THE INFLUENCE OF *IT* ON THE DEVELOPMENT OF MATHEMATICS AND ON THE EDUCATION OF FUTURE TEACHERS

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Abstract

Today, computers strongly influence the entire human society. Mathematics has also experienced great changes. The author introduces three areas, where computers have had a strong influence on the development of mathematics in the 20th century. Based on these changes, the author contemplates the methods of education of future teachers of mathematics, and, based on a specific case study at the Faculty of Education of the Charles University in Prague, he describes possible changes to the future teachers' theoretical knowledge base.

1 INTRODUCTION

The invention of the computer was one of the greatest achievements of the 20th century. Already, given our short history with computers, we can claim that computers have changed our world tremendously. Computers influence our lives every day, from dawn to dusk, and only rarely have human activities remained unaffected by them. Computers, so to speak, guide our every step. They are ever more the part of our daily appliances — be it cars, mobile phones, microwaves, or refrigerators.

The world of mathematics is no exception in this respect. Computers influence the lives of mathematicians, but they also influence their daily bread: mathematics itself. The aim of this paper is to introduce some basic changes that were adopted as a result of the introduction of computers into mathematics and the teaching of mathematics, and to hint at some amendments in the education of future mathematics teachers that would reflect these changes.

2 THE HISTORY OF COMPUTERS

Generally, we take the year 1822 for the birth-year of the computer, when it was Charles Babbage (1791–1871), an English mathematician, philosopher, and mechanical engineer, who originated the idea of a programmable computer (Hyman, 1982). A computer based on his blueprint was constructed some time later, but it was he who laid down the foundations of computer history.

Some of the first computers with the functionalities we know today were built about half way through the 20th century during the Second World War (see table 1). Some of them had influence on the course of the war, some more than others (Dohas, 2002).

Table 1

Name	Date	Programming
Zuse Z3	May 1941	By punched film stock
Atanasoff-Berry Computer	Summer 1941	No
Colossus	1943/1944	Partially, by rewiring
Harvard Mark I/IBM ASCC	1944	By punched paper tape
ENIAC	1944	Partially, by rewiring
ENIAC	1948	By Function Table ROM

These computers, which are categorised in the 0^{th} generation, had very limited performance capabilities. During the second half of the 20^{th} century however, the power of computers skyrocketed. Man gave birth to computers of the 1^{st} to 4^{th} generation. Currently, engineers are working on computers of the 5^{th} generation — machines with artificial intelligence and quantum computers. The speed and availability of computers was boosted mainly by the invention of the microprocessor (Intel 1971) and the production of the first microcomputers and personal computers (Allan, 2001).

2.1 The History of Computer Languages

Together with the development and advancement of computers, programming languages flourished (Bergin, 1996). An overview of some of the most important events in this particular field is listed in table 2.

Table 2

1949	Short Code, the first language
1951	A-0 first widely known compiler
1952	AUTOCODE, a rudimentary compiler
1957	FORTRAN — Mathematical FORmula TRANslating system
1960	ALGOL 60, the first block-structured language
1966	Logo — “turtle graphics”
1968	Niklaus Wirth begins work on Pascal
1970	Work on Prolog begins
1975	Bill Gates and Paul Allen write a version of BASIC
1976	Design System Language
1983	First implementation of C++
1983	ADA
1994	Visual Basic for Applications in Excel.

3 The Influence on Society

Towards the end of the 20^{th} century, computers ceased to be the privilege of the few, and became an integral part of virtually everybody’s lives. The big boost occurred when the Internet, a digital information highway, was opened to the public.

These changes have had great impact on the mathematics community as well: communication is easier; thanks to the Internet, information is now readily available, for example worldwide papers on mathematics; some mathematical journals are available in an electronic form. Computers have also changed the format of the new standard paper on mathematics.

The standard is being set by the TEX mark-up language, which was developed by Donald Knuth in the 1970s with the help of the American Math Society.

4 How Computers Influence Mathematics

To evaluate the events occurring in the recent years in such a broad field, which modern mathematics undoubtedly is, would be very difficult. We will therefore try to outline three areas where computers have had the greatest impact on mathematics.

4.1 The Birth of Theoretical Information Theory

Already in the first half of the 20^{th} century, many mathematicians were working on the theoretical aspects of the later physical "piecing together" of a computer. For example, the theoretical model of the computer was emerging at that time. From the many mathematicians taking part in these projects, let us name a few of the most important ones: John von Neumann, Alan Turing, Alonzo Church, Moses Schönfinkel, Andrei Markov, Noam Chomsky, Emil Post, Stephen Cole Kleene or Lila Kari. Each one of them relates to an independent area in modern mathematics (c.f. Turing, 2004).

In the second half of the 20^{th} century, probably the most important theoretical results are associated with Claude Elwood Shannon, the Nobel Price laureate and the so called "father of information theory" (e.g. Shannon, 1998).

Thanks to the work of the above, and many other mathematicians, new disciplines of mathematics emerged that are today listed under computer science or formal information theory — e.g. formal logic, Theory of computation, or Code theory.

4.2 The Influence on Applied and Numerical Mathematics

Other significant areas under a strong influence of computers are numerical mathematics and, to a large extent, applied mathematics. Thanks to computers, we are able to carry out complex calculations at near-real-time speeds. In many areas, all work has moved on from theoretical research onto real-time simulations. This change consequentially imposes new demands on the calculation algorithms used, and allows us to ask a whole new range of questions. Typical examples of this include the Mathematical fluid dynamics, Dynamical systems, or the Chaos theory.

Results in the number theory field form a separate area of results which includes, for example, the methods of finding large prime numbers and factorization. These results are closely tied to the creation of modern Cryptography (Singh, 2000).

4.3 Automated Proofs

The last major area we will present here is the use of computers to prove mathematical statements. The best known example is the solution to the four colour problem (Fritsh, 1998). In this case, computers were used to research the finite number of cases (Appel, 1989).

Thanks to the development in formal logic, however, computers can also be used nowadays to create formal proofs of mathematical statements, especially in predicate logic of the first order. Perhaps the greatest achievement in this respect is the proof of the Robbins hypothesis concerning Boolean algebra (McCune, 1997).

5 Computers at School

Some of the first computers were often created at universities, or in cooperation with universities. In the early 1960s, computers started to appear at high schools. The core of the "computer education" then was mainly programming basics. With the rise of the personal computer, the content of the classes began to change. Programming and the related mathematical skills were pushed into the background and computer literacy became the new "computer science" at schools. The focus of the subject, mainly the ratio of mathematical topics to computer literacy, varies across countries. See (Impagliazzo, 2004), (Tailor, 1980) for more information on the history and use of computers in education.

6 Changes in Future Mathematics Teachers' Education

Every university training mathematics teachers must look for its own ways to reflect the rapid changes in society and mathematics that occurred towards the end of the last century. The following text attempts to describe changes that were met in the Bachelor programme for future teachers at the Faculty of Education of Charles University in Prague.

6.1 Computer Literacy

A modern teacher must be fully acquainted with the possibilities of using computers to do every day's work, he/she must be able to use computers to communicate with pupils and their parents, use them to tackle everyday administrative agenda and to prepare for lessons. A teacher should be able to use a computer as a demonstration tool every time when such a demonstration is appropriate and effective.

All student teachers at the Charles University in Prague must complete at least two subjects of the following subject base:

- Introduction to ICT
- Internet as Information and Communication Environment
- Data Presentation on PC
- Computer Graphics

In addition, the Department of Mathematics and Mathematical Education offers an optional e-learning seminar in computer literacy, based on the European Computer Driving Licence standard. Since a mathematical text has its peculiarities, the curriculum also includes a subject called Writing a mathematical text where students learn to create texts using the TEX language.

Due to the rapid development in IT, one must acknowledge that a degree of flexibility in using new technologies is much more important than mastering a single program or platform. That is why, throughout the education programme for future mathematics teachers, three different e-learning systems are used — Moodle, ActiveMath, and Microsoft Class Server. A narrow focus on only one of the above could prove counterproductive in the years to come.

6.2 A Computer as a Calculator Tool

Today, a lot of software is available to perform various mathematical calculations. These range from the simplest calculator and spreadsheet programs, to geometry programs (e.g. Cabri), to highly sophisticated programs like CAS (Maple, Mathematica) or programs for statistical calculation (e.g. Statistica). A future mathematics teacher should be aware of these tools and be able to use them to perform some of the basic tasks or operations, and to use them as a teaching tool in his/her lessons. For example, thanks to computers, statistical calculation methods can be demonstrated on real data sets of hundreds of thousands or millions of items. The trustworthiness of the data obtained is thus much greater than that of a set of 10 items processed by pen and paper.

At the Faculty of Education, the Bachelor programme focuses mainly on acquainting students with theoretical subjects. Mathematical education forms the core of the Master programme. For this reason, an independent subject called Mathematical Software B-I was included in the programme in addition to the demonstration of using various programs in other subjects, especially in applied mathematics (e.g. Applied Calculus, Numerical Methods, or Statistics and Probability) and geometry (mainly using the Cabri software).

6.3 Computer Science

The last major area we think future mathematics teachers should be made aware of is the field in modern mathematics that has the most to do with computers — computer science. Thanks to applications in information theory, students can be taught a practical use for mathematical theorems. A good example of the "usefulness" of mathematics is using Fermat's little theorem in the RSA algorithm, which is the most commonly used algorithm in public keys systems.

In the past, the studies at the Faculty of Education focused on teaching PASCAL programming. However, based on our experience, the students focused on handling the programming language itself, and the presented thoughts and algorithms were often left "untouched". In the newly accredited programme, the subject Computers and Programming was omitted and replaced by Algorithms B-I. In this subject, students are introduced to the basics of algorithm theory, like complexity or computability, and to some basic algorithms — irrespective of the programming language used. Our goal is to provide the students — future teachers — with a "meaning" for things like the Turing machine, Grammar, or recursive algorithms.

Apart from the compulsory subject Algorithms, students can choose two other optional-compulsory subjects, where they can learn about modern mathematics' achievements, and improve their skills in applying mathematics in information theory. That is the role of the subjects Applied Algebra and Computer Science, and Number theory and Cryptography.

6.4 The Master Programme

At the Faculty of Education the master programme includes advanced subjects the students became familiar with in their Bachelor programme — Algorithms M-II, and Mathematical Software M-II — as well as subjects on the didactic use of computers in teaching (it will be taught in the form of specialized seminars of Mathematical Education).

7 Conclusion

The computer is the first tool that strengthens the human mind and not the body. Thanks to computers, we have an easier access to information — we can search, process and evaluate. This tremendous potential enables man's advancement in mathematics, especially in applied mathematics. However, research in artificial intelligence and automated proofs is yielding results in a field that was solely the domain of man — thinking — and is giving rise to completely new philosophical questions, like "what is the definition of a personality", or "is a computer-computed proof acceptable".

The future teachers of mathematics, our students, will live and work in a world where computers will play an increasingly crucial role. Our goal should be to prepare them for such a world, both professionally and socially, so that not only can they use modern technology well in their lessons, but also understand the principles, the mathematics, that are applied in IT, and are able to pass that knowledge on to students of their own.

The changes described herein that were adopted for future mathematics teacher's education at the Faculty of Education of Charles University in Prague are, in our opinion, a step towards that goal.

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