Various Materials for Primary School Teacher Training

Or: Can you do *something* even if you can't do much?

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Abstract

In my pre-service courses for primary and lower secondary school teachers, I include history of mathematics in several ways. In this workshop, I give examples of several of these, and discuss the choices I have made. In particular, I discuss to what extent it is possible to include bits of history of mathematics even to students with no prior knowledge of history of mathematics.

1 Introduction

In this paper, I will describe my context (Norwegian pre-service teacher training) and give some examples of different ways I work on history of mathematics. A major part of the paper will be spent on looking at some of the materials I have used with students and discussing these materials.

As subtitle of this talk, I have chosen "Can you do *something* even if you can't do much?" In conferences such as this, we get to see wonderful examples of how rich a resource the history of mathematics can be, but often I am left with the question "Will I have time to do this with my students?" A dedicated history of mathematics course would have been great for prospective mathematics teachers – but when they can't have that, what can they have?

2 Background

I teach a course in mathematics for prospective primary and lower secondary school teachers. The course lasts for two years, and is supposed to occupy a fourth of the students' time for that period. After doing this course, students are expected to be able to teach mathematics from grade 1 to 10 in the Norwegian school system — in itself an optimistic expectation.

There are certain important factors that have to be taken into account when planning such a course. When it comes to history of mathematics, students usually know very little in advance. The time is so limited that we can't give an overview of the history of mathematics – everything we do on history of mathematics must be part of a broader treatment of mathematics. The mathematics we study is mostly at the level of lower secondary and lower. Moreover, my students generally do not enjoy working in other languages than Norwegian. On the other hand, my students are to become teachers, so they should be interested in anything that can enhance their teaching.

Nonetheless, I would like to include some history of mathematics and try to reach the following goals: I want my students

- to see that the problems they have or their pupils have, also have been present for the mathematicians of past history
- to get a general sense that mathematics has developed and give mathematics a human and cultural dimension
- to see different ways in which history of mathematics can be included in teaching (even as games!)
- to know that questions about the origin of mathematical words, usually have an answer, often even an interesting one (etymology)

Even though my students do not get a course in history of mathematics, I want them to get a taste of history of mathematics and a wish to learn more.

Previous studies that I've done, give me two important insights:

1. History of mathematics easily becomes just biography when prepared for the classroom. This is shown both in my analysis of Norwegian textbooks (Smestad 2003) and in my analysis of 638 mathematics lessons in 7 countries from the TIMSS Video Study (Smestad, 2004). Therefore, it is important for me to work on "real mathematics", not just to give anecdotes or biographies.
2. History of mathematics is sometimes seen as "taking time away from the mathematics". This view is expressed by teachers in an interview study I am doing. Therefore, it is important to show the prospective teachers how history of mathematics can add value to the mathematics teaching, also from a purely mathematical point of view.[1]

I should add that I have no ambition of being original – except in a purely local sense. I am happy to pick up ideas from conferences and articles to enrich my teaching, as long as my students have not seen the material before. Therefore, many of the examples in this paper may be familiar.

3 Ways of working with history of mathematics

In my teaching, I have included history of mathematics in several different ways. I have included historical information in lectures. I have been working on original sources. My students have done projects in which they have connected the history of mathematics to activities for pupils. I have given my students tasks from history and I have also created an etymology game. I will give examples of all of these, but mainly, we will look at tasks I've given my students.

There are, of course, several other possibilities which I have not explored, for instance having historical/mathematical plays or historical/mathematical exhibitions (see Funda Gonulates' and Oscar João Abdounur's presentations at this conference). Many possibilities are also described in the ICMI Study (Fauvel/van Maanen, 2000).

3.1 As part of a lecture

I mention history of mathematics in many of my lectures, and spend some time on Al-Khwarizmi (see Michael Glaubitz' presentation at this conference), on the history of measurements, on Platonic solids, on Erathostenes' calculation of the circumference of the Earth, on equations (for instance Tartaglia and Abel) and on Florence Nightingale and the use of statistics. However, here I will give an example from a lecture on the history of perspective

[1] See Siu (2004) for more reasons *not* to use history of mathematics in the classroom.

drawing. In Norway, perspective drawing is a part of the mathematics curriculum in both primary and secondary school.

First, I show an example of Egyptian art, for instance from the 4 000 year old tomb of Khnumhotep in Beni Hasan. The students will notice that although some parts of the painting are very naturalistic, other parts are not – and it is clear that it was never the intention of the artist to depict the world exactly as it is. Two men have very different height, even though they are standing in the same boat. Obviously, this is not because of the artist's lack of skill, but because the artist wanted to show that one of the men was more important than the other. I show this example to make the students aware that it would be misguided to judge paintings based on what we think is "right" or "wrong".

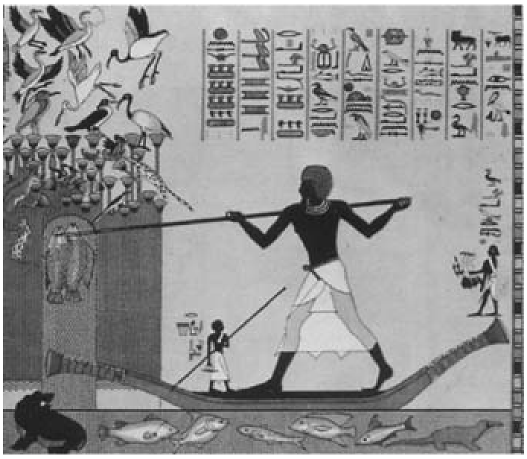


Figure 1 – From the tomb of Khnumhotep

Figure 2 – Upper Rhenish Master: The Little Garden of Paradise

Then I go on to a few other paintings, for instance "The Little Garden of Paradise" by an unknown artist ("Upper Rhenish Master"). This painting, which is in the Städel Museum in Frankfurt, is painted about 1410. By that time the intention had changed. It is not unfair to say that the painter wanted to portray the world as it is, or rather, as it could have been in such a garden. When asked if there is something odd about the picture, the students immediately say that there is something a bit "wrong" with the table, or with the well or with the walls. The table, for instance, is seen slightly from above, while the rest (including the glass on the table) is seen more from the side.

Figure 3 – Raphael: The School of Athens

Figure 4 – Max Beckmann: Synagogue

Then, of course, we look at a few paintings which are "perfect", for instance Raphael's "The School of Athens" (1509–1510) in The Vatican Museum. By this time, those students who don't know (or remember) how to draw in perspective, are intrigued. Therefore, I give them a copy of a painting (or a simpler perspective drawing), and ask them to figure out what is going on – which geometrical properties are the same in the drawing as in the real world it portrays, and which are not? This leads to discussion on concepts such as lines, parallels, angles and so on. Even if the students already know the laws of perspective, they will probably see that this is a possible way of introducing it to pupils in school – to let the pupils "discover" the rules.

After looking at the rules a bit closer, seeing a few more examples and doing a few drawings, we go on to looking at some examples of later art. Picasso rejected the single viewpoint, and instead painted objects as seen from several points of view at the same time. Escher played with perspective to create "impossible" drawings. My favourite, however, is Max Beckmann's "Synagogue" from 1919 (which is also in the Städel Museum). Max Beckmann knew very well how to draw a building in perfect perspective. The whole point was, however, that after the terrible war, nothing was perfect anymore. The painter consciously breaks the rules to get the effect he wants. These examples illustrate another important point: to draw in perspective, you need certain skills. Having these skills, however, does not mean that you have to use them. The skills give you more choice, also to create new effects.

What is the point of including the history when teaching perspective? In my opinion, there are many points. For instance, the students see that this particular part of mathematics has developed over time as people met artistic and mathematical problems that they needed to solve. They also see that there are important connections between mathematics and arts. This last point may be particularly important for some of the pupils or students who feel that mathematics is too "sterile". Moreover, the pre-perspective paintings give a motivation to learn how to avoid those "mistakes" – to become "better than these ancient masters".

3.2 Working on original sources

As mentioned earlier, my students prefer texts in Norwegian, which means that there are not many authentic original sources to choose from. Moreover, working on original sources tends to take more time than we have available (even though I acknowledge the great value in doing it).[2] Consequently, I don't do this very often. The original source I use most often is the beginning of Leonardo Pisano (Fibonacci)'s "Liber abaci" (1202). The text shows how Fibonacci had to explain the Hindu Arabic numerals to great length to make them understandable to his contemporary public. Students tend to have a feeling that numerals and number systems are simple, perhaps because they have understood them for such a long time. Working on Fibonacci makes the students see that these topics indeed are difficult — teachers should not be surprised that pupils have to struggle to understand them. Moreover, students see how mathematics has developed. Fibonacci also gives a good opportunity (among many) to illustrate the central role of non-European cultures in the development of the mathematics we do in school.

There is time for a little warning on translations, which I will also come back to later: I give my students a translation which I have done from an English translation. This is certainly not optimal, but the alternative is to wait for some scholar to do the translation into Norwegian. In this particular context, I think the main points will be kept in the translation, and that the outcome for the students is better than if I did not use Fibonacci at all.

[2] See Bekken et al (2004) and Clark et al (2006) for more on using original sources in teaching.

3.3 Projects

I will not say much about projects. That is not because projects are not valuable — they are — but because I find them to be difficult in my context. At times, my students have done wonderful projects where they have connected the history of mathematics to teaching in imaginative ways, but it does take quite a lot of time and tends to involve colleagues in parallel classes and preferably also my students' pupils in schools. So in the spirit of the subtitle ("Can you do *something* even if you can't do much?"), projects may not be the place to start.

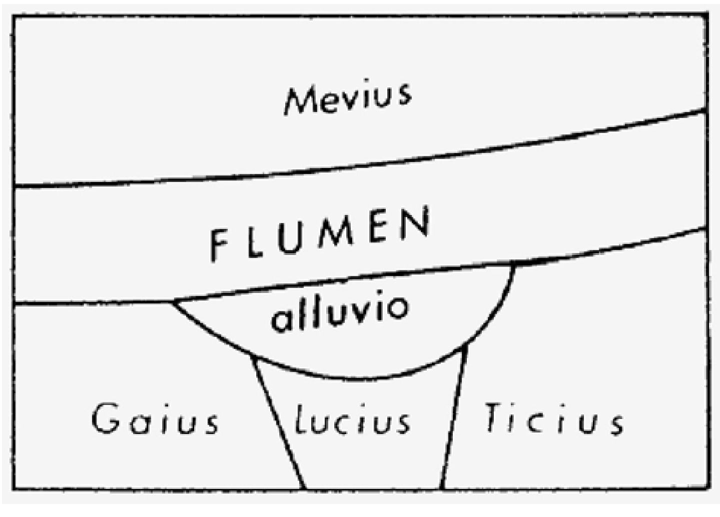
3.4 Tasks

Most of the rest of this workshop will concentrate on tasks given to students as part of their normal work in mathematics. The tasks all have mathematical content, which means that they can not be seen as "taking time away from the mathematics", and they give the students an opportunity to discuss the problems in groups. Here are some selected examples. Many more are in the worksheets handed out in the workshop, and even there I chose not to include topics such as numeral systems or unit fractions, for instance. Please note that the tasks have been translated from Norwegian for the workshop. This may have added inaccuracies compared to the worksheets actually used with students.

Geometry and medieval lawyers

Before this activity, students have been given a translation of Jan van Maanen's article "Teaching geometry to 11 year old 'medieval lawyers'" (van Maanen 1992), in which he describes pupils working on a juridical document from 1355. In this document, division of new land (for instance formed by alluvial deposits) is discussed, and the following general principle is established: New land belongs to the owner of the nearest old land. The article goes on to explain that the borders can be found by bisecting angles, and gives a few examples of tasks.

Task 1
This figure is taken from the article by van Maanen. Mark the new borders in the new land (marked as "alluvio").



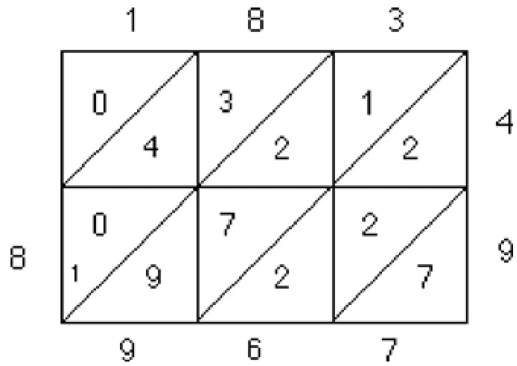
Comment: This well-known task is good for several reasons. Many Norwegian pupils learn to bisect angles mechanically, as a procedure, without ever discussing what the properties of the points on the line that bisects the angle are. By working on this kind of problem, the properties are in focus. Problems such as these also show that even this kind of geometry (ruler and compass constructions) is useful, and they present possibilities for connections to other subjects. Even discussions on what is a "fair" division can be included.

I also give a non-historical task about five farms scattered on an island, where the perpendicular bisector is useful for dividing the island between the farms in a "fair" way (by one definition of the word fair). Here, however, there is no historical component, and I think there are signs that the students find the problem a bit too "constructed".

Algorithms for multiplication through time

Task
On this page are different algorithms for multiplication. For every algorithm, I want you to try to understand the procedure. Use the same algorithm to calculate $265 \cdot 38$. Try to understand *why* the algorithm gives the right answer, and what may be the advantages and disadvantages of the algorithm.

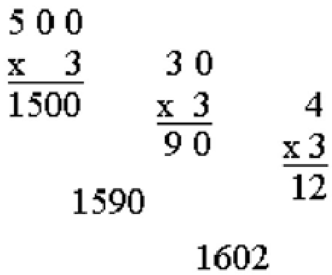
a) "Gelosia method": This method is found in *Lilavati* (by the Indian mathematician Bhaskara, who lived around 1150). The method came to Europe via Arab manuscripts, and was found in printed textbooks until the 1700s. Here, $183 \cdot 49$ is calculated:



b) "Russian Peasant Multiplication": The method is called "Russian peasant multiplication" because it was used in rural communities in Russia all the way into modern time. But this is essentially the same method as the old Egyptians used, four thousand years ago. Here, $183 \cdot 49$ is calculated:

Halves	**Doubles**
49	183
~~24~~	~~366~~
~~12~~	~~732~~
~~6~~	~~1 464~~
3	2 928
1	5 856
	8 967

c) Here is a third method, used by Eutocius of Ascalon (ca. 500 AD). Here, $534 \cdot 3$ is calculated:



Comment: Many students enter my mathematics course with an opinion that there is only one way of doing multiplication — the algorithm they have been taught in school. It is essential that they learn to appreciate other methods and to understand the unfamiliar, as they will later meet pupils that are doing things in their own ways — either through their own invention or through education elsewhere. The first and the third algorithm also help us see clearer why "our" algorithm is working. The second algorithm is interesting because it is not "basically the same" as ours. The algorithms also show how multiplication has been done in other cultures, and make it possible for us to discuss what in our culture makes our algorithm a good one for us. (For Egyptians, doubling and division by two were the basic operations, which meant that their algorithm was good for them.)

Problems from probability theory

For the work on probability theory, I have written a booklet which includes historical notes and problems. I have chosen some problems from this booklet for discussion here. I chose to avoid the "problem of points", which has been discussed in detail by Chorlay and Brin at this conference.

Task 4–7

"Supposing a tree fell down, Pooh,
when we were underneath it?",
Piglet asked.
"Supposing it didn't",
said Winnie-the-Pooh.

a) Is there anything so improbable that we don't worry about it, even though the probability is not equal to zero?

b) In his 1777 *Essai d'Arithmétique Morale*, Buffon argues that probabilities less than 1/10 000 cannot be distinguished from a probability of 0. He argues that the probability that a 56 year old man will die in the course of one day (according to his tables) is about 1/10 000, while such a man in reality regards the probability as 0. (A similar argument was given by d'Alembert earlier.) What do you think of such a reasoning?

Task 4–12 (The St. Petersburg problem)
A classical problem from probability theory is the following:

a) A throws a coin. If head turns up on the first throw he gets one ducat from B, if head does not turn up until the second throw, he gets 2 ducats from B, if head does not turn up until the third throw, he gets 4 ducats from B and so on (getting 2^{n-1} ducats if head doesn't turn up until the nth throw). Calculate the expected value for A.

   (The problem is called the St. Petersburg problem simply because it was first published in St. Petersburg — by Daniel Bernoulli. It is, however, Nicolas Bernoulli who is credited for first posing the problem, in 1713.)

b) How much should A be willing to pay to play this game? Would you?

c) What will the expected value be if we assume that B has only a limited sum to give to A — for instance 10 000 ducats?

d) Some of the "problem" with this problem is probably that we would rather not pay a very large amount of money to have a tiny chance of winning an incredible amount of money. Some (for instance Cramer) have tried to cut the knot by saying that for all practical purposes, winning ten million ducats is not ten times as valuable as winning one million ducats — I won't get that much happier by the additional nine million ducats. Do you see why this "cuts the knot"?

e) Others (for instance d'Alembert) have cut the same knot by saying that probabilities less than for instance 1/10 000 can just as well be regarded as 0. Do you see how this "cuts the knot"?

f) Still others (for instance Buffon) argued that there are limits to how many throws you have the time for in the span of one life — therefore the number of throws must be limited. Do you see how this "cuts the knot"?

g) Why have so many good mathematicians used so much energy on "explaining away" the result in a), do you think?

Comment: Students often have an intuitive feeling that small probabilities are unimportant, and that probability theory is about saying if something is "probable" or "not probable". These two problems show that small probabilities may be very important — that is also part of the reason for the opposition to nuclear energy, for instance. However, this problem also shows that there may at times be a gap between the theoretical and the practical (because of the formulation of the problem), and this gap, which leads to counterintuitive results, needs to be bridged. We see how mathematicians struggled to bridge it. More mathematically: when students really understand why each of the modifications of the original problem leads to a finite answer, they have surely understood important parts of the concept of expected value.

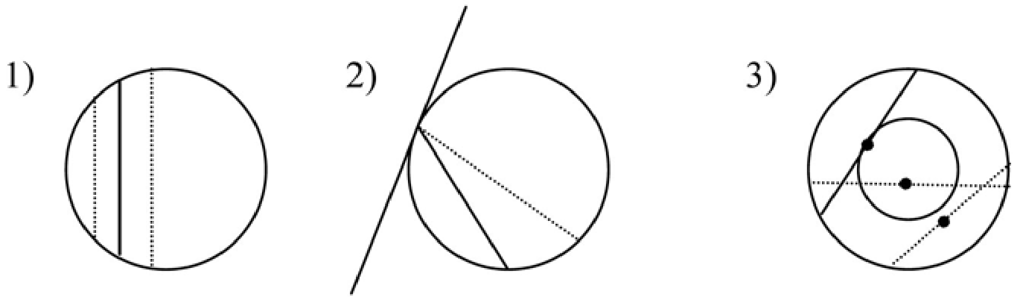
Task 2–22

Bertrand has a paradox, the so-called "chord paradox", which was published in 1888. The question is simple: You have a circle of radius 1 cm, and choose a random chord. What is the probability that this chord has a length greater than $\sqrt{3}$ cm? You can try to answer before reading the following three alternative answers:

1. Because of symmetry, we can assume that the chord has a particular direction, for instance that it is vertical. With a little calculation, we see that only the chords which are less than 1/2 cm from the centre of the circle, have a length greater than $\sqrt{3}$ cm, while the ones that are more than 1/2 cm from the centre, will have a shorter length. Therefore, half of the distances give the length we want, so the probability is 1/2.
2. Because of symmetry, we can choose a point on the circle, which the chord should touch. The question is then only which angle the chord should have to the tangent of the circle. A little calculation shows that only the chords which have angles greater than 60 degrees to the tangent, has a length greater than $\sqrt{3}$ cm. Out of 180 degrees, there is only a sector of 60 degrees that gives the length we want, so the probability is 1/3.
3. To choose a chord randomly is equivalent to choosing the midpoint M on the chord. The chord will only get a length of $\sqrt{3}$ cm (or more) if M is inside a circle with radius 1/2. This circle has only a fourth the area of the bigger circle. Therefore, the probability is 1/4.

Which of these alternatives is correct? (Or may all of them be correct?)

Comment: This problem is important in showing that "randomness" is not an easy concept — it must sometimes be carefully defined. It is not always obvious what it means to be "picked randomly". Probably a bit more context would be useful to see if the mathematicians at the time of Bertrand found this puzzling or just entertaining.



Task 4–5
In the saga of Olaf the Holy, chapter 97, the following story is told:

Figure 5 – Drawing by Erik Werenskiold

"Thorstein Frode relates of this meeting, that there was an inhabited district in Hising which had sometimes belonged to Norway, and sometimes to Gautland. The kings came to the agreement between themselves that they would cast lots by the dice to determine who should have this property, and that he who threw the highest should have the district. The Swedish king threw two sixes, and said King Olaf need scarcely throw. He replied, while shaking the dice in his hand, 'Although there be two sixes on the dice, it would be easy, sire, for God Almighty to let them turn up in my favour.' Then he threw, and had sixes also. Now the Swedish king threw again, and had again two sixes. Olaf king of Norway then threw, and had six upon one dice, and the other split in two, so as to make seven eyes in all upon it; and the district was adjudged to the king of Norway. We have heard nothing else of any interest that took place at this meeting; and the kings separated as the dearest of friends with each other."

a) What is the probability of getting a double six three times in a row, as is related here?

b) Do you think that the kings wanted chance to decide, or did they see some other significance in the throws of dice?

Comment: A basic assumption in our work on probability in school is that some events are random — such as throwing dice. Obviously, not everybody agrees on this. The outcome of a throw may be ascribed to gods or to what is perceived as "fair". (Even today, we talk of "fair dice", even though the outcomes often seems unfair to the loser.) Discussing such competing assumptions will be important for the students when they become teachers, so they should be made aware of them. Of course, Olaf the Holy's saga has the added benefit of being available in Norwegian.

Pascal's triangle

Because of space restrictions I cannot include the three pages of worksheets on Pascal's triangle. However, the main point of interest may be that students are (again) made aware that far from all mathematics is European, and that even mathematics bearing European names may in fact have other origins.

3.5 A game

Teacher education (almost) always serves two purposes: to improve the students' content knowledge and to provide examples of how teaching can be done. I try to provide my students with examples of different ways of including history of mathematics, even by making an etymology game. The game is quite simple, and the main point is that the player is given part of the etymology of a word, and is to guess which word it is. The reaction of students have been interesting: some students complain that they can't do it, because we have never studied etymologies, while others get fascinated. I have no ambition that my students will learn the etymology of lots of words during my course, but I want them to remember that every word has a background, and many of these backgrounds cast light upon the concept — sometimes from a surprising angle. A teacher should not be uninterested in the origins of the words he's teaching.

For instance, students are interested to see that the word "trigonometry", which they mostly associate with abstract functions, comes from Greek words for "triangle" and "measure". The word "interval" comes from Latin and means something like "a place between the walls" — that is immediately understandable. That "asymptote" has its root in something meaning "not to meet" is also quite reasonable.

4 Discussion

In the discussion, it was pointed out to me that one of the etymologies given in the game was wrong. That error is regrettable. But it also points back to the subtitle of this workshop. For me as a teacher to make an etymology game, I have to rely on sources such as etymological dictionaries, which themselves have errors. For me to research every word would make the process too time-consuming. The same goes for almost everything else I do — when including history of mathematics in my teaching, I rarely go back to the primary sources, but instead often rely on secondary sources (although preferably not only one). I think that trying to include history of mathematics with as few errors as possible is better than not including it at all. When teachers show such attempts to each other, the materials will both be used more widely and, we might hope, corrected.

My answer to the question in the subtitle ("Can you do *something* even if you can't do much?"), is "yes". I do believe that it is possible to include history of mathematics that may light the interest of some students, even if you don't have the opportunity of doing everything you would have wanted. And it is possible to create resources based on others' ideas, removing the need to research everything from scratch. I have tried to show some of my work, and hope that the ideas that are available will multiply in the years to come, so that still more teachers who are not experts in the history of mathematics, will feel able to enrich their teaching in this way.

My PowerPoint presentation for this workshop (and my other workshop materials) can be found at http://home.hio.no/~bjorsme/prague.htm.

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