Today's Mathematical News Are Tomorrow's History

Interweaving Math News Snapshots in the Teaching of High School Math

Nitsa MOVSHOVITZ-HADAR

Technion – Israel Institute of technology, Haifa 32000, Israel

nitsa@tx.technion.ac.il

Abstract

School mathematics generally reflects neither the ever growing nature of the field, nor the steady struggle of mathematicians for establishing new results. Consequently, high school graduates leave school having the wrong image of mathematics as a discipline in which all answers are known, leaving little room for further exploration. This non-constructive conception of mathematics is henceforth spread around to the public and keeps the majority hating it on the one hand, while blindly admiring those weird ones who find it intriguing, on the other. Interweaving snapshots of mathematical news in the daily teaching of high school mathematics is proposed as a cure. This paper presents five different types of math news illustrated by fascinating and accessible examples for considering their interweaving as snapshots in the teaching of high school mathematics. ESU5 Prague workshop focused on this proposal. Participants collaborated looking for updated math news on the web, discussed the need, values and appropriate pedagogy for introducing math news in the classroom, and considered the dilemma and efforts involved in interweaving snapshots of mathematical news in the daily teaching of high school mathematics. This paper shares the main ideas and calls for international collaboration in coping with the dilemma. It claims the proposed idea to be worth the effort as it fits ESU Aim and Focus statement. Moreover, it is believed that the suggested approach can help boost teachers' ego and self esteem as well as fight speedy burnout, so common among teachers after several years in the profession.

1 The Ever Growing Nature of Mathematics

Mathematics has been for long, a highly prolific discipline. Beyond its glorious past, it has a vivid present and a promising future. New results are published on a regular basis in the professional journals; new problems are created and added to a plethora of yet unsolved problems, which challenge mathematicians and occupy their minds.

These facts come across in a vivid way in the June/July 2007 issue of the Notices of the American Mathematical Society published just before the Prague ESU5 meeting convened. In a paper by Agnes M. Herzberg and M. Ram Murty entitled: *Sudoku Squares and Chromatic*

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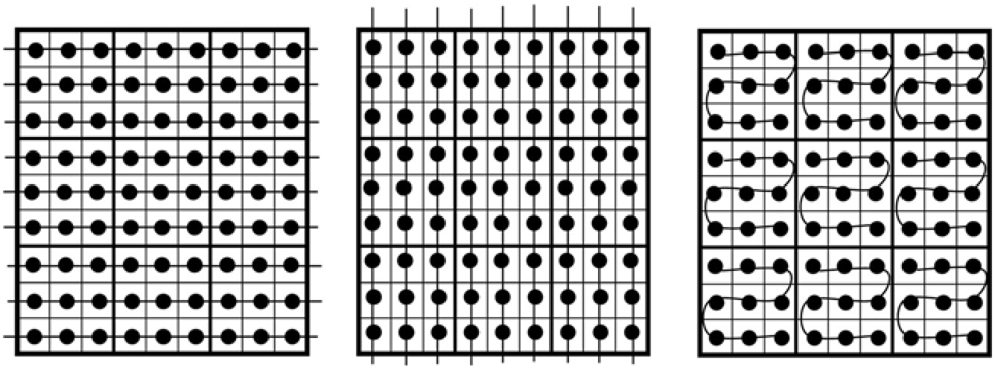
Polynomials, the authors present an amazing idea which leads to some surprising results and a few open problems about this popular puzzle[1].

Herzberg and Murty represent the problem of solving a Sudoku Puzzle, in the language of Graph Theory: The 81 squares in the grid correspond to vertices in a mathematical graph. A line connects vertices that appear in the same row (Fig. 1a), column (Fig. 1b), or sub-grid (Fig. 1c). Finally, nine different colors replace the 9 digits.



a) b) c)

Figure 1

From here Herzberg and Murty get that (i) A Sudoku Puzzle, in Graph Theory terms, is a *partial* coloring, as at the start, just a few vertices (i.e. squares) are colored (i.e. numbered). (ii) A Sudoku puzzle is solved, once *all* vertices (squares) are colored, such that no two line-connected vertices have the same color (number). This is called *proper* coloring. Thus, in Graph Theory terms, Sudoku means: Extending a partial coloring to a proper coloring of the vertices.

Using tools from *Graph Theory* and the general *Latin Squares* studies, the two Canadian mathematicians proved, among other things, that the number of *different* solvable Sudoku Puzzles is in the billions (Solvable meaning — having at least one solution.) They also proved that given a partial coloring of a graph, the number of ways of completing the coloring to obtain a proper coloring, using at least the number of colors in the partial coloring, is determined by a polynomial in this number of colors. Interestingly, their work is related to a few *unsolved* problems about Sudoku. Two of them are:

1. A constructive existence proof

   As mentioned above, Herzberg and Murty showed that there exists a polynomial which determines the number of possible solutions (extensions to proper coloring) for a given

[1] A Sudoku puzzle is a $3^2 \cdot 3^2$ $(9 \cdot 9)$ grid forming 81 squares, subdivided into nine 3x3 sub-grids. A few numerals between 1–9 are positioned, one in each square. For example:

5	3			7				
6			1	9	5			
	9	8					6	
8				6				3
4			8		3			1
7				2				6
	6					2	8	
			4	1	9			5
				8			7	9

The task is to insert the numerals 1–9 one in each of the 81 boxes such that no row, column or sub-grid includes 2 equal numbers. In the given example, see if you can justify that the number to be placed in the center box (row 5, column 5) must be 5.

Sudoku Puzzle. What a relief this could provide to a persistent but tired Sudoku-solver, who wishes to make sure that a certain puzzle really has at least one solution and that there isn't more than one. Unfortunately, although they proved that such a formula exists, they were unable to figure it out. Who will do it? When? And most important: how? — This is yet unknown.

2. The Minimum Sudoku problem

   What is the minimum number of given entries needed to ensure that a Sudoku Puzzle has a unique solution? A Sudoku Puzzle with just 17 given entries, that has exactly one solution, is known. (In fact there are people who collect only these Sudoku and one of them has almost 50 000 of them on file[2].) Hence, the minimum number is at most 17. But could it be 16? Or even less maybe? — This is yet unknown. By the way: Is it true that the more entries are given, the likelier it is for a puzzle to have a unique solution? Not really. Herzberg and Murty show a puzzle with 29 given entries, and prove that it has two *different* solutions. This is really counterintuitive and hence surprising, as one would be tempted to claim to the contrary and attempt a proof by mathematical induction...

It is worth noting that Herzberg and Murty treat the ordinary $3^2 \cdot 3^2$ Sudoku, as a particular case of the $n^2 \cdot n^2$ grid.

2 The Yet Unknown in Mathematics

As (almost) nothing becomes obsolete in mathematics, the ever growing, accumulative nature of this discipline has an enormous impact on its learning and its teaching. We'll discuss recently solved problem in the following sections. At this point, if you wish to familiarize yourself with a few yet unsolved mathematics problems (new or old) you may be glad to realize that surfing the web is a good vehicle. Here is a (partial) list of useful URLs which are updated periodically for this purpose:

- http://www.answers.com/topic/unsolved-problems-in-mathematics?cat=technology/
- http://www.claymath.org/millennium/
- http://mathworld.wolfram.com/UnsolvedProblems.html
- http://en.wikipedia.org/wiki/Unsolved_problems_in_mathematics
- http://www.mathsoft.com/mathsoft_resources/unsolved_problems/
- http://www.math.fau.edu/locke/Unsolved.htm

Note: ESU5 workshop participant shared their web-surfing findings with others in their group, reflected upon their experience and reported to the whole group.

3 School Mathematics vs. Mathematics

School mathematics all over the world does not reflect the ever growing open ended nature of the field. Nor does it expose students to the steady struggle of mathematicians for solving open problems, and establishing new results. Consequently, students graduate high school having the (wrong) image of math as a "dead end" discipline, in which all answers are known, and nothing curious is left for their creative exploration. The logician and math-educator at U.C. Berkeley, Prof. Leon Henkin (1921–2006) put it in his witty style:

[2] See for example, http://people.csse.uwa.edu.au/gordon/sudokumin.php by Gordon Royle of The University of Western Australia

> One of the big misapprehensions about mathematics that we perpetrate in our classrooms is that the teacher always seems to know the answer to any problem that is discussed. This gives students the idea that there is a book somewhere with all the right answers to all of the interesting questions, and that teachers know those answers. And if one could get hold of the book, one would have everything settled. That's so unlike the true nature of mathematics. (Steen, L. A., Albers, D. J., 1981)

The wide gap between school curriculum and the true nature of contemporary mathematics poses a cause for concern. As one way for bridging between the two, **this paper (and the ESU5 workshop it is based upon) proposes interweaving snapshots of mathematical news in the daily teaching of high school mathematics**. This proposal stands on the shoulders of giants — e.g. the notable member of the French Academy of Science and Professor at the Sorbonne, Henry Poincaré (1854–1912) who opened "The Future of Mathematics", his 1908 address to the 4-th international congress of mathematicians in Rome by saying:

> The true method of forecasting the future of mathematics lies in the study of its history and its present state. (Poincaré, H. 1908).

Surely, the study of present state mathematics may take various modes. The one advocated here, interweaving snapshots of mathematical news in the daily teaching, assumes that a snapshot is a short intermezzo, taking a part of a lesson or an entire lesson at most, linked to the particular topic in the curriculum that occupies the class during that week. It does not change the flow of the ordinary curriculum. It does not interfere with its continuity. Needless to say, a unit in a selected topic in contemporary mathematics, which is another alternative for exposing school students to modern mathematics, may have neither the same attributes nor the same impact as a collection of snapshots, interspersed in the curriculum. While there are many topics in contemporary mathematics which can be developed into a learning unit of a week or several weeks, this mode and appropriate topics for it, are not discussed in this paper[3].

In his plenary address to ESU5 participants (See E. Barbin's panel: Mathematics of yesterday and teaching of today, in this volume) Luis Radford's argues for making students sensitive to the changing nature of mathematics and reconnecting Knowing and Being. Also, Frank Swetz advocates continually expanding the exposure to the *scope* of mathematics. He recommends (referring to Morris Kline): "Teach more *about* mathematics first, and then teach mathematics". Their views about the history of mathematics are no less relevant to the issue raised in this paper.

4 Several Kinds of Mathematical News

In order to open-mindedly examine the possibility of integrating mathematical news in the ordinary teaching of high school mathematics, and to carefully search for methods to act upon it, we first make an attempt to identify various kinds of news in mathematics, briefly giving an example or two for each category.

4.1 A recently presented problem of *particular interest* and possibly its solution

Herzberg and Murty's Sudoku paper (2007) provides a good example of this kind of news. Sudoku puzzles have been a challenge that attracts non mathematicians as well as profession-

[3] The study mentioned at the end of this paper includes efforts in that direction as well, using Berman (2006) survey of applications of nonnegative matrices to Transmission Control Protocol and Google Search Engine.

als in the past decade. Many papers on various levels were published about the mathematics of Sudoku (E.g. Keh Ying Lin 2004; Felgenhauer and Frazer 2005; Russel and Jarvis 2006; Felgenhauer and Jarvis 2006). The treatment of Sudoku as a graph and employment of coloring to its study is new and fascinating. One of their results is accessible to all: Among others they showed that for a Sudoku Puzzle to have exactly one solution, it is necessary that its initial presentation includes 8 of the 9 digits (or else the two missing digits can be switched in the final solution to get an alternative solution.)

4.2 Long-term open problems *recently* solved

Let us agree on a period of 30 years as a definition for "recently solved" and at least 100 years for "long-term". For an example in this category of news we bring the proof of Kepler conjecture. Its time-line is briefly as follows[4]:

1591: Thomas Harriot, a British astronomer, intrigued by Sir Walter Raleigh, A British explorer, published a study of various-patterns of stacking canon-balls.

1606: Johannes Kepler (1571–1630), a German astronomer corresponded with Harriot. This yielded a study of the question: Given a sphere in 3-d Euclidean space, how many identical spheres can possibly touch it?

1611: Kepler proposed a conjecture: The arrangement of equal spheres filling space, with the greatest average density (i.e. the relative portion of the occupied space), is the so called hexagonal close packing: Around any given sphere there are six sphere around it in the plane, three touching it from above and three below it. The density of this arrangement is nearly 75 % ($\pi/\sqrt{18}$ to be precise).

1998: Thomas Hales, (U. of Pittsburg, USA) submitted to Annals of Mathematics, a computer-aided proof, a proof by exhaustion of all possible arrangements.

2005: Hales' proof was accepted for publication (with reservations), and published soon afterward.

Although the solution of this problem is far beyond high school level, students can understand the problem itself, and attempt to look into the difficulties it raises or at least acknowledge the huge time lag between its posing and its solution. Additionally, this particular problem, like a few others solved in a similar way, brings up the notion of computerized proof which can be discussed and compared with a traditional logic-based proof. (Hales himself started in 2002 a project named: project Flyspeck, aimed at bridging between computerized and formal proof[5].)

The interested reader may wish to explore further more this kind of news. Here is a (partial) list of Long-term open problems solved in the period 1977–2007. The internet contains various levels of descriptions of each of these problems and their solutions.

- Lie Group: Mapping of E8, (David Vogan et als., 2007)
- Combinatorics: Stanley-Wilf conjecture (Gabor Tardos and Adam Marcus, 2004).
- Topology: Poincaré conjecture (Grigori Perelman, 2002).
- Number Theory: Catalan's conjecture (Preda Mihăilescu, 2002).
- Operator Theory: Kato's conjecture (Auscher, Hofmann, Lacey, and Tchamitchian, 2001).
- The Langlands program for function fields (Laurent Lafforgue, 1999)

[4] For more details see for example: http://www.math.pitt.edu/~thales/kepler98/

[5] For more details go to http://code.google.com/p/flyspeck/wiki/FlyspeckFactSheet

- Elliptic Curves: Taniyama-Shimura conjecture (Wiles, Breuil, Conrad, Diamond, and Taylor, 1999) .
- Discrete Geometry: Kepler conjecture (Thomas Hales, 1998).
- Algebra: Milnor conjecture (Vladimir Voevodsky, 1996).
- Number Theory: Fermat's last theorem (Andrew Wiles, 1995).
- Complex Analysis: Bieberbach conjecture (Louis de Branges, 1985).
- Knot Theory (Topology): Vaughan Jones Invariants (1984).
- Fractals: The Mandelbrot Set (Benoit Mandelbrot 1980).
- Graph Theory: Four color theorem (Appel and Haken, 1977, proved differently in 1995 by Neil Robertson, Daniel P. Sanders, Paul Seymour and Robin Thomas.)

4.3 A recently *revisited* problem

This category of news includes a new proof to a known theorem, or new findings in an already solved problem, or a new solution to a previously solved problem, or a generalization of a well established fact, or even a salvaged error.

One example is the four color problem for which a computer-assisted proof was provided in 1976, and about 20 years later a formal proof was suggested (see no. 14 above, and also: http://www.math.gatech.edu/∼thomas/FC/fourcolor.html).

Another example is the endless race for higher prime numbers. The Great International Mersenne Prime Search (GIMPS) revealed on September 4, 2006 the discovery of the 44th Mersenne prime: $2^p - 1 = 2^{32\,582\,657} - 1$. This is an almost 10 million digit prime, but not quite, hence the \$ 100 000 prize for getting over this size is still waiting for its winner![6] Prime numbers have been a challenge to mathematicians just because they are intellectually interesting. For centuries they had no application beyond pure mathematics. In the 20$^{\text{th}}$ century they became the basic tool for modern cryptography. High school students can be assigned related problems to cope with, and enjoy the satisfaction their solution brings about.

Yet another example is Tom Apostol's (2000) geometric proof of the irrationality of $\sqrt{2}$, published in 2000, which interestingly he said "I discovered this proof because I wanted something that could be presented in animated form in the Project Mathematics! Video." (Personal communication 2007)

4.4 A mathematical concept *recently* introduced or broadened

In this category, as in 4.2 above, we take "recently" to mean the past few decades. To illustrate this category let us look at the changes occurred in the last century in the notion of dimension, and are both surprising and accessible to high school students.

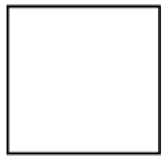
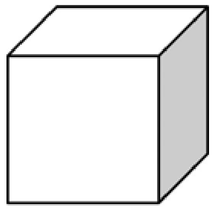
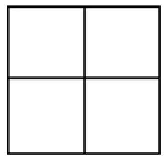
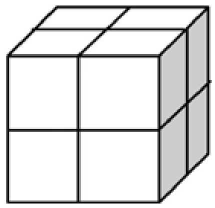
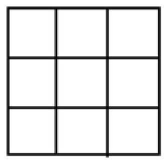
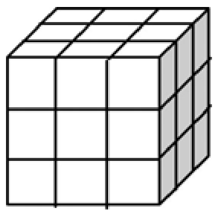
Euclidean space dimension d assumes the integer values 1, 2, or 3. If we take a Euclidean object (a line segment, a square, a box) of dimension d, and reduce its linear unit size by r (namely making it $1/r$ of its original size) in each spatial direction, its measure (length, area, or volume) becomes $M = r^d$ as shown in Table 1.

In 1918 the German mathematician Felix Hausdorff treated $M = r^d$ algebraically, as follows: $M = r^d \Rightarrow \log M = d \log r$. Consequently he made an intellectually courageous move suggesting that d need not be an integer. Since $d = \dfrac{\log M}{\log r}$ it could be a fraction, he claimed ! Non integer dimension is "visible" in the so called Koch Snowflake, first introduced

[6] For more details go to: http://www.mersenne.org/

by Hegle von Koch in 1904[7], and later on found to have fractal dimension between 1 and 2. Cantor No-Middle-Third Set whose Lebesgue measure is zero[8] has dimension a little higher than 1/2, and Sierpinski gasket[9] described in 1915 has dimension of about 1.5. Benoît Mandelbrot, who was born in 1924 into a world that had recognized Hausdorff dimension, employed this generalized treatment of dimension for his 1977 publication: *Fractals: Form, chance and dimension.* (Mandelbrot, M. 1977).

Table 1 – The measure (M) of a Euclidean shape of dimension (d) if reduced by r in each direction

Reduction factor r	**Euclidean Dimension d**		
	1	**2**	**3**
1 (Original size)	$M = 1$	 $M = 1$	 $M = 1$
2	 $M = 2^1 = 2$	 $M = 2^2 = 4$	 $M = 2^3 = 8$
3	 $M = 3^1 = 3$	 $M = 3^2 = 9$	 $M = 3^3 = 27$

4.5 A new application to an already known piece of mathematics

Mathematics develops as a result of human curiosity, quite often independent of the physical real world. The history of mathematics knew many cases of mathematical results that had no application whatsoever, developed by some intellectually intrigued mathematicians. It is quite fascinating to find out that a piece of pure mathematics becomes utterly useful for some real application. Perhaps the ultimate example in this category is the employment of prime factorization to modern Public Key Cryptography.

It is relatively easy to generate large prime numbers and find their product. However, the reverse isn't easy at all. In fact, it is practically impossible to find the prime factorization of a very large number that is a product of only 2 primes. Almost thirty years ago this asymmetry and the related parts of Number were announced by Rivest, Shamir and Adleman (1978) applicable to modern Cryptography to provide safe delivery of encrypted secret messages in open communication networks and much more.[10]

[7] For more details see for example http://mathworld.wolfram.com/KochSnowflake.html
[8] For more details see for example http://mathworld.wolfram.com/CantorSet.html
[9] For more details see for example http://mathworld.wolfram.com/SierpinskiSieve.html
[10] For more details see for example http://www.claymath.org/posters/primes/

5 Resources for Mathematical news Snapshots

Integrating snapshots of mathematical news in the ordinary teaching of high school mathematics is not an easy task. To be able to do it, it is necessary for a mathematics teacher to become familiar with resources for mathematical news, the appropriateness of which for a particular group of students s/he may consider. Only then one may start developing a didactic plan for exposing students to the news. Unfortunately, the professional journals that publish regularly new findings of prolific members of the mathematics community, are usually written symbolically and abstractly so that even a professional mathematician finds it difficult to follow the findings in a field of mathematics that is not exactly his or her own expertise. Fortunately, there are websites devoted to, and constantly updated about new findings achieved by creative professional mathematicians. Some of them attempt to bring the results in a non technical style so that mathematicians working in other fields of mathematics can follow. Yet others attend to non professional readers. Here is a mixed sample of mathematical news websites:

- http://www.answers.com/topic/unsolved-problems-in-mathematics?cat=technology
- http://www.geocities.com/ednitou/
- http://www.claymath.org/millennium/
- http://www.mersenne.org/prime.htm
- http://www.math.princeton.edu/~annals/issues/issues.html
- http://www.ams.org/ams/press/home.html
- http://www.ams.org/featurecolumn/
- http://www.ams.org/dynamic_archive/home-news.html
- http://mathforum.org/electronic.newsletter/
- http://www.eevl.ac.uk/mathematics/newsfeed.htm
- http://www.topix.net/science/mathematics
- http://web.mit.edu/newsoffice/topic/mathematics.html
- http://www.sciencedaily.com/news/computers_math/mathematics/
- http://www.nature.com/news/archive/subject/mathematics.html
- http://www.maa.org/news/mathnews_scinews.html
- http://camel.math.ca/Future/future.html
- http://plus.maths.org/latestnews/index.html
- http://www.sciencenews.org/

There are also printed resources, of which we mention here only the series "What's happening in the mathematical Sciences" a periodic survey of recent developments by Barry Cipra published since 1993 by the American Mathematical Society, and Piergiorgio Odifreddi's 2004 book "The mathematical Century, The 30 Greatest Problems of the last 100 years".

Note: ESU5 workshop participants worked in small groups, searching for a few pieces of news they felt they could find interesting, shared their findings and picked up one piece of news they thought might be worth introducing to high school students of a specified age/ability level. They then prepared "a snapshot" — A brief presentation of that piece of news to the whole group.

The reader is encouraged to stop here and give it a try. The end part of this paper will be much clearer for a reader who possesses such an experience.

6 Exposing HS Students to Math News — the Dilemma

The road to exposing high school students to mathematical news is strewn with difficulties. Web and journal resources such as mentioned above do not readily lend themselves to implementation in the classroom. In order to convince a high school teacher that the effort involved in preparing news snapshots for his/her class is worthwhile, many questions have to be addressed:

- How can one tell that a particular piece of news is *worth* introducing to HS students?
- Can we set up a list *criteria* for selecting news for high-school age-level?
- What about *accessibility* and other pedagogical issues such as *connectivity* to the current topics dealt with in class?
- What is the proper "prescription" — the *duration* of each snapshot, and their interweaving *frequency*?
- What means might be used to make HS students get *interested* in a piece of news?
- To what extent do we want them to *understand* it?
- Reflecting upon the goals — how would a teacher *evaluate* such an intervention (a goal-oriented evaluation)?

Participants in ESU5 workshop suggested the following as criteria for selecting a piece of mathematical news for developing a snapshot and bringing it to a high school class (Original quotes. The order has no significance):

- Importance of the news to mathematics/science;
- Importance of the news to the wider society;
- The problem has an appeal to every day situation;
- Can be embedded in a mathematical topic familiar to student; or is related to known/ understandable ideas;
- Involves some level of Mathematics that students can understand. The problem is accessible by relatively elementary methods. There is a possibility of explaining findings at a level not completely superficial.
- Has relevance to student's experience Connected to other relevant pieces;
- The problem is rather easy to understand (not necessarily the solution);
- Relatively short;
- Provokes curiosity for learning more;
- Is within students' ability to appreciate;
- Is interesting to the teacher;
- Students can do some work on it; at least a little progress; It includes a possibility to do some experimental work;
- There is an opportunities for further work on topic;
- Existence of Partial Results;
- There is a long human story of working on the problem.

In considering the pro and cons for interweaving snapshots of mathematical news in the ordinary teaching of mathematics, ESU5 workshop participants listed the following (Original quotes. The order has no significance):

Pro	**Con**
• Away of the daily routine; Positive Changes in the curriculum; Refreshing the routine curriculum. • Stimulating for teacher and students; Challenging for teacher • Motivate students; Attracting students to a scientific career • Makes Mathematics more Interactive • Influence students' view of Mathematics Changing a view of Mathematics; Gives adequate Image of Mathematics; • Show that Mathematics is not dead; Show Mathematics as a living developing subject; Mathematics is an ongoing process "not a dead end"; • Show that Mathematics is done by people and is done everywhere; • Shows how Mathematics is relevant in life; • Students (and teachers...) realize Mathematicians are Normal people, even interesting; • Shows Mathematics as related to other disciplines as: art, music, everything! • Answers the Question: What is the purpose of Learning Mathematics; • Increases Teacher's awareness of News; • Show Open Problems that request Mathematics; • Opportunity to teach **about** Mathematics.	• Some Topics may not be well managed by (novice) Teachers; • Not in Curriculum — Lack of Time. Time is needed to teach the basics; • Students (and Parents) would not accept too many "diversions" from main aim: The Exam; • Students expect the teacher to know the answers but here the teacher may not know them. • Readiness level of Student may not allow introducing most of the news; • Could intimidate students; • Time consuming for teachers to prepare even one piece under the school year pressure; • Difficult to evaluate students' achievements; • Lack of Sources for it.

7 Concluding remarks

As we have seen, interweaving math news snapshots in the daily teaching involves facing some true dilemma. This paper does not pretend to have all the answers. Teachers need to mull over the pro and cons for each and every idea that becomes a candidate for a snapshot, and make their own decision so as to fit it to their teaching style, the curriculum, the load of assignments in a particular class, their students' background as well as their own, and much more.

Moreover, to meet the challenge of modifying high school *students*' perception of mathematics, *teachers* in service and pre-service, should be equipped with *tools* to bridge between high school mathematics and contemporary mathematics. Such tools may include sample snapshots like the Sudoku and other examples given earlier; Resources for locating mathematical news expositions and more.

Readers of this paper are invited (as were Prague workshop participants) to join in an on-going research and development project conducted at Technion, addressing some of the basic questions involved in the proposed idea thus changing mathematics teaching at the high school level to reach ESU aims to:

> ... lead to a better understanding of mathematics itself and to a deeper awareness of the fact that mathematics is not only a system of well-organized finalized and polished mental products, but also a human activity, in which the processes that lead to these products, are equally important with the products themselves. (From ESU aims and focus statement, http://class.pedf.cuni.cz/stehlikova/esu5/01.htm)

Beyond the influence it could have on students' conception about the nature of mathematics, it is believed that the suggested approach of interweaving mathematical news snapshots in the ordinary teaching of mathematics, can also help boost high school teachers' self esteem and status, as well as fight speedy burnout, so common among teachers after relatively small number of years in the profession.

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