mathematics. Rather, the history of the mathematics is only one among several resources that we can use in our mathematics lessons, but, when using this one, we have to guarantee, clearly, that the learning of mathematics is not relegated to the second place. The point that we want to defend here is that, when integrating history into the process of teaching and learning mathematics, we must teach and learn a variety of other things beyond the mathematics.

Still focusing attention on the aspects related to the learning of the mathematical concepts, we can point to some items that we consider indicators of the students' learning:

1. The students correctly deduced the relations that resulting from the variation of length of one of the legs of a right triangle while the other leg remains fixed.

2. Starting from initial unfamiliarity and difficulty in dealing with trigonometrical tables, the students gradually become familiar with this resource to the point of using them in the resolution of problems. Moreover, they demonstrated that they had developed a good understanding of the meaning of the tables by voluntarily proposing suggestions to reformulate the statements of the problems in order to diminish the differences in the numerical results obtained by each member of the group.

3. In the process of construction of the cross staff, via the trigonometrical tables, there were initial difficulties in the calculation of the first values. Such difficulties were overcome and moreover the process of construction via the tables continued without difficulty.

4. Another indication that the students were learning was the resolution of proposed challenges. Such challenges demanded, for their resolution, surpassing the direct information that had been given during the lesson. One such challenge was to obtain the value of the tangent of an angle of 43° making use only of a ruler, compass and protractor (without using tables or a calculator). There was a general persistence in the search for the solution, but only two groups solved the problem. However, when the solutions were presented, the entire class participated in ways that led some groups to propose improvements on the solutions presented. That experience led some students to make important pedagogical reflections.

The analysis above allows us to say that recourse to the history of the mathematics can be integrated into mathematics lessons without loss of mathematics learning. Although this is a point of extreme importance, we want go to beyond this and examine other aspects of the lessons that, in our opinion, were made possible by the recourse to history.

7 Other important elements in the formation of the mathematics teacher

The development of views on the nature of mathematics and mathematical activity was one of the benefits of the course. In fact, in the oral comments presented by the students at the end of the course, some affirmations were made that had the character of personal discoveries that had occurred during the course. One of them was that "mathematics is not alone, it is with other disciplines". Another surprise was that the expression "to measure height" when related to one celestial body, means to measure the angle (in degrees) and not it linear length of height in meters. The perception of the connection of mathematics with other areas of knowledge and the perception that the relation of a celestial body with the Earth not only involves measuring lengths (distance) but also measuring inclination (value of the tangent) reveals, in fact, a new way to conceive the nature of mathematics and mathematical activities.

The acquisition of a new view on the meaning of the history of the mathematics can be observed in the comments of the students who emphasized that the importance of history is to be found "not only in the form of biographical stories". This discovery was surprising for some students whose only contact with the history of the mathematics was through text books (for primary and secondary school) in which only history that the authors include is biographies of past mathematicians.

The comments of some of the students expressed that the approach we used in the course provided them with a larger variety of pedagogical choices in their everyday teaching performance. With respect to trigonometry, they observed that the introduction of the tangent concept does not necessarily demand the previous introduction of the concepts of sine and cosine. Moreover, they observed that it is possible to work with the values of the tangents of the angles without restricting themselves exclusively to the angles of 30°, 45° and 60°. A point that the students valued and emphasised was that the course brought them a historical context of application of the trigonometrical concepts. To attain this, we used readings of introductory text, maps, terrestrial globes and some samples of cross staffs. The students recognized the great importance of such activities, but they pointed out that its implementation demands much reading, research and planning. In their evaluation of these activities, they stated this was beyond the conditions that they normally have in the schools.

But, in contrast, they perceived the real possibility of using maps and a globe in their mathematics lessons in order to enrich it. They feel themselves capable of using such resources not in the historical approach, but as a possibility in developing of interdisciplinary studies or projects. The exploration of the longitude and latitude concepts was cited as an example.

Some of the students' comments stressed the importance of courses of this type to help mathematics teachers immerse themselves in interdisciplinary projects. We consider this aspect very important, because, when interdisciplinary projects are proposed in the school, the mathematics teacher frequently has great difficulty in participating in such projects.

We also found that the course provided a certain **affective predisposition towards trigonometry**. In fact, when asked about relevant subjects for future courses, the participants suggested that we continue the study trigonometry stressing the sine and cosine concepts and their applications in an historical approach. Taking into consideration that at the beginning of the course there was a certain rejection in dealing with trigonometry, the predisposition now presented in relation to the continuing of study of the subject indicates a change of position.

Our investigation shows that the approach to teaching mathematics by means of the historical application of a concept allows **a wider formation of the teacher** by means of insertion of knowledge from other content areas. Our experience indicates that mathematics

teacher trainees don't usually like disciplines from other content areas. For example, physics, education and the mother language courses are seen as extraneous to their major.

In contrast, in the approach we used, the knowledge of other content areas was introduced when it was made necessary and the students accepted it as intrinsically related to the object of study. In this way, the students naturally returned with interest to the history of Brazil in the Age of Discovery. This included knowledge that was outside of their previous experience, such as astronomy and aspects of the historical development of the Portuguese language.

8 Conclusions

The data collected during our study allow us to affirm that it is viable to teach mathematical concepts by means of an historical approach, since the teacher can reach an equilibrium between the emphasis that she will give to the historical aspects and to the introduction of the proper mathematical concepts.

Moreover, when this equilibrium is reached, we can detect certain implications of this teaching approach in the formation of the school mathematics teacher. Some of these are:

- improvement in the understanding of the mathematical concepts, while at the same time providing knowledge of applications of these same concepts;
- an increase in the possibilities of useful pedagogical choices for the teacher in her math classroom;
- providing the teacher the chance to extend her knowledge into other fields of knowledge beyond mathematics.

However, it is important to say here that the preparation of a course such as the one presented above demands that the teacher of the course: chooses an historically located problem that involves mathematical concepts; has knowledge of the disciplines involved; has enough time to dedicate to the necessary research and planning; makes use of auxiliary texts that contribute to elucidate the context of the problem; creates, elaborates and makes use of audiovisual resources that contribute to the understanding of the problem; finally, that the instructor has the necessary pedagogical experience to co-ordinate the use of the several elements involved.

These highlighted requirements are hardly included in the context of the activities of a basic school math teacher. Neither has her initial education been directed to this, nor does her daily routine foresee or stimulate such types of activity. Our opinion is that it is the university-level researcher who has the conditions to prepare and offer instruction in the use of such courses. However, and it is important to state this, even though the course as a whole is rather complex in its elaboration, elements or parts of it are understood and assimilated by the student-teachers with such promptness that they consider it an easily applicable resource in their classroom.

Finally, we want to talk about one remaining question: Why the results were positive? Our opinion is that there were many factors that contributed to the success of the course as we exposed above. However, we want to stress the fact that the Portuguese navigations were taken by participants of the course as a part of our national history and therefore they should know and understand all the facts related to. So, in that meaning, we can say that the historical approach applied to a theme embedded in the cultural context of the participants of the course strongly contributed to the positive results.

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