The Platonic Anthyphairetic Interpretation of Pappus' Account of Analysis and Synthesis

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Abstract

In the present paper we outline a novel interpretation of Pappus' famous account of Analysis and Synthesis, suffering none of the shortcomings of the earlier interpretations (such as forced to discard or even to consider as later additions parts of Pappus' account, or forced to assume some confusion on Pappus' part, or forced to assume some confusion on ancient commentators such as Proclus), based (a) on the connection of Analysis to the Platonic method of Division and Collection, and (b) on the anthyphairetic interpretation of Division and Collection, developed earlier by one of the authors.

1 Pappus' account of Analysis and Synthesis

The most authoritative ancient description of the geometric method of Analysis and Synthesis at our disposal is due to Pappus, the eminent geometer of the fourth century a. d., in his work *Sunagoge* (= *Collectio*) 7,634.2–636.18. For purposes of easier reference, we divide the account into three parts, and we further identify some subparts. Except for a general introduction (P 1) which we omit, Pappus' account, consists of parts (P 2), itself being subdivided in (P 2a) and (P 2b), and (P 3), containing (P 3 theor-neg) and (P 3 probl-neg) and reads as follows:

(P 2): 634,11–23

(P 2a) 634,11–13: *'Analysis is the way from what is sought, admitted [as true], through its* ***successors in order*** *('hexes akolouthon') to some entity admitted [as true] in synthesis.'*

(P 2b) 634, 13–23: ***For*** *('gar') in analysis we suppose what is sought as something* ***generated*** *and we inquire the entity* ***from what it results*** *('to ex hou touto sumbainei') and again the entity* ***antecedent*** *('to proegoumenon') of the latter,* ***until*** *('heos an'), proceeding backwards, end at some entity already* ***known*** *('ton gnorizomenon') or* ***being first in order*** *('taxin arches echonton'). And we call such a method analysis, namely backwards ('ana') division ('lusin'). In synthesis conversely we assume that which was last reached by analysis to be already* ***generated****, and arranging in their natural order* ***as next those that were previously prior****, we arrive at the end of* ***construction*** *for the entity sought. And this we call synthesis.'*

(P 3): 634,24–636,14

'Analysis is of two kinds. One seeks ***the truth*** *('talethous'), being called* ***theoretical****. The other serves to carry out ('poristikon') what was desired to do, and this is called* ***problematical****.*

(P 3 theor) 634,26–636,7: *In the* ***theoretical*** *kind we suppose the thing sought as* ***being*** *('on') and as* ***being true*** *('alethes), and then we pass to its successors in* ***order*** *('hexes akolouthon'), as though they were* ***true*** *and* ***existent*** *('hos estin') by hypothesis, to something admitted; then, if that which is admitted be* ***true****, the thing sought is* ***true****, too, and the proof ('apodeixis') will be the reverse of analysis.*

(P 3 theor-neg): *But if we come upon something false, the thing sought will be false, too.*

(P 3 probl) 636,7–14: *In the* ***problematical*** *kind we suppose the desired entity to be* ***known*** *('gnosthen'), and then we pass through its successors in order ('hexes akolouthon'), as though they were* ***true****, up to something admitted. If the entity admitted is* ***possible****, and****constructible*** *('poriston'), that is, if it is what the mathematicians call* ***given*** *('dothen'), the desired thing will also be* ***possible****. The proof will again be the reverse of analysis.*

(P 3 probl-neg): *But if we come upon something* ***impossible*** *to admit, the problem will also be* ***impossible*** *('adunaton').'*

2 Existing Interpretations of Pappus' account

Early researchers have assumed that Analysis consists in deductive steps from antecedents to consequents, and I fact in steps that are fully convertible. This is the case of the interpretations of Duhamel (1865), Hankel (1874), Zeuthen (1874), Heath (1926), Robinson (1936), Cherniss (1951), Mahoney (1968), and lately Menn (2002). This interpretation, based on the rendering of the term 'hexes akolouthon', appearing three times in Pappus' account, as 'logical consequences', seems to provide an interpretation of part (P 3), since there are both positive and negative outcomes there, but it fails in part (P 2), since in (P 2b), Analysis is explicitly described as**an upward movement'** (i.e. as a movement from the consequent to the antecedent). In addition, Gulley (1958), as Hintikka and Remes (1974), p. 12, correctly point out, 'has presented a most convincing case against' an interpretation of analysis as a downward deductive movement', since, according to the external evidence he presents, the prevalent idea both in writers earlier than Pappus and in later ones was that of **analysis as** an upward movement. Mahoney tried to get rid of this 'troublesome' part (P 2b), by arbitrarily declaring it an interpolation 'by some later editor'.

There is an opposing interpretation, expressed primarily by Cornford (1932), secondarily by Mugler (1948), and later by Mueller (1992). For them the steps of analysis were in an upward movement from a consequent to an antecedent. This interpretation succeeds in part (P 2), but seems to fail when it comes to the case of the two negative outcome in (P 3). The same is true for the Hintikka-Remes interpretation, although it is based on a different interpretation, relating ancient Analysis with modern mathematical logic.

More recent interpretations, starting with Gulley (1958), and including those of Hintikka-Remes (1974), Knorr (1986), and Jones (1986), try to solve the problem by admitting the simultaneous presence, in Pappus' account, of two different forms of Analysis, one, in (P 2), being upward and inverse deductive, and another, in (P 3), consisting of logically equivalent fully convertible steps. But in this way the responsibility for the inability to find a satisfying interpretation is made to fall upon Pappus himself, who is essentially held responsible for some type of inconsistency or error. Thus, according to Gulley, "Pappus, although apparently presented a single method with a single set of rules, is really repeating two different accounts of geometrical analysis, corresponding to two different forms of this method...". Knorr, essentially agrees with the presence of two, mutually incompatible, versions, coexisting in Pappus' account, additionally believing that the convertible version of Analysis (P 3) reflects mathematical practice, while the upward version of Analysis (P 2) has philosophic, vaguely platonic, sources. Maenpaa (1997) and Panza (1997), although proposing different interpretations, are equally unable to come in terms with the totality of Pappus' account.

Jones (1986), the modern editor and commentator of Book 7 of the *Sunagoge*, **epitomizes perfectly this interpretative impasse**, because:

a) in part (**P 2a**), he translates 'dia ton hexes akolouthon', which he calls 'the short definition', as **'by way of its consequences'**, thus momentarily subscribing to the Heath-Cherniss approach;

b) in part (**P 2b**), he states that 'the logical operation used in analysis is the inverse of inference', and in effect Pappus 'corrects **a flaw** in the short definition', thus reverting to the Cornford interpretation; and,

c) when he comes to part (**P 3**), he states that there **'this kind of analysis proceeds by direct, not reversed, inference'**, thus at the end agreeing with the compresence of two, mutually incompatible, versions of Analysis, as proposed by Gulley and Knorr.

Another central question regarding Pappus' account is its **relation to philosophy**. Heath noticed that Proclus, in his *Comments to the First Book of Euclid's Elements* 211.19–212.1, is connecting directly Analysis with the Platonic dialectical process of **Division and Collection**. Heath believes that here Proclus is in **confusion**, and there is no connection between these two processes — and Cherniss fully agrees. On the other hand, Cornford believes that Analysis is closely connected with Collection (and Synthesis with Division). However both Cherniss and Cornford, holding directly opposing views, nowhere show that they possess a clear notion of what Division and Collection really is. (In fact Cornford bases his conclusion on an obviously mistaken interpretation of Platonic Collection).

It thus seems that Pappus' account has been interpreted, by modern researchers, as confusing and seemingly self-contradictory, while the relation of Analysis to Division and Collection, attested not only by Proclus but by a large number of ancient commentators, must wait for an essential clarification of the Platonic process of Division and Collection. It will turn out that understanding Pappus' account rests crucially on its relation to Platonic philosophy. The clarification of the Platonic method of Division and Collection will be described in Section 4, below, but, since this clarification will be expressed in terms of the geometric concept of anthyphairesis, we must deal first with this in Section 3. Once we have understood the meaning of Division and Collection, we will be able, in Section 5, to provide a fully satisfying and internally consistent interpretation of Pappus' account, without any of the difficulties and shortcomings besetting the previous attempts, described in Section 2. A Platonic interpretation of Pappus' account of Analysis and Synthesis gains in plausibility, if Platonic credentials can be established for Pappus; such credentials are indeed found to be existing in the *Sunagoge*, as shown by Mansfeld (1998), and prominent in the *Commentary to the Tenth Book of Euclid's Elements*, as shown by Thomson (1930) and Negrepontis preprint (d).

3 Geometric Anthyphairesis

We outline here the mathematics of 'anthyphairesis', developed by the Pythagoreans, Theodorus, and the geometers, principally Theaetetus, in Plato's Academy, and presented, albeit in highly incomplete manner, in Books VII and X of Euclid's *Elements.*

3.1 Definition

Let a, b be two magnitudes (line segments, areas, volumes), with $a > b$; the **anthyphairesis** of a to b is the following, infinite or finite, sequence of mutual divisions:

$$\begin{aligned} a &= I_0 b + e_1, \text{ with } b > e_1, \\ b &= I_1 e_1 + e_2, \text{ with } e_1 > e_2, \end{aligned}$$

$$\begin{aligned}
&\dots \\
e_{n-1} &= I_n e_n + e_{n+1}, \text{ with } e_n > e_{n+1}, \\
e_n &= I_{n+1} e_{n+1} + e_{n+2}, \text{ with } e_{n+1} > e_{n+1}, \\
&\dots
\end{aligned}$$

We set $\mathbf{Anth}(a, b) = [I_0, I_1, \dots, I_n, I_{n+1}, \dots]$ for the sequence of successive quotients of the anthyphairesis of a to b.

3.2 Definition (Definitions X.1, 2 of the *Elements*)

Let a, b be two magnitudes with $a > b$; we say that a, b are **commensurable** if there are a magnitude c and numbers n, m, such that $a = mc$, $b = nc$, otherwise a, b are incommensurable.

The fundamental dichotomy for anthyphairesis is contained in the following

3.3 Proposition (Propositions X.2, 3 of the *Elements*)

Let a, b be two magnitudes, with $a > b$. Then a, b are incommensurable if and only if the anthyphairesis of a to b is infinite.

3.4 Anthyphairetic definition of proportion of magnitudes

Aristotle, in the, justly celebrated and extremely important for the history of Greek mathematics, *Topica* 158b–159a passage, refers to a period where no rigorous theory of proportion existed, while in the *Metaphysics* 987b25–988a1, explicitly states that the Pythagoreans were not conversant with dialectics and "logoi" (cf. Becker (1961)). In the same *Topica* passage Aristotle tells us that an astounding for its mathematical content (pre-Eudoxian, before Book V of the *Elements)* theory of proportion of magnitudes was discovered, based on the following

Definition. Let a, b, c, d be four magnitudes, with $a > b$, $c > d$; the analogy $a/b = c/d$ is defined by the condition $\text{Anth}\,(a, b) = \text{Anth}\,(c, d)$.

3.5 The Logos Criterion for periodicity in Anthyphairesis

An immediate consequence of the anthyphairetic definition of proportion (3.4) is the following

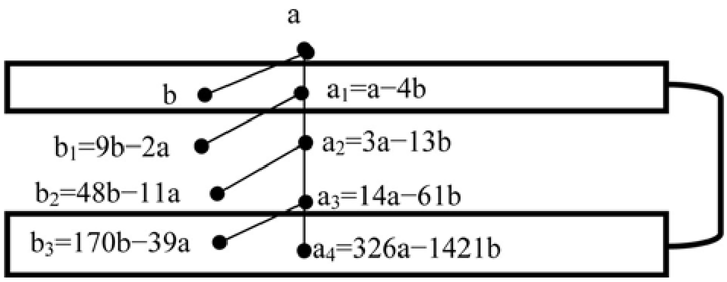
Proposition ("the logos criterion" for the periodicity of anthyphairesis"). The anthyphairesis of two line segments a, b, with $a > b$, with notation as in the definition and setting $a = e_{-1}$, $b = e_0$, is **eventually periodic**, with period from step n to step $m - 1$, if there are indices n, m, with $n < m$, such that $e_n/e_{n+1} = e_m/e_{m+1}$.

3.6 Reconstruction of proof of quadratic incommensurabilities by the Logos

There are good arguments, not to be given here, that the proofs of incommensurabilities given by Theodorus, reported in Plato's *Theaetetus* 147d3–148b2, of square roots of 3, 5, …, up to 17, are anthyphairetic, and employ the Logos Criterion (3.5). Anthyphairetic reconstructions, employing the Logos Criterion, has been proposed by Zeuthen (1910), van der Waerden (1954), Fowler (1999), Kahane (1985), Artmann (1994), Negrepontis (1997), a non-anthyphairetic one by Knorr (1975). We outline, in Table 1 below, a reconstruction of the proof of the incommensurability of the line segments a, b, with $a^2 = 19b^2$, the first one that Theodorus refrain from giving (abbreviated in the sense that we have omitted the even indexed division steps)

Table 1 is to be understood as follows: we first proceed with the steps of the anthyphairetic **Division** of a by b, employing elementary computations and expressing at the same time the remainders generated in terms of the initial line segments a and b:

Table 1 – Anthyphairetic Division and Logos Criterion for $a^2 = 19b^2$



$a = 4b + a_1$, with $a_1 < b$ (hence $a_1 = a - 4b$), (and $b = 2a_1 + b_1$, $b_1 < a_1$ (hence $b_1 = 9b - 2a$)),

$a_1 = b_1 + a_2$, $a_2 < b_1$ (hence $a_2 = 3a - 13b$), (and $b_1 = 3a_2 + b_2$, $b_2 < a_2$ (hence $b_2 = 48b - 11a$)),

$a_2 = b_2 + a_3$, $a_3 < b_2$ (hence $a_3 = 14a - 61b$), (and $b_2 = 2a_3 + b_3$, $b_3 < a_3$ (hence $b_3 = 170b - 39a$)),

$a_3 = 8b_3 + a_4$, $a_4 < b_3$ (hence $a_4 = 326a - 1421b$); and

we next verify the **Logos Criterion** (indicated in the Table by the coupling of the two expressions in the rectangles), employing the expressions found for the remainders:

$$\frac{b}{a_1} = \frac{b_3}{a_4}.$$

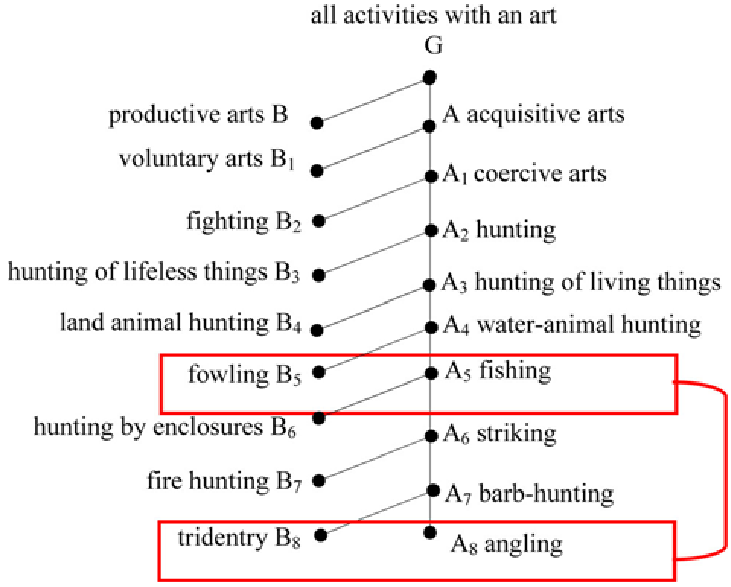
It follows that, after the initial ratio a/b, the sequence of successive Logoi b/a_1, a_1/b_1, b_1/a_2, a_2/b_2, b_2/a_3, a_3/b_3, forms a complete period of Logoi, repeated ad infinitum, and provides **full knowledge** of the initial ratio a/b, i.e. of the quadratic irrational square root of 19, and proving incidentally, the incommensurability of the ratio a/b.

4 The anthyphairetic interpretation of Division and Collection

Periodic anthyphairesis and the Logos Criterion has been shown by one of the authors to be at the center of Plato's dialectics (Negrepontis (2000), (2005), preprints (a), (b), (c)). The simplest way to see this is to correlate anthyphairesis with the Platonic Division and Collection, a method, by which Platonic Beings become known to the human soul, described in the Platonic dialogues *Sophistes*, *Politicus*, *Phaedrus*, *Philebus*; and the simplest way to grasp the close connection between Division and Collection and periodic anthyphairesis is to examine the examples of this method provided by Plato in the *Sophistes*. For lack of space, we restrict attention to the Division and Collection of the Angler, given in the *Sophistes* 218b–221c, and summarized in Table 2.

The Division, thus, starts with the Genus G, and this is divided into two species B and A, of which A is clearly the one containing the Angler. In the next step B remains undivided, but species A is turned into a Genus and is divided again into species B_1 and A_1. After a number of such binary division steps we arrive at the species A_8, the Angler. So far we have only performed Division, obtaining the Name ('Onoma') of the Angler. We maintain that this division process is but a philosophical version of the anthyphairetic division, as in Section 3 and Table 1, for $a^2 = 19b^2$. There is, additionally, need for the philosophic analogue of the Logos Criterion, what Plato calls Logos or Collection, described in the *Sophistes* 220e3, 221a2, 221b5, 221b7 and summarised as follows:

Table 2 – Division and Collection for the Angler



tridentry B_8/angling A_8 =
from above downward barb-hunting/from below upwards barb-hunting,
fowling B_5/fishing A_5 =
from above downward water-animal hunting/from below upwards water-animal hunting,
so that
tridentry B_8/angling A_8 = fowling B_5/fishing A_5.

In Table 2 the Logos-Collection $B_5/A_5 = B_8/A_8$ is indicated by the coupling of the two expressions in the rectangles. We see that the Platonic Logos-Collection is the philosophic version of the Logos Criterion for anthyphairetic periodicity, as in Section 3.

We conclude that a Platonic Being becomes known to us as a periodic anthyphairesis (in abbreviated form, with the even numbered steps omitted, for a philosophical reason, related to limited 'participation', we have no time to explain).

We will need another aspect of Plato's dialectics: Plato equates Platonic Being with Truth and Not-Being with Falsity (cf. *Theaetetus* 160a5–e1); thus, according to our anthyphairetic interpretation of a Platonic Being, Truth is associated with the periodic philosophic anthyphairesis, while Falsity with the non-periodic one. A remarkable consequence is that in a binary division scheme, Falsity of a final tail of the whole scheme implies Falsity of the whole scheme; this will be exploited in dealing with the troublesome negative outcomes of Analysis, in 5.4 below.

5 The anthyphairetic interpretation of Pappus' account

5.1 The relation of Analysis with Division and Collection

Plato was greatly interested for the method of Analysis (cf. Diogenes Laertius, in *Vitae philosophorum* 3, 24, 8–10, and Proclus, in *Commentary to the first Book of Euclid's Elements* 211, 18–23), and various ancient commentators, including Heron, Albinus, Iamblichus, Proclus, Ammonius, connect Analysis with Division and Collection; thus Albinus (in *Didaskalikos* 5, 1, 1–5, 6, 6) states that both aim at Platonic Being, Division and Collection from above, Analysis from below, presumably because, as Plato criticizes in the *Politeia* 509d1–511d5, the geometers do not provide Logos. Thus Analysis is rather closely related to Division and Collection, but it lacks Logos. Indeed Plato, in his concluding description of the Division and Collection of the Angler (*Sophistes* 221a7–c3), focuses on the right-hand side of the given Division, going only from the Genus to the Species which will be further divided, till we arrive at something which, on account of the presence of Logos, is known:

'of the art as a whole half was acquisitive, and of the acquisitive half was coercive, and of the coercive half was hunting, and of hunting half was animal hunting, and of animal hunting half was half was water hunting, and of water hunting [half] was fishing, and of fishing half was striking, and of striking half was barb-hunting, and of barb-hunting [half] was angling'.

A similar Genus-Species scheme is induced from the Division and Collection of the Sophist (*Sophistes* 268c5–d5). In general, we will say that this Genus-Species scheme is **the Analysis induced by the Division and Collection** of a Platonic Being

The induced Genus-Species Analysis scheme has the following features:

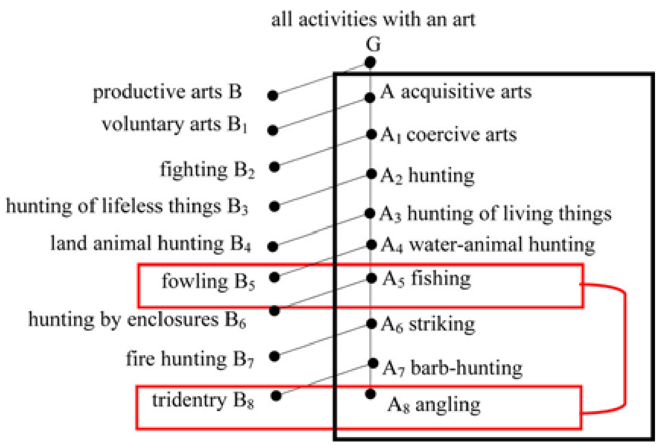
a) each entity in the induced scheme plays the role of a Genus to the immediately next entity which plays the role of a Species, hence each step is like a logical consequent followed by a logical antecedent; for example, in the case of the scheme for the Angler, a Genus-consequent is the art of hunting, while the immediately next entity, the Species-antecedent, is the art of animal hunting, and, indeed, every 'animal hunting', is certainly a 'hunting. Hence every movement from an entity in the induced scheme is an inverse implication, while the inverse scheme, the corresponding Synthesis, is a chain of logical implications, and, thus, has the structure of a mathematical proof.

b) the scheme is however something more that just the counter of a sequence of logical implications, since the steps in it, being determined by the Division process of a Platonic Being (the Angler in this case), are in natural order and succession; and,

c) the Logos, present in the Division and Collection scheme, is **lost** in this scheme, since the successive difference of each genus or species is missing, and so the induced Genus-Species scheme does not have, by itself, the power to provide true knowledge, but, with proper dialectical ingenuity and heuristics, logos and knowledge may be recaptured. *Anonymous Scholion* 4 to Euclid's *Data* provides a Platonic interpretation of the term 'given' ('dothen'), occurring in Part (P 3) of Pappus' account, relating it to the Platonic principle of the Finite, and thus to Collection and Logos in the method of Division and Collection, and connecting it to Pappus' *Commentary.*

Plato's criticism of the geometers (they treat hypotheses without providing Logos for them) suggests that Plato believes that EVERY Analysis is the Analysis Scheme induced by the Division and Collection of a Platonic Being, thus subsuming Geometry to his Dialectics and showing that mathematical proof, the essence of mathematical reasoning, is UNDER the umbrella of dialectics, an imperfect image of dialectics. Such a proof can be found by the heuristic method of Analysis; it consists in a chain of inverse implications $A \Leftarrow A_1 \Leftarrow A_2 \Leftarrow \ldots \Leftarrow A_{n-1} \Leftarrow A_n$. The way in which Analysis and Synthesis is embedded in Division and Collection is shown in Table 3.

Table 3 – Locating Analysis and Synthesis in a Division and Collection



5.2 Interpretation of (P 2b) and of the positive outcomes of (P 3) as inverse implications

In the Platonic interpretation of Analysis, outlined in 5.1, every Analysis is induced by the Division and Collection of a Platonic Being, as in the paradigmatical case of the Angler. This interpretation supports the description of Analysis as a process moving from the consequent-Genus to the antecedent-Species, precisely as described by the expression 'from what it results' ('to ex hou touto sumbainei') in (P 2b).

5.3 Interpretation of (P 2a) as steps in Platonic Division

The expression 'the successors (or followers) in order' ('ta hexes akoloutha'), occurring in (P 2a) and in (P 3), is known to have Platonic roots, going back to the *Phaedo* 101d3–5, 107b4–9 We have seen in Section 2 that the meaning of this expression cannot be 'the logical inferences'; our interpretation, according to which every Analysis is the Analysis induced by the Division and Collection of a Platonic Being, provides the natural meaning of this expression: 'the successors in order' refers to the steps, anthyphairetic in our interpretation, in the Division process; thus every such step results in the division of the Genus at this step into two species, of which one contains the Species to be defined, and as such it is indeed, as explained in (P 2b), an upward motion from the consequent to the antecedent.

5.4 Interpretation of the negative outcomes of (P 3) in terms of dialectical implication

The observant reader will notice **something peculiar in part (P 3)** of Pappus' account:

- for the case of the positive outcome a proof, by synthesis, is claimed, in both theoretic and problematic Analysis.

But

- for the case of the **negative** outcomes, **no such proof is claimed**, in both theoretic and problematic analysis.

If such a proof could be given, say because steps were fully convertible, Pappus would have absolutely no reason not to say so, but in fact, strangely enough, he doesn't.

This distinctly different treatment of the negative cases by Pappus strongly suggests that the movement
from false derived result to false searched for result
is realized not by proof and inference, but by some wider philosophical method.
Indeed, suppose
that the thing sought is A,
that by performing Analysis we come after n steps
A is implied by A_1 is implied by $A_2\dots$ is implied by A_{n-1} is implied by A_n, and
that A_n is false, and
we are to conclude that A is false.

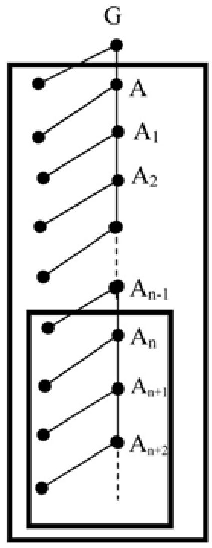
We are at a total loss to **prove** the falsity of A by mathematical implication, since the falsity of A_n in general does not imply the falsity of A. But there is a window of hope in that Pappus is very careful **not to claim** in either of the two negative outcomes, as he explicitly does in the two positive outcomes, that the conclusion of falsity would be the result of a mathematical proof. The possibility remains open that falsity of A is established not by a mathematical, method, but by a dialectical, as described at the end of Section 4. This may mean essentially one thing: we must show that if the thing we come upon by analysis is a Falsity, a Non-being, namely an entity that does not possess periodicity by Logos, then the thing sought is also a Falsity, a Non-Being, namely an entity that does not possess periodicity by Logos either.

But according to our Platonic-anthyphairetic interpretation, given in 5.1, the Analysis of A consists not only of a finite chain $A \Leftarrow A_1 \Leftarrow A_2 \Leftarrow \ldots \Leftarrow A_{n-1} \Leftarrow A_n$ of converse implications, but of a dialectical Division scheme, containing, beside the analysis chain, an initial genus G, and entities B, B_1, B_2, ..., B_{n-1}, such that

G is divided into B and A,
A is divided into B_1 and A_1
A_1 is divided into B_2 and A_2,
...
A_{n-1} is divided into B_n and A_n.

The falsity of A_n, namely the fact that A_n is non-being, implies that the Division of the dyad B_n, A_n, has no Collection, no Logos for instituting periodicity. It is then clear that the Division of the Dyad B, A has no Collection, Logos, and periodicity either, simply because the Division of the dyad B_n, A_n is a final tail of the Division of the Dyad B, A. Thus, A is a non-being, and hence false, in full accordance with Pappus' account. The situation is indicated in Table 4.

Table 4 – Falsity of A_n implies dialectically falsity of A



It is remarkable that Plato separated mathematical from philosophical-logical Truth, something that occurred, under quite different terms, in the epoch making work of Godel (1930) (cf. Paris – Harrington (1977)). Taking into account this separation, we have arrived at an interpretation of Pappus' account that does not have any of the defects of previous interpretations, outlined in Section 2. In particular we do not have to account for an inconsistency on the part of Pappus, who supposedly is accounting for two mutually contradictory versions of Analysis and Synthesis, one upward philosophical and the other fully convertible mathematical, nor do we have to try to argue that a part of the text is a later interpolation. Nor do we have to assume that Proclus, and in fact a large number of ancient commentators were confused about the close relation of Analysis with Division and Collection (cf. 5.1). Such a connection between mathematical proof (identified with Synthesis and discovered by Analysis) is indeed necessary, if Mathematics is to be subsumed under Plato's dialectics and Platonic Ideas. The second component in that scheme, namely the generation of the fundamental definitions and postulates of Mathematics from the Platonic dialectical principles, will be the content of a forthcoming work by Farmaki-Negrepontis.

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