The Use of Original Sources in the Classroom

Theoretical Perspectives and Empirical Evidence

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Abstract

Many teachers and educational researchers firmly believe in the value of historically enriched mathematics teaching. However, history of mathematics does not seem to have a permanent place in the ordinary classroom and very little is known about the real effectiveness or possible drawbacks of historical teaching. As a matter of fact, historical material can be used in various ways. In this presentation I am going to discuss the traditional genetic method and compare it with the rather unfamiliar hermeneutic approach. Furthermore, I report on a large-scale empirical research project that was based upon the hermeneutic approach and involved the reading of original sources.

1 Introduction

It is well-known that distinctive didactical values have always been attributed to the historical aspects of mathematics. Not only teachers and educational researchers but also leading mathematicians (e.g. Clairaut, Abel, De Morgan, Poincaré, Klein) often expressed their views in this regard accordingly (Fasanelli et al. 2000: 33 ff.]. In the German-speaking countries, above all, Felix Klein (1849–1925) and Otto Toeplitz (1881–1940) supported the use of historical elements in teaching. The didactical ambitions that are associated with such efforts indeed appear enormous. Historically enriched mathematics instruction is usually supposed

- to communicate technical contents in a more comprehensible way,
- to correct the image of a rigid and dry science,
- to stress the human and individual dimensions of the subject,
- to strengthen learners' motivation, etc. [Furinghetti et al. 2006: 1–4]

As early as 1913 M. E. Barwell wrote on the use of historical elements in her teaching:

> There can be no doubt that it is a great gain to the young student, when he can look upon Mathematics as living and growing, rather than as a crystallised thing from a text-book. Does not even a rock appeal more to our imagination when we realise that it has a story? The subject is humanised; it takes a place in the pageant of our race's history. The student begins to take up a right attitude towards it. He realises what it is that makes progress possible, — how the first impulse came from practical need; how ideas can be extended from the purely concrete to the abstract; how necessary it is to have, besides the thought, a compact and adequate means of expressing that thought [...] [Barwell 1913: 72]

These expectations have again and again been expressed in similar ways throughout the following decades [Fasanelli et al. 2000: 36 ff.]. In the German-speaking countries a phrase by Otto Toeplitz became very popular, according to which “the dust of time, the scrapes of long wear“ would drop from the mathematical objects and procedures, if one went back to their historical roots, so that they would resurrect as fresh, “vivid beings” before our eyes [Toeplitz 1927: 92].

In view of such immense hopes it appears amazing that historical issues still have not found a permanent place in the ordinary mathematics classroom [Smestad 2006, Siu 2005, Fraser/Koop 1987]. With regard to the use of history in class teachers often express scepticism:

- They question the actual use of historical elements for the learning,
- they point to the tremendous time pressure in school as well as to
- their insufficient training in the history of mathematics,
- they assume that historical interventions are unpopular with the majority of the pupils and
- they are worried about the testability of the learning results.

Doubts like these are thought-provoking, since, as a matter of fact, very little is known about the actual effects of historical enrichments in mathematics teaching. Of course, Barwell and Toeplitz did report on good personal experiences with their respective concepts. Also, some explorative research does point in this direction [Glaubitz/Jahnke 2003, Jahnke 1995]. However, systematic large-scale studies from which stronger and, above all, *statistically significant statements* could be derived are missing so far. Only in the year 2005 was an appropriate study published [van Gulik-Gulikers 2005]. In it several hundred pupils in the Netherlands participated in two large projects on the use of historical sources in geometry teaching. Interestingly enough, the study could *not* confirm the general hypothesis, according to which historical enrichments positively affect the understanding and the motivation of the learners [op. cit.: 222]. This result shows the necessity of further, differentiated research as to the use of historical elements in mathematics teaching.

Such a study has been conceived, conducted and evaluated as a thesis project at the University of Duisburg-Essen, Germany. Its goal was to contribute to the further closing of the aforementioned research gaps. In particular, the study was to explore the effects that could be expected from a certain type of historico-mathematical intervention — the reading of original sources in class. The data and findings from this kind of experimental teaching were explicitly to be compared with and measured by the standards and results of conventional teaching. Therefore, the study was set up as a comparative experiment, in which two analogous teaching units (on quadratic equations) were devised, carried out and analyzed: one historical, including the reading of original sources and the other quite conventional, assembled from various standard textbooks and without any historical references.

The theoretical part of the study was concerned with the development of a thorough philosophical and didactical frame for the use of historical elements and the reading of original sources in class. In order to accomplish this goal several relevant approaches were examined, deepened and related to corresponding concepts from other content areas (language teaching, history etc.)

2 he theoretical frame of the study: History of mathematics in the classroom — genetic or hermeneutic approach?

In the tradition of Felix Klein and Otto Toeplitz in the German-speaking countries historical elements are mostly used within a genetic perspective. This approach has been proposed by Felix Klein, on the assumption that

> by nature, the learner will pass in small stages through the same development as science has done on a grand scale."[1]

This view — which represents a transfer of Ernst Haeckel's (1834–1919) questionable theory of recapitulation to an educational context — was indeed very common among mathematicians and educators of Klein's period. Its value and particular appeal consisted of the possibility of aligning the ontogenetic development of individuals (pupils) with an allegedly objective model — namely the scientific phylogenesis that mankind had run through. According to Klein mathematical education was to

> build on the natural disposition of young people and slowly lead them to higher matters and, eventually, to abstract formulations very much in the same way that all mankind has struggled upwards from a naive and primitive state to higher knowledge."[2]

This idea was taken up by Otto Toeplitz, who refined it by saying that teaching which is based on historical developments should not follow each and every blind alley or detour:

> I wish to extract from history only the motives for those matters that have proved to be successful and make use of them in a direct or indirect way [...] It is about the *genesis* of problems, facts and proofs, not about their *history*."[3]

In this context Toeplitz proposes to follow

> the genetic development, that all mathematical mankind has gone through, basically according to its rough, ascending line."[4]

The purpose of such an approach was

> clarification of didactical difficulties, I would like to say: didactical diagnosis and therapy on the basis of historical analysis that is only used to direct the attention to the appropriate issues."[5]

[1] "der Lernende naturgemäß im Kleinen immer denselben Entwicklungsgang durchlaufen (wird], den die Wissenschaft im Großen gelaufen ist. (Klein 1896: 148)

[2] "an die natürliche Veranlagung der Jugend anknüpfend, sie langsam auf demselben Wege zu höheren Dingen und schließlich auch zu abstrakteren Formulierungen führen, auf dem sich die ganze Menschheit aus ihrem naiven Urzustande zu höherer Erkenntnis emporgerungen hat. (Klein 1968: I, 289)

[3] "Ich will aus der Historie nur die Motive für die Dinge, die sich hernach bewährt haben, herausgreifen und will sie direkt oder indirekt verwerten. [...] Nicht um die *Geschichte* handelt es sich, sondern um die *Genesis* der Probleme, der Tatsachen und Beweise [...] (Toeplitz 1927: 93)

[4] "die genetische Entwicklung, die die gesamte mathematische Menschheit gegangen ist, sinngemäß in ihrer großen, fortschreitenden Linie (op. cit.: 95)

[5] "Aufhellung didaktischer Schwierigkeiten, ich möchte sagen didaktische Diagnose und Therapie auf Grund historischer Analysen, die nur dazu dienen, die Aufmerksamkeit auf die richtigen Punkte zu lenken (op. cit.: 99)

This historico-genetic approach as developed by Klein and Toeplitz (cf. table 1) has never gained much influence — although Toeplitz himself gave a remarkable example with his book on the genetic approach to calculus (published only posthumously) [Toeplitz 1949]. However, the opinion has prevailed, according to which historical elements or references were particularly suitable for the *introduction* of new ideas and procedures or to supply evidence for a presumed organic, coherent and continuous growth of mathematics from elementary, initial roots.

Table 1 – The genetic and the hermeneutic approach, a comparison

genetic approach (Toeplitz/Klein)	**hermeneutic approach (Jahnke)**
global concern: *reconstruction of whole developments*	local concern: *treatment of limited historical episodes*
reading and analysis of original sources has been done by teachers or by publishers and is *not* part of the teaching	reading and analyzing original sources is integral part of the teaching
lecture-style	learners are to develop independent and self-determined activities
leads to understanding by retracing a smoothed and rectified historical development	modern understanding is a precondition; the historical episodes serve as means of deepening and reflecting
scientific standards of today represent the consummation of an organic (continuous, linear) development	scientific standards of today partly contradict certain stages of their development
attaches little value to detours or peculiarities	discontinuities, detours and contradictions are appreciated as keys to deeper understanding
experiences of strangeness are to be minimized or avoided; history is to provide affirmative evidence for today's standards	experiences of strangeness and oddity are desirable — they give reason for deeper consideration
no context	context is important
declarative concern (explanation of facts, history as an instrument for getting the "real" mathematics across)	hermeneutic concern (technical understanding *and* understanding of human signification)

Although such an approach occasionally produces beautiful results some doubt seems appropriate. Mathematicians very early criticized its insufficient consideration of scientific progress [Pringsheim 1898]. Educational experts disapproved of its poor connection to students' mental processes and their daily life [Lietzmann 1919, I: 135; Klafki 1963: 273; Wittmann 1976: 101]. From a philosophical viewpoint objections against the outdated idea of continuism can be raised [Mehrtens 1976, Jahnke 1991]. Finally, in classroom practice the genetic approach does not seem very feasible and all too often cannot fulfil its very ambitious expectations [Glaubitz/Jahnke 2003: 71].

However, the suggestions of Toeplitz and Klein are not the only method of integrating historical references into mathematics education. An interesting alternative, which may be called the historico-hermeneutic approach, was put forward by Jahnke [1991]. This approach is not concerned with 'continuistic' reconstructions of whole developments but rather with local and episodic historical interventions. These are not utilized for the motivating *introduction* of mathematical ideas or procedures but rather serve as a means of deepening and reflecting (cf. table 1). Reading original sources is the most important methodical aid of the hermeneutic approach. The pupils work on them only *after* they have acquired an understanding of relevant ideas and procedures in a conventional way.

The fact that original sources possibly convey contradictions or 'discontinuities' to the standards of today is not regarded as negative. To the contrary — it is appreciated as a key to understanding:

> It is the comparison with one's own conceptions that makes history educationally valuable."[6]

The experience of strangeness and oddity prepares the ground on which pupils' consideration may grow. Hence, they may begin to think about some new and hitherto disregarded aspects of mathematics and, in consequence, review their own beliefs about the subject. This idea is in general accordance with traditional notions of "Bildung" (educatedness), as put forward by Georg Wilhelm Friedrich Hegel (1770–1831) and Hans Georg Gadamer (1900–2002) [Gadamer 1990: 20].

3 The empirical design of the study

The hermeneutic approach served as a theoretical basis for an empirical in-depth study of possible effects of teaching with original sources. 'Quadratic equations' was chosen as the all-embracing subject matter of the two analogous teaching units that the experiment consisted of. This choice represented a core element of the syllabus and ensured the desired comparability between historical and conventional teaching. The material was organized in two specially designed workbooks for the participating pupils. The whole project was conceived in accordance with figure 1.

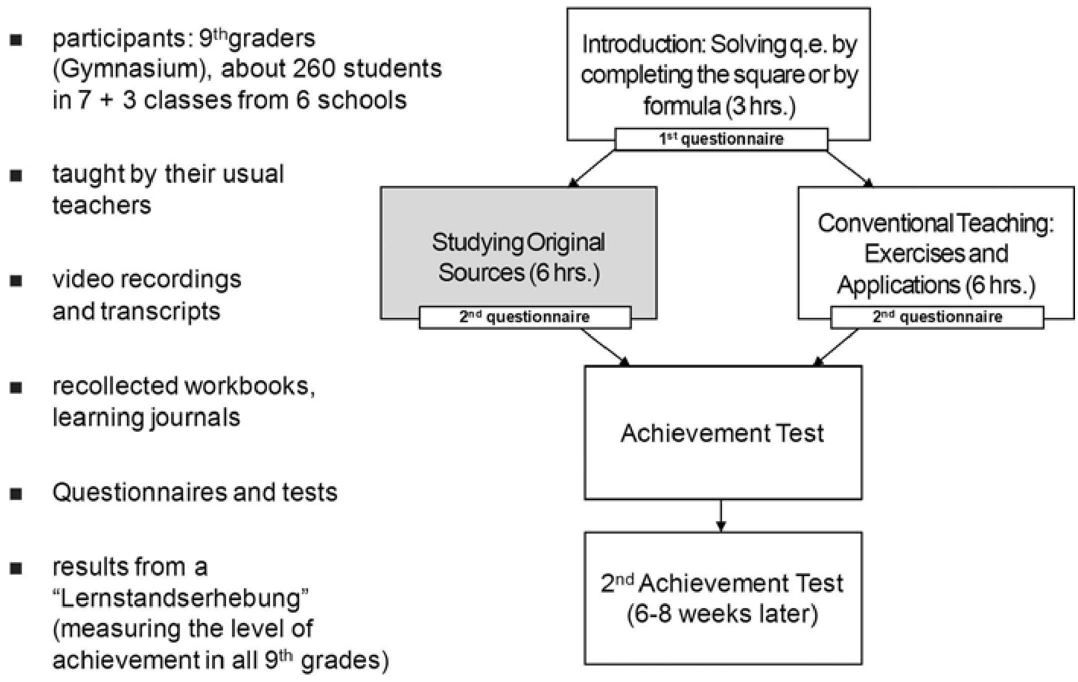

Figure 1 – Overview of the teaching — and researchproject

The project was carried out with 260 9[th]-graders in ten classes from six schools. Each class got an identical and quite conventional introduction to quadratic equations and learned to solve them by completing the square and by using the formula.

[6] "Im Vergleich mit den eigenen Vorstellungen liegt der bildende Wert der Geschichte. (Jahnke 1991: 12)

Seven of these classes then studied the historical material that consisted of excerpts from Al-Khwarizmi's 'al-jabr' (820 A. D.) in which he introduces his famous rhetoric solving method along with its geometric proof [Rosen 1831]. The pupils read and discussed the source, initially in small groups, subsequently in class. Then they tried to solve some typical quadratic problems with the ancient method. By doing so they discovered some of its advantages (e. g. "clearness", "comprehensibility") and its problems (e g. "lengthiness", "incompleteness") as compared to the modern method. The activities then gave rise to a critical discussion concerning negative numbers and today's use of formulas. Furthermore the pupils explored the differences in context between mathematics of today and mathematics in medieval Arabia by reading and discussing Al-Khwarizmi's preface to his "al-gabr" and comparing it to the preface of their ordinary textbook.

In the meantime, three control classes pursued the conventional treatment of quadratic equations and concerned themselves with standard exercises and applications.

The overall methods of data sampling were: identical achievement tests (right at the end of the unit and six to eight weeks later), video recordings and transcripts, questionnaires, recollected workbooks and learning journals. In the first questionnaire (in advance of the experiment) the pupils were asked about their achievements in mathematics, their self-assessments and their beliefs on mathematics as science and as school subject. These questions were repeated in a second questionnaire at the end of the experiment in order to find some possible shifts or changes.

The main research questions were:

1. How do the achievements of pupils in the experimental group compare to those of pupils in the control group?
2. In which way and to what extent did the historical enrichment and the reading of original sources have effect upon the beliefs on mathematics and upon the perceived methodical and general focus in class?

Also, the interrelations between the pupils' in-advance-dispositions and their respective profit (or disadvantage) from the historical teaching unit were investigated.

4 Major results

152 boys (58.5 %) and 95 girls (36.5 %) participated in this experiment (13 pupils forgot to reveal their sex in the questionnaires). The experimental and the control group comprised 172 and 88 pupils, respectively. An exhaustive testing of any significant in-advance-differences between features of both groups that might have been relevant for this study amounted to negative results. Thus, the experimental group and the control group were indistinguishable with respect to related statistical values. In detail the following results were found.

4.1 Pooled in-advance features of the experimental and the control group

Mathematics is a popular subject with the pupils of both groups. It accordingly reaches a value of 3.08 on a 1 to 4 popularity-scale (with 4 being the highest value). In particular, it is its applicability that is very much appreciated. In the list of favourite school subjects mathematics takes the second place of 21 (18.2 %), behind physical education (26.2 %) and ahead of art (11.2 %). In the list of most unpopular subjects mathematics takes the ninth place of 19 (5.8 %) while physics (18.8 %), history (14.2 %) and chemistry (13.5 %) top this list. Interestingly enough, all language subjects (German, English, French, Latin) received worse rankings than mathematics.

When asked about their skills in mathematics, pupils say that they feel competent at routine (calculating, transforming equations, drawing) or ritual activities (listening), whereas they think that they are rather weak at analyzing mathematical problems and proving theorems.

The dominating activities in class seem to be “doing algebraic transformation”, “calculating” and “working with the pocket calculator”. Moreover, mathematical reasoning plays an important role. On the other hand, activities involving language (reading texts etc.) are rare. Also, pupils hardly ever work with a computer in class.

Pupils generally think that mathematics has to do with solving problems, calculating and using symbols. Obviously this is not a nuisance to them. Furthermore, the subject is appreciated as one in which learning by heart is not very important and, what is more, does not help very much. Pupils think that mathematics is a subject in which you (have to) learn logical reasoning instead.

4.2 Effects of the historical intervention upon the experimental group

The historical teaching unit was appreciated very much and even exceeded the good popularity value of mathematics as a subject (3.23 vs. 3.08). It could be demonstrated that this appreciation did not correlate with individual test results or marks in recent school reports. However, pupils with a positive attitude towards mathematics and little or no difficulties in the subject were significantly more appreciative of the unit than those pupils who do not like mathematics or have serious problems with it. These pupils did not think that the historical intervention could help them. Maybe this is a kind of ‘Matthew-effect’ (cf. Mt XXV: 29, in essence: the rich get richer and the poor get poorer). The vast majority of learners would in principle (but not enthusiastically) welcome more teaching with historical elements.

Table 2 – Results of the achievements tests (as average marks) and average marks in advance of the experiment. In Germany the mark scale is from 1 to 6, with 1 meaning “excellent”

	in-advance mark	**1st test**	**2nd test**
experimental classes	3,16	2,89	3,04
control classes	3,29	3,30	3,59

With regard to the first research question it was found:

- The pupils of the experimental group performed significantly better than those of the control group in both achievement tests.
- Even pupils who did not like the historical unit very much, achieved better results than they had done before. The most sceptical class experienced the largest increase.
- In every experimental class the effect upon memory was significantly better than in any control class.

As for the second research question, 56 % of the pupils in the experimental group said that the historical teaching unit made them think about mathematics and their own attitude towards the subject. By comparison, in the control group only 5 % agreed with this statement. It could be shown that the positive effect was rather limited to pupils who are interested in mathematics anyway.

Furthermore, many pupils in the experimental classes felt that the methodical focus in the historically enriched lessons had changed. Routine activities like ‘calculating’, ‘working with formulas’, or ‘proving’ had become less important in their view, while the main stress had been put on hermeneutical and communicative activities like ‘reading mathematical texts’, ‘discussing with others’ or ‘varying the modes of representation’.

As a consequence, some of the pupils' beliefs were questioned. For example, mathematics was no longer regarded as a subject in which the main concern is (or should be) calculating or doing schematic problems. Instead, many students said that the importance and necessity of understanding contexts and reasoning became more apparent to them. On the other hand, they did not believe that the contents of the historical unit were of any use for later classes or for their professional careers.

In the control group no significant changes or shifts could be found in the aforementioned areas (focus in class and beliefs). This result was in accordance with the expectations.

5 Conclusion

This study demonstrates the possibility of elaborating and conducting a successful teaching unit based on the reading of historical sources by Al-Khwarizmi. In particular, the historically enriched teaching could contribute to the positive development of learners' motivations, achievements and beliefs. The study by van Gulik-Gulikers, however, shows that these results cannot be generalized undisputedly. The pupils in her experiment, e.g., experienced tremendous discomfort with the original sources they had to read and work with [van Gulik-Gulikers 2005: 222]. These problems did not occur with the Al-Khwarizmi texts used in the present study. In this context it would surely be an interesting and deserving research task to find out and specify those factors that reliably contribute to the success or failure of historically enriched teaching. For example, a thorough analysis of appropriate original sources will be one of the necessary subtasks. In the medium term a catalogue of criteria for the integration of historical elements into mathematics teaching, *based on statistically significant empirical findings*, should be a desirable goal. In addition to this, history of mathematics and its use in the classroom should become an integral part of pre-service and in-service teacher education. In particular, this could help the hermeneutic approach to attract the closer attention that it seems to deserve.

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