History of Mathematics as a Part of Mathematical Education

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Abstract

We will present some of the findings obtained during the past nine years at the Faculty of Transportation Sciences of the Czech Technical University in Prague while teaching special courses on the history of mathematics and development of mathematical thinking. We will also discuss the role of a teacher of mathematics and history of mathematics in the motivation and education at technical schools.

1 Contemporary Situation

At present, our educational system is facing a number of severe and urgent problems. Apart from continuous financial difficulties that can be solved neither by the teachers nor students, there are and always were those that will be discussed forever. Let us mention some of them: a proper structure of instructions, standards of students' knowledge at every level of their studies, uniform "state final examinations", uniform admission examinations for particular university studies, excessive demands on the students, responsive instructions, discipline of both pupils and teachers, new didactic methods. The educational methods were already under discussion during the Austro-Hungarian empire, as well as during the so called "First Republic" in the post-war I period. In fact, they have been discussed ever since. All mass media, television, radio, newspapers and magazines talk about and examine the education level of our secondary and university graduates. A number of comparative studies, both national and international, have dealt with these questions. They usually rate our school system optimistically, although some of them are rather cautious. Our seniors, our parents, those of former generations would typically say: "In our times the situation was quite different. The students, and the university graduates in particular, had expert knowledge of high quality and general insight. They were both proficient in their specialities, as well as highly knowledgeable and broadly educated."

Currently, at the time of rapid developments in all areas of human endeavour, our university graduates are usually fairly good at having a factual knowledge, however they lack very often a deeper insight of the material they study. They lack the ability to formulate their ideas both verbally and in a written form. In fact, they have difficulties just to express and communicate their suggestions or findings even in their mother tongue. This has always been a problem for the teachers at the secondary level, however now it has become a very serious

problem for the teachers at universities. What is the origin of this problem? Clearly, it is not only the problem of the students who may have little interest in their studies or who see their education in a narrow specialization. Obviously, the performance of teachers also plays a very important role. Do all teachers request their students to present their ideas verbally? Do they ask their students to submit their homework, their essays, their critical studies, their unaided creative works in a meticulous written form? These activities, their evaluation are both time consuming and demand a lot of expertise. In many cases these aspects of teaching are very restricted or completely missing. It is imperative that the teachers put a lot of effort into preparation of homework, comprehensive marking and result analysis.

2 Teaching "Humanities" at the Faculty of Transportation Sciences

Various studies point out to the issues that we have mentioned earlier. Individual schools are trying to react to these problems. Thus, the Faculty of Transportation Sciences of the Czech Technical University has incorporated a block of courses in "humanities" (i.e. in non-technical subjects) in its program since 1996. Each term, students of the third, fourth or fifth year classes must complete at least one course of two hours a week from this block. These courses, supplemented by seminars and tutorials, are organized by individual departments. Their objective is to contribute to a cultural, historical and philosophical literacy of the future technical intelligentsia and, at the same time, to strengthen inter-disciplinary relations among various fields of study. The following courses in this category were offered at our Faculty during the 2005/2006 academic year: History of Physics, History of Mathematics, Logic and Semantic of Technical Reasoning, History of Germany, Europe in the International Relations of Modern Period, Roots of European Integration, Legislature in the Czech Republic and European Union — Protection of Health and Transport, Sociology of Production, Introduction to Sociology, Sociology of Human Resources, Psychology of Transportation in the German Speaking Countries, Culture of Speech, History of Modern Germany, World War II and its Consequences, European Integration after the World War II, Critical Moments of the Czech Nation, Democracy and Totalitarian Systems.

3 My First Experience in Teaching of *History of Mathematics*

A new one-semester course "History of Mathematics" was introduced in the elective block of "humanities" in the academic year 1998/1999.[1] The subject of the course extends the basic four-term course in Mathematics and is available to all students of the third, fourth and fifth year of study in all specialities. 80 students enrolled in the course in 1998/1999 academic year (44, 30 and 6 students from the third, fourth and fifth year, respectively). The course focused on the formation and development of the basic mathematical disciplines. It also addressed the most significant achievements of Mathematics and underlined close connections between development of Mathematics and evolution of other sciences, philosophy, medicine, architecture, music, painting and technology. The aim was to introduce Mathematics as a method, a way of exact reasoning and viewing of the world. The course tried to outline motivations that had lead people to study certain mathematical problems, to sketch ways how solutions had been tried, how the problems had been solved and applied afterwards. Finally, the presentation was enriched by pointing to specific interesting historical incidents

[1] The following topics were discussed in the 14 lectures and seminars: First traces of mathematics, birth of mathematics. Mathematics in Egypt. Mathematics in Mesopotamia. Birth of Greek mathematics — Pythagoras and his school. First crisis of mathematics and its solution. Euclid's Elements. Archimedes, Eratosthenes, Apollonius. Mathematics in the Roman Empire. Mathematics in Arabian world. Mathematics in the Medieval Europe. Medieval counting algorithms. Mathematics in the 15th and 16th century. Medieval textbooks of mathematics. Visit of the Czech National Library and Czech Technical Library.

and by providing information concerning important events or personalities. In this course, the priority was not learning the facts, but rather understanding of the progress in reasoning.

The course has its origin in need to broaden mathematical education at the Faculty of Transportation Sciences of the Czech Technical University. It contributes to strengthen the cultural element in mathematical education. Applications of Mathematics to demanding technical problems require that the students become broadly educated individuals.

The marking scheme, as well as the requirements for receiving their final course marks, were given to the students in the first class. To pass the course, students are required to submit a short paper of 6 to 10 pages on a topic from the history of Mathematics – an essay, a compilation, a critical analysis or alike. The paper should be presented in a typewritten form, with proper references to the literature used in preparations of the paper. The students were given 53 topics to choose from, all related to the material presented in the course. However, students were free to choose their own topics, as well. In the course of the term, the students were given a list of both Czech and foreign literature followed by a list of web sites dealing with the history of Mathematics, a list of relevant libraries both in and outside of Prague, as well as detailed abstracts of all thirteen classes.

The rules for receiving their final course marks astonished most of the students. Many of them would have prefered another form of final assessment. Some proposed a mandatory attendance and a test, or any other method, to examine and appraise their knowledge. They felt that such a way of examination would be easier and less time consuming. They remarked that hardly anybody has ever requested an independent written work from them.

4 History of Mathematics at the Faculty of Mathematics and Physics

The very good situation for teaching history of mathematics is at the Faculty of Mathematics and Physics of Charles University where future teachers of mathematics and physics for secondary schools are prepared because its students are interested in mathematics and its history much more than students at the Technical university. At the Department of Education of Mathematics there are taught special lectures from history of mathematics for students of Faculty of Mathematics and Physics (MFF UK), Faculty of Science (PřF UK) and Faculty of Physical Education and Sport (FTVS UK). The first course *History of mathematics I.* which is obligatory for all students — future teachers is devoted to the history of mathematics in the old ages. These topics are discussed during the 14 lectures and seminaries: 1. The beginning of the Greek philosophy and mathematics. 2. The discovery of incommensurability and its consequences. 3. The first crisis of mathematics. The way out of this crisis. 4. The famous problems of Greek antiquity. Squaring of the circle, trisection the angle, duplication of the cube. 5. "Nonclassical" solving of clasical problems. Hippokrates, Hippias, Archytas, Menaechmus, Dinostratus. 6. The problems with infinity. Zeno of Elea and his arguments about motion. Theodorus of Cyrene and Theaetetus, Eudoxus and his method of exhaustion. 7. Eudoxus, theory of proportion. 8. Socrate, Plato, Aristotle. 9. Archimedes, his life, work and activities. 10. Eratosthenes and his work. Apollonius, Claudius Ptolemy. 11. Diophantus of Alexandria and his Arithmetica. Pappus and his Mathematical Collection.

The first special optional lectures *History of mathematics* for students — future teachers from the MFF UK, PřF UK and FTVS UK are devoted to the development of mathematics in the Antiquity. These topics are discussed during the 14 lectures and seminaries: 1. Teachers of mathematics and history of mathematics. First traces of mathematics, birth of mathematics. 2. The history of ancient Egypt, Mathematics in ancient Egypt — writing, hieroglyphics, counting. 3. Arithmetic operation — addition, subtraction, multiplication, division, counting with fractions, arithmetic series and geometric progressions. 4. Algebra — the fundamental methods using by mathematicians in Egypt for solving the linear, quadratic.

5. Geometry — the fundamental methods using for solving the problems form plane and space geometry. 6. Daily life problems. 7. The history of ancient Mesopotamia, Mathematics in ancient Mesopotamia — writing, cuneiforms, notation, counting. 8. Arithmetic operation — addition, subtraction, multiplication, division, counting with fractions, arithmetic series and geometric progressions. 9. Algebra — the fundamental methods using by mathematicians in Mesopotamia for solving the linear, quadratic and cubic equations and their systems. 10. Geometry — the fundamental methods using for solving the problems form plane and space geometry. 11. Theory of numbers, calculus of interest, daily life problems. 12. Mathematics in ancient China. 13. Mathematics in ancient India.

The second special optional lectures *History of mathematics II.* for students — future teachers from the MFF UK, PřF UK and FTVS UK are devoted to the development of mathematics in the Middle Ages. These topics are discussed during the 14 lectures and seminaries: 1. The extinction of Antique World, its reasons and consequences. The last mathematicians of classical Antiquity. 2. The Middle Ages. 3. Septem artes liberales, trivium and quadrivium. 4. Church, culture and education. 5. Mathematics at the end of the 8th century. Alcuin of York, his life and activities. 6. Mathematics at the end of the 10th century. Gerbert of Aurillac — pope Silvestre II., his life and activities. 7. Mathematics in the Arabic World. The development of Arabic science. Al-Khwarizmi, Abu Kamil, Omar Khayyam. 8. Transfer of antique knowledge from Arabic World and the Byzantine Empire to Europe. 9. Mathematics at the beginning of the 13th century. Leonardo of Pisa — Fibonacci, his life and work. 10. The Middle Ages counting algorithms. 11. Universities. 12. Mathematics at the second half of the 14th century. Nicole of Oresme, his life and activities. 13. Mathematics at the 15th century. Johannes Muller — Regiomontanus, his life and activities, Luca Pacioli and his Summa de arithmetica, geometria, proportioni et proportionalita.

The third special optional lectures *History of mathematics III.* for students — future teachers from the MFF UK, PřF UK and FTVS UK are devoted to the most important events from the development of mathematics from the 16th to the 20th century. These topics are discussed during the 14 lectures and seminaries: 1. Algebra in the 16th century. 2. The development of the algebraic notation. 3. René Descartes and his era. 4. The beginning of the modern number theory. 5. The birth of the calculus. 6. The further development of the calculus. 7. Beginnings of linear algebra. 8. Complex and hypercomplex numbers. 9. Algebra in the 18th and 19th century. 10. Non-euclidean geometry. 11. Analysis in the 19th century. 12. Set theory. 13. Mathematics at the beginning of the 20th century.

These special lectures from history of mathematics are much-frequented by students, PhD students and teachers from secondary schools and universities. The detailed syllabus (in Czech) is on the lecture www page where the extensive list of references is added. The students do not write any seminar paper on a history of mathematics, they pass a classical written examination test (90 minutes).

The special optional *seminar — didactics and history of mathematics* is open to all students and PhD students. The mathematicians, specialists on the history and didactic of mathematics, teachers from secondary schools give their lectures on interesting topics. The seminar is open to all who are interested in the mathematics, its history and teaching and it is very much-frequented by teachers from practice. The students do not write any seminar paper on a history of mathematics and they do not pass any classical written or oral examination tests. I believe that their works during the seminars for example discussions, questions, presentations their ideas and experiences are the most important activities.

At the Department of Education of Mathematics at the Faculty of Mathematics and Physics Charles University the PhD studies in *General questions of mathematics and information science* were opened in the school year 1992/1993 which are very popular in our country now because of the changing in our educational system the young teachers make an effort to improve and enlarge their qualification and knowledge. The Department of Educa-

tion of Mathematics at the Faculty of Mathematics and Physics Charles University is one of two places in the Czech republic where the history of mathematics can be studied as a deeper specialization. The studies are divided into three areas: Elementary mathematics, History of mathematics and information science, Teaching mathematics and information science. The PhD studies in *General problems* are aimed at secondary teachers who graduate with mathematics or information science as a teaching subject and at university teachers who teach mathematics or information science or didactics of these subjects. An individual study plan is prepared for each student. It contains the common elements of all three areas, deeper studies in the chosen area and a section directly connected with the proposed thesis topic.[2]

5 My Experiences and Outcome

During the summer examination periods of the 1998/1999 academic year and of the winter semester 1999/2000, I received from the students 87 project; I read and marked each of these projects very carefully. After taking about 30 to 40 minutes to do that, I spent another 10 to 15 minutes with every student to check orally his or her knowledge of subject. During this discussion, I learnt why the students chose particular topics, what determined the way they had treated them, how much time they spent on the projects and what they found most satisfying in the process.

Only 59 projects received a passing mark. Indeed, I found 29 projects (i.e. 33,3 %) entirely unsatisfactory. They were either reproductions of easily accessible documents or copies of some other student's work. Only 8 projects could have been classified as excellent. They were characterized by a perfect theme handling, excellent language and form, and they contained new ideas. There are the topics of the best projects:

1. The birth of counting or why do we express the names of numbers as we do?
2. Can we solve the quadratic equations with the rule and compass?
3. The Pythagoras's theorem in the Mesopotamia, Egypt and Greece.
4. The oldest Czech textbook on Arithmetic — historical analysis of teaching of arithmetic series.[3]
5. Banking and counting in Mesopotamia — analysis of the texts with problems of calculus of interest.

The seminars papers are in the teacher's archive, they are available for future students.

Other projects, although correct in content and mathematics, were incredibly full of spelling errors (errors in words spelt with "y" rather than "i"; ignorance of subject and predicate match, mistaken syntax, errors in capital letters etc.), stylistic mistakes, typing errors etc. It's a wonder how many spelling errors can be done by a high school or a university student. It was evident that some students wrote their project in a hurry, often even without any final reading.

There were students who did not submit a project at all (9 from the third and 4 from the fourth year class, i.e. 15,5 %); they may have left the course altogether. Some of the unsuccessful students have written a second project (16 students, i.e. 20,0 %), one of them has written even a third project (1 student, i.e. 1,25 %). The following table provides a summary of the statistical data.

[2]For more information see http://www.karlin.mff.cuni.cz/∼becvar.

[3]Ondřej Klatovský: *Nové knížky vo počtech na cifry a na liny*, Praha, 1558.

Year 1998/1999	Students	1st work	2nd work	3rd work	Got credit	Did not get credit
3. class	44	35	7	1	28	16
4. class	30	29	8	0	25	5
5. class	6	6	1	0	6	0

It is interesting that most of the best projects were written by students from the fifth class who, at the same time, worked on finishing their diploma work and who had already finished several seminary or class projects earlier. The worst projects were written by students from the third class. They admitted that this had been one of their first written project that required an individual study of literature and independent formulation of their own ideas. It was simply not sufficient just to copy a part of the lectures or teaching materials.

6 Later School Years — Some Changes and Improvements

I was surprised how many students signed up to the History of Mathematics in the summer semester in 1999/2000 academic year. In view of the fact that correcting of the final projects required a lot of time I had to allow only 24 students to register. Let me point out that these students chose the course knowing from the faculty web site very well what they may expect in the way of course requirements. However, I hasten to add that, due to ever increasing demands of the students, I have allowed, since 2000/2001 academic year, that the final project be replaced by a regular attendance of the classes, an active participation in discussions and a short final test. The students should pass a classical written examination test (60 minute). They had to choose three questions from the test or their three best answers were classified. They could bring and use all materials which they prepared themselves (notes and comments, copies of my lectures, some books or papers from www pages). They could pass the test only once. There is one example of the test written in 2001/2002:

1. Describe and explain in detail counting algorithms for multiplication and division using by Egyptian scribes. Use some typical examples.

2. By the rule and compass solve the equation $8x - x^2 = 4$. Explain and prove your method.

3. Give the short description of fundaments of numerical systems using in Egypt, Mesopotamia, Greece, Roman Empire and the Middle Ages.

4. Calculate $1\,324 \times 589$. Explain three methods using by Middle Ages European calculators. Compare them with our algorithm.

5. The Pythagoras' theorem — its history and at least two proofs.

The following tables present the statistical data of the later academic years.

Year 1999/2000	Students	1st work	2nd work	3rd work	Got credit	Did not get credit
3. class	13	13	2	0	13	0
4. class	7	7	1	0	7	0
5. class	4	4	1	0	4	0

Year 2000/2001	Students	Test	1st work	2nd work	Got credit	Did not get credit
3. class	10	3	5	1	6	4
4. class	5	0	5	0	5	0
5. class	3	1	2	1	3	0
6. class	1	0	1	0	1	0

Year 2001/2002	Students	Test	1st work	2nd work	Got credit	Did not get credit
3. class	9	7	1	0	6	3
4. class	1	0	1	0	1	0
5. class	1	0	1	0	1	0

Year 2002/2003	Students	Test	1st work	2nd work	Got credit	Did not get credit
3. class	8	5	3	0	8	0
4. class	5	2	2	0	4	1
5. class	0	0	0	0	0	0

The History of Mathematics was not offered in the summer semester in 2003/2004 academic year.

Year 2004/2005	Students	Test	1st work	2nd work	Got credit	Did not get credit
3. class	4	0	4	0	4	0
4. class	6	0	5	0	3	3
5. class	4	2	0	0	1	3

Year 2005/2006	Students	Test	1st work	2nd work	Got credit	Did not get credit
3. class	0	0	0	0	0	0
4. class	19	0	15	0	14	5
5. class	9	1	6	1	4	5

The History of Mathematics was not offered in the summer semester in 2006/2007 academic year.

7 Web Site Available to the Students

Since 2000/2001 academic year, the students of my course History of Mathematics can find all information concerning the course, including weekly course summaries and suggested topics for the final project, on my web site

http://www.fd.cvut.cz/personal/nemcova/qhm.htm.

The site provides also motivations and justification for individual subjects covered in the classes, as well as brief exposition on relationships between Mathematics and other fields of human endeavor. The web site has proved to be of a significant assistance to the students.

8 Concluding Observations

The above experiment demonstrates that the work in the course and mainly writing the project, the teacher's work with the students on developing their projects contribute to students' ability to formulate their ideas and theories fairly well, both orally and in a written form. Moreover, they can get a better preparation for their future, often ambitious jobs that will indisputably require both good expert knowledge and good managerial experience. One has to acknowledge candidly that this form of work has been very demanding both for the students, and above all for the teachers. However, I believe that all the efforts made in this venture are highly worthy and laudable and will bear their fruits. Thus, it depends on each of us whether we choose to follow this route.

References

– Bečvářová, M., 2000, "Humanizace technického školství aneb výsledky malého experimentu", *Aula* **8**, 21–24.

– Personal www page of Martina Bečvářová:
http://www.fd.cvut.cz/personal/nemcova/qhm.htm. [26. 8. 2007]

– Personal www page of Jindřich Bečvář:
http://www.karlin.mff.cuni.cz/∼becvar [26. 8. 2007]