How Much History of Mathematics Should an Elementary Mathematics Teacher Know?

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Abstract

Students in the Elementary Mathematics Education program at Brooklyn College often take the History of Mathematics elective because they think it will be an easy course that does not involve any mathematics. Some voice the complaint that they only need to know the mathematics, which is taught in elementary school. The 2000 ICMI Study entitled History in Mathematics Education provides a contemporary, international view of these issues. My paper discusses what material is appropriate for these students and the purpose for studying them.

How much history of mathematics should an elementary mathematics teacher know? Well, the answer to this question is certainly a function of whom you ask. If you ask me, I would say as much as possible. What better preparation can there be for a teacher than to know the origins of what he or she is teaching? On the other hand, if you ask the beleaguered elementary school teacher who has worked at his or her school all day long and takes an evening class once a week for three hours, the answer is, frequently, as little as possible. Some of the students who come to me are poorly prepared for such a class. Not only are they deficient in basic mathematical skills, but they have no idea of the flow of history. Some are only interested in an easy elective which will allow them to complete the degree required for their continued employment. Fortunately, the exposition above merely describes one end of the spectrum of students that I meet in this class. Many students wish to learn as much as they can in such a class. They appreciate anything that will improve their ability to teach mathematics. In this paper I will examine what can be done for both types of students.

The impetus of this study comes from a correspondence from the first type of student. We will refer to him only as JSV.

> Dear Professor,
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> I am one your students taking Math 604.4T. I am a first grade teacher. Elementary teachers are not used to high level math. So I hope that the focus of the test will be the "History of Mathematics" not "Mathematics" per se.
>
> JSV

This correspondence is telling in many ways. JSV's essential worry is the content of a test and has given himself an out by describing himself as a mere first grade teacher. My response to him contained the following points:

1. As to the issue of whether the focus of the test will be mathematics or history of mathematics, I can assure you there will be both. May I remind you that the class is entitled MAT because it is a mathematics class.

2. As to the fact that you are a first grade teacher, I believe the degree you will receive is in Elementary Mathematics Education which includes grades up to 6. You are not getting a degree in first grade education.

3. As to the level of mathematics you will need to understand in order to be an elementary mathematics education teacher, you will need to know much more than your students. You will need at least a high school level of mathematics.

It was extremely gratifying to discover that many of these issues had already been addressed in the 2000 ICMI Study on *History in Mathematics Education* edited by John Fauvel and Jan van Maanen. In the preface they outline the aim of the study:

> The movement to integrate mathematics, history into the training of future teachers, and into the in-service training of current teachers, has been a theme of international concern over much of the last century. Examples of current practice from many countries, for training of teachers at all levels, enable us to begin to learn lessons and press ahead both with adopting good practices and also putting continued research effort into assessing the effects. (p. xvii)

In Chapter 4, *The History of Mathematics for Trainee Teachers* (Fauvel & van Maanen, 2000), Victor Katz relates that in 1998 the majority of US state certification programs require history of mathematics courses for secondary teachers. While a 1962 ICMI study on the history of mathematics concluded that history of mathematics should be required for secondary teachers, this study proposes that "teachers in primary schools are now seemed to be helped as well" (p. 93). Unfortunately, elementary school teachers in most of the United States face no requirement to be certified in an academic field. The teachers that I come into contact with are being certified as specialists in elementary mathematics. In Chapter 6, Karen Dee Michalowicz reminds us that "many of these teachers would not be comfortable with secondary school mathematics content" (p. 173). Certainly, JSV is a case in point. However, one suggestion of this new study is that "the overall didactic aim is to understand mathematics in its modern form" (p. 210). A study from Cyprus (see Chapter 4.3.1.2) concluded that a guided journey through the history of mathematics would enable students to construct mathematical meanings and support their new conceptions about mathematics by changing their attitudes and beliefs. In Chapter 8, Gispert and Siu state that historical studies can help a teacher understand not only the way of teaching the syllabus, but also the origin and reason for its content.

After concluding that such training is worthwhile, the next question that needs to be addressed is what should be taught. Chun Ip-Fung has pointed out the need for a compromise between historical and pedagogical aims (see chapter 4.3.1.1). I would add to this the need to compromise between the density of the subject and the understandability of the material presented. We would like to give the best possible overview in a finite length of time without creating a curriculum which is a 'kilometer wide and 2 centimeters deep'. A thirty-five hour course for elementary teachers has been described by David Lingard of the UK (see Chapter 4.3.1.3).Torkil Heiede described an in-service course for primary teachers(see Chapter 4.3.2.1) similar to the one I teach. The course covers the following areas: Egypt, Babylon, Greece, India, China, Arabia, Medieval and Renaissance Europe and non-Euclidean geometry. Although Heiede had specifically proscribed discussions about calculus and series, I feel that some exposure to these topics is essential. To this list I would add a preliminary section

on ancient numeration, a final section on the relation of the search for a general solution to the quintic equation to modern algebra, and another on transfinite numbers.

How can one accomplish such a daunting feat? I have found that using a threaded approach works best (see Laubenbacher & Pengelly, 1999). Most of the material I cover will fall under one or more of the following threads: number, equations, area and volume, right triangles, and proof. While the typical elementary teacher cannot be expected to master integration, it is still possible to expose them to that concept using the method of exhaustion (area) and geometric series (number). After exposure to the work of Hippocrates of Chios, Eudoxus, Archimedes, Wallis and Fermat, the teacher will gain an appreciation of that skill.

One NCTM publication on the history of mathematics for elementary school students defines calculus as "a hard kind of math". I think we can do much better than that for both the teachers we are training and the students that they will come in contact with. Through the use of worksheets containing "guided sets of questions" (see Chapter 7.4.4) for group discovery activities we may construct a reasonable facsimile of what the ancients knew. The activity for discovering the Babylonian nine times table (see Chapter 8.3.1.2) is a particularly good example of this and one that I usually begin the course with.

Chronology is important as it helps the learner organize mathematical development. However, the emphasis should not be on memorizing dates but rather ordering the flow of ideas. At any stage "use may be made of concepts, methods, and notations that appeared later than the subject under consideration" (Fauvel & van Maanen, 2000, p. 210). It is not necessary to teach Egyptian duplation using hieroglyphics. Once students have been made familiar with the symbols it is perfectly sound to express 12 as 10,2 for the purposes of computation. We should never let historical purity override our ability to express the mathematical ideas within. However, students should be constantly reminded that these are simplifications. We would never want to lead them to believe that the Babylonian quadratic formula was originally written with variables.

More importantly we must show primary teachers how to teach themselves about the history of mathematics. I think it is particularly important for teachers to be able to separate history from rumours and fables. There is a cornucopia of material out on the market which is written at this level of thinking. One only needs to spend some time "surfing the net" to see just how bad it can get. Our teachers need training for "a critical use of historical sources and to judge the value of secondary literature" (Fauvel & van Maanen, 2000, p. 141). On the first day of class I introduce a maxim of my illustrious predecessor at Brooklyn College, Carl Boyer: "mathematical formulas and theorems are usually not named after their original discoverers" (Kennedy, 1972). We then spend much of the semester discovering examples of this theorem. Much can be said of the "constructive role of errors" (Fauvel & van Maanen, 2000, p. 219) in the work of the mathematicians we study. The discovery of both scribal errors in source documents and conceptual errors from the Egyptian formula for the area of a quadrilateral to the 'problem of points' in probability give students confidence in their own ability to understand mathematics. Learning about the difficulties with acceptance of such concepts as negative numbers (see Chapter 9.2.3) give teachers the "knowledge that much of what is taught today as a finished product was the result of centuries of groping or of spirited controversy" (Fauvel & van Maanen, 2000, p. 38).

My students in the course at Brooklyn College come from a wide variety of ethnic groups: Italian, Jewish, Russian, Haitian, Puerto Rican, Pakistani, and Chinese, to mention only a few. While this may at first seem a handicap, it is clearly a motivation for multicultural approaches to the subject. An overarching theme of the ICMI study was that we need to humanize the subject. Finding examples of mathematical excellence in their own culture can be extremely rewarding for most students. I have screened the discussion of the Mayan concept of zero in the film *Stand and Deliver* with good results. California programs require students to understand chronological and topical development of mathematics, including in-

dividuals of various racial, gender and national groups. The National Council of Teachers of Mathematics (NCTM) has stated that we should "prepare prospective teachers who have a knowledge of historical development in math that includes the contributions of underrepresented groups and diverse cultures." (Fauvel & van Maanen, 2000, p. 106) Interdisciplinary approaches foster connections with physical sciences, geography, economics, art and music, religion and philosophy (see Chapter 2.4.1ff). Each of the above approaches lend themselves nicely as the basis of topics for the research projects that each of the teachers present at the conclusion of the course. These projects give the teachers a chance to 'shine' in front of their colleagues. For some of them it is the highlight of the course.

My student JSV has given the following assessment for my course:

> Dear Professor Kiernan,
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> I can say that I have benefited from attending Math course 604.4. I enjoyed it very much. I think I can successfully now teach the "History of Math" to elementary students as a component of social studies, thanks to you. I have struggled hard to try to get at least a B because of my standing probationary status at the college. I am not going to tell you to e-mail me my grade because a C would be very devastating and would affect me during the summer session.
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> Otherwise, you may do so.
>
> JSV

So, while JSV's motivations did not reach the lofty goals we wished he could have, it is clear that he and many others did indeed benefit from the course.

A speaker at a recent conference said that the great thing about teaching a course on the history of mathematics is that you can teach exactly what you want to teach. While I understand his delight at the prospect, I feel that it is certainly more appropriate to concentrate on what our new teachers need both to gain sufficient background in mathematics and to be exposed to the available materials, which will improve their teaching. Yes, a history of mathematics course can be what we want it to be. In particular, it can serve the needs of elementary mathematics teachers.

References

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