# Reflection and Revision

## First Experiences with a *Using History* Course

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**Abstract**

*In this paper I share initial results from a study conducted at Florida State University (FSU) in the United States in which I analyzed students' experiences with the capstone project in the course, "Using History in the Teaching of Mathematics." The course, required of all undergraduate secondary mathematics education majors, has in recent years at FSU been structured as a survey course, which included a biography paper for a final project. The course I designed focused on presenting various middle school and high school topics from an historical perspective, while emphasizing essential mathematics and pedagogy related to such a perspective.*

*The focus of the study was to investigate how pre-service mathematics teachers (PSMTs) draw upon their experiences with various course activities to consider a topic (or collection of related topics) historically and subsequently develop a teaching unit or model lesson (the capstone project in the course) for use in future secondary mathematics teaching. In the capstone project, students were required to examine their topic along several dimensions. For example, the teaching unit might ideally include cultural and humanistic influences and historical texts and problems. The study's data sources included the students' completed teaching unit or model lesson assignment and accompanying documentation required for the assignment, as well as student journal reflections documenting their historical, mathematical, and pedagogical progress during the course. I used my own weekly reflections on course activities, as well as the evaluation of the content of the capstone projects produced to consider potential revisions for future course offerings.*

**Keywords:** history of mathematics, pre-service teacher education, pre-service mathematics teachers, capstone project

## 1 Introduction

As with the construction of any secondary mathematics education course, a course on the history of mathematics for teaching can assume many different forms. For example, if the secondary mathematics education major resides in a Department of Mathematics, the course may tend to be more of a pure mathematics course instead of one with explicit attention to pedagogical ideas. Alternatively, if the course is a College of Education offering, it may shed some of its mathematical features and concentrate more on biographical, anecdotal, or pedagogical information. In recent years, what constitutes a history of mathematics course has become the subject of discussion for different audiences focused on undergraduate mathematics teaching (Rickey, 2005). Given the professional discussion taking place about the content of history of mathematics courses in general, I conducted a study to investigate

undergraduate mathematics education students' learning in the course, *Using History in the Teaching of Mathematics* (or, *Using History*). A natural consequence of the research has been to reflect on each offering of the course in order to revise course topics and assignments for the purpose of fulfilling course objectives, which are designed to create opportunities for pre-service mathematics teachers (PSMTs) to consider using the history of mathematics in their future teaching.

As part of a larger line of inquiry, I began with the following research questions:

1. In what ways does the study of the history of mathematics impact pre-service mathematics teachers' mathematical, historical, and pedagogical knowledge?
2. What do pre-service mathematics teachers report as being significant to their engagement with and influential on their learning of the history of mathematics?
3. What kinds of learning experiences are most promising for increasing critical knowledge (mathematical, historical, and pedagogical) of pre-service mathematics teachers?

The presentation given at the Fifth European Summer University focused primarily on the first research question, in an effort to investigate and understand the impact of prescribed experiences that call for pre-service mathematics teachers to obtain or demonstrate historical knowledge of the topics they will be called upon to teach (Conference Board of the Mathematical Sciences, 2001; National Council for the Accreditation of Teacher Education, 2003).

### 1.1 Perspectives for consideration

Both my own perspective about how prospective teachers will realistically consider the use of history of mathematics in teaching and the pre-service mathematics teachers' perspectives on why the use of history may be beneficial were made explicit before the start of each semester of *Using History*. Many students, in reflecting in their journal about taking the required course, stated that they did not understand why they needed to take such a course and more strongly, they asked why one would ever need to include the history in their teaching. One student shared the following:

> I heard something interesting in one of my classes today. I heard about a teacher in a local school who doesn't understand the proper placement of history in a math class. When he attempted to teach his students the Pythagorean Theorem, he first introduced them to Greece, then to Pythagoras, and on and on until he had completely lost his students. I don't think this is the place of history of math in the classroom. In the same sense, I don't know that I've placed it in the right place either. (That is, at the end of a class to catch the last of students' attention.) I think that I am really lost as to its real roots. (Sharon, Fall 2006).

So, even though Sharon was engaged with and positive toward studying the history of mathematics, she struggled with finding its "proper placement in a math class." Other students, however, were not so sure of the need to study the history of mathematics in the first place — either from their own perspective or that of their future students:

> Before taking this class, I did not understand why I needed to learn the history of mathmatics for teaching. Do not get me wrong, I was very interested in it; however, I just did not know how it would make me a better teacher. (Kristie, Spring 2007)
>
> Upon entering this course, I had a hard time understanding how incorporating history into a math class is necessary for a student's education. (James, Spring 2007)

With the knowledge that pre-service mathematics teachers do not understand (or, in many ways appreciate) the requirement of a *Using History* course, I approached the course with the hypothesis that if they experience the benefit of learning mathematics through the study of the history of mathematics, prospective teachers can envision the use of an historical perspective in their future teaching. In planning the course, *Using History*, I designed activities and tasks that I hoped would provide prospective teachers with learning mathematics in ways that would motivate them to plan for the use of history in teaching.

## 2 Course context

*Using History* is a required mathematics education course for all prospective middle grades (students aged 10–13) and high school (students aged 14–18) teachers in the secondary mathematics education program at Florida State University. In addition to *Using* History, the pedagogical preparation includes courses in using technology, how adolescents learn mathematics, instructional methods, classroom management and planning, and student teaching. In the last decade, *Using History* has most often been delivered in one of two formats. Most recently, the course has been conducted as more of a mathematics course, with an emphasis on the mathematical contributions of more prominent mathematicians (i.e., Archimedes, Euler, Pascal). Prior to this manifestation, the course included a combination of mathematics content with a culminating course project in which students developed or located a collection of classroom activities containing some historical significance. It is not clear (due to lack of institutional records), however, to what extent students either participated in or had modelled for them the various ways to engage in the study of the history of mathematics both for personal understanding of mathematics and potential instructional practice.

The mathematical preparation of the students enrolled in *Using History* is a student contextual characteristic worth noting. Secondary mathematics education majors at Florida State University do not complete the same mathematics courses that mathematics majors do — unlike any other secondary mathematics education major within the state. Instead, prospective middle grades teachers complete up through Calculus I and take three mathematics courses in the College of Education (courses in algebra, geometry, and problem solving). Students preparing to teach high school must complete through Calculus II, take four prescribed courses beyond the calculus requirement (Applied Linear Algebra, Modern Algebra, College Geometry, and an elective with Calculus II as the prerequisite), in addition to the three College of Education mathematics courses. Prospective middle grades teachers represent approximately one-third of the *Using History* enrolment each semester, creating a diversity of level of mathematical preparedness among the students taking the course. Indeed, each semester half the students pursuing middle grades mathematics certification claim they are doing so because the undergraduate mathematics courses required for high school mathematics certification are too difficult. Furthermore, the variability of student experience with mathematics content courses may impact student participation in *Using History*, particularly with respect to completion of the capstone assignment in the course.

### 2.1 Course goals and foci

The goals and foci of the current course were developed from the philosophy that, "the beauty of the study of the history of mathematics is that it can give a sense of place. . . from which to learn mathematics, rather than merely acquiring a set of disembodied concepts" (Pimm 1983: 14). The goals of the course ask for students to engage in the study of the history of topics that prospective mathematics teachers are expected to teach in the content areas of number, algebra, geometry, precalculus, and calculus and to consider alternative perspectives when teaching mathematics. In addition, a significant aspect of the course is to provide students opportunities to gain expertise in identifying and creating appropriate resources for the

purpose of integrating an historical perspective in teaching mathematics. The three course foci include (1) working with mathematical ideas that evolved over time; (2) studying and discussing the historical and cultural influences on and because of the mathematics being developed; and (3) developing the pedagogical knowledge needed to integrate an historical perspective in the teaching of school mathematics.

## 3 The capstone project

The culminating task in *Using History* gave students the opportunity to create an instructional unit or lesson that enabled them to apply their experience with each of the course foci.

### 3.1 First iteration of the course: The Teaching Unit Assignment

For the first semester I taught the course I planned for students to draw upon the examples of content, tasks, resources, and readings throughout the semester to create a teaching unit that could be used in a middle or school classroom. The *Teaching Unit Assignment* was composed of several parts, including (1) a brief history of the topic selected; (2) the student's mathematical interpretation of the topic; (3) a scope and sequence of the unit they designed; (4) lesson plans, accompanying activities, and necessary materials; (5) a rationale for why history was infused in the lessons selected from within the scope and sequence; and (6) a bibliography containing at least 12 resources, several of which were required (e.g., the *Dictionary of Scientific Biography*).

For several reasons, the *Teaching Unit Assignment* as I originally planned was overly ambitious. In one sense, many of the undergraduates had formed a negative opinion about having to take *Using History.* Ten of the 19 undergraduates enrolled during Fall 2006 had failed or withdrawn due to poor performance at mid-term when taking the course in Spring 2006. [Note: Only 16 of these 19 undergraduates were considered for the discussion that follows. Three students did not complete the capstone assignment in the course during Fall 2006.] In addition, because of the previously unsuccessful students' prior experience with *Using History* was primarily as a mathematics course, it was difficult to fully engage them in two of the three course foci (i.e., cultural and historical aspects of mathematics and the pedagogical knowledge necessary for infusing history of mathematics in teaching). Several students' aversion to mathematics — originating from their difficulty with pre-calculus and calculus concepts and their lack of success in a previous version of *Using History* — was evident in the overall lack of inclusion of mathematical tasks within the teaching units created. Table 1 displays the content areas and topic choice descriptors for the teaching units created in Fall 2006. In addition to the fact that 81 % of the topics chosen were beginning topics (number, beginning algebra, and some geometry) only five of the submitted teaching unit assignments included significant mathematics content. Two of these contained mathematical errors in either the lessons or accompanying materials (e.g., answer keys).

The hypothesis I originally approached the course guided my reflection of the results of the students' work on the *Teaching Unit Assignment.* If students were not conceptualizing the use of the history of mathematics in teaching as much more than a few historical anecdotes or timeline activities, I believed that the assignment was not providing students with the opportunity to envision the use of the history of mathematics to include mathematics. Consequently, I modified the capstone project and for Spring 2007 required a *Model Lesson Assignment* as the capstone project in the course.

### 3.2 Second iteration of the course: The Model Lesson Assignment

The modification of the *Teaching Unit* into the *Model Lesson Assignment* was conducted to enable students to think more deeply on one lesson of a unit, as opposed to trying to

Table 1 – Teaching Unit Topic Choices: Content Areas and Topic Descriptors (Fall 2006)

| Content area (number of *Teaching Units* created) | Topic descriptors |
|---|---|
| Number (4) | multiplication; fractions; square roots; distributive property |
| Beginning Algebra (3) | slope; linear equations; quadratic equations |
| Geometry (5) | similar triangles; area and perimeter; parallel lines; Pythagorean Theorem |
| Advanced Algebra (2) | combinatorics; matrices |
| Trigonometry (1) | Vectors |
| Other: beginning topic (1) | central tendency |

conceptualize the use of history of mathematics across an entire unit of instruction in middle or high school teaching. In many ways, this modification was motivated by the fact that the secondary mathematics education students at FSU take *Using History* at different times during their two years to complete the program. Consequently, if students have not taken one of the two methods courses, it is difficult to combine the mathematics history knowledge with instructional planning knowledge across an entire unit — especially if they have not had such experience prior to the *Using History* course. A reasonable compromise entailed requiring students to create a model lesson as opposed to a model teaching unit. In addition, I anticipated that students' attention to one lesson would engage them in developing mathematics with which they could be successful and that would impact their view that benefits gained from learning mathematics from an historical perspective were worth seeking in their future teaching.

The *Model Lesson Assignment* asked students to spend more time with their topic of choice and use fewer historical resources more deeply in the work of creating a model lesson. Students were tasked with creating a model lesson for which the history of mathematics provides a significantly enhanced perspective in teaching the topic and one which would challenge pre-service teachers' own thinking and understanding. The required elements for the *Model Lesson* included (1) an historical background piece, including basic biographical information about mathematicians who contributed to the development of the idea or topic; cultural and societal aspects of the places, people, and events of the major time periods involved; and historico-mathematical information sufficient for "setting the stage" for the topic; (2) the lesson plan and supporting documents, including all of the items needed to complete the lesson, such as maps, copies of original sources, student worksheets, notes to students, PowerPoint presentation slides, and solution guides; and (3) a bibliography containing at least seven resources, several of which were required (e.g., the *Dictionary of Scientific Biography*).

In addition to the concentration on a single model lesson as opposed to an entire unit, the new requirement of seven resources instead of twelve (Table 2) was included to encourage students to be more selective in the resources that they used in the creation of their model lesson and to spend more time using those resources in its development. This modification emerged from the distinction between *learning the use of* resources and *learning from* resources. In the construction of the teaching units, *Using History* students certainly showed evidence of their ability to access and use a wide variety and a greater number of resources. The intent of the requirement, however, was that students *learn from* the research that they conducted. In reducing the number of resources required for the construction of the model lesson I hoped that students would spend more time with the resources that they did access and consequently this deeper study would impact their mathematical and historical understanding in meaningful ways.

Table 2 – Required Teaching Unit Resources versus Required Model Lesson Resources

| Minimum resources required for *Teaching Unit* (Fall 2006) | Minimum resources required for *Model Lesson* (Spring 2007) |
|---|---|
| 7 text resources (one of which must be the *Dictionary of Scientific Biography*) | 3 text resources (one of which must be the *Dictionary of Scientific Biography*; not all three can be encyclopedias) |
| 2 website resources (author must be identified) | 2 website resources (author must be identified) |
| 2 journal article (e.g., *Mathematics Magazine, Mathematics Teacher, ISIS*) | 1 journal article (e.g., *Mathematics Magazine, Mathematics Teacher, ISIS*) |
| 1 "alternative format" resource (e.g., portraits, maps, media files, novels | 1 "alternative resource" (e.g., portraits, maps, media files, novels |

The outcomes of the *Model Lesson Assignment* in Spring 2007 were generally more successful than the *Teaching Unit Assignment* in Fall 2006. Neither capstone assignment description included the requirement that the students emphasize a mathematical component within the unit or model lesson. In Fall 2006 approximately 11 % of students chose to include significant mathematics (framed by historical problems) within the content of their teaching unit. In contrast, 46 % of Spring 2007 students decided to incorporate significant mathematics informed by historical problems into their model lesson.

An example can highlight the contrast in quality and content of model lessons submitted in Spring 2007 with lessons submitted within teaching units in Fall 2006. In Fall 2006, no student selected a topic that was related to the concept of infinity. In Spring 2007, however, three students focused on topics that included some aspect of the concept (development of $\pi$; special constant e; concept of infinity). Mark decided to examine the development of the constant e based upon developing interests in Euler and the concept of infinity while taking *Using History*. His model lesson included historical information to be given to students that focused on "exploring the transcendental number e" (Model Lesson, April 2007), as well as exercises for students to explore the approximation of e and application of the constant in mathematical models. For Mark, it was important to use the history of mathematics to aid in making sense of two concepts that were difficult for him to explore, learn, and accept. Mark now possessed concrete knowledge of the existence of e, as opposed to viewing it as a mysterious constant stored in calculator's memory. In addition, Mark viewed his knowledge — enhanced by the study of the history of the concept — would in fact impact his future students' learning in similar ways.

Table 3 displays the content areas and topic choice descriptors for the model lessons created in Spring 2007. Fifty-four percent of topics chosen by Spring 2007 students were considered beginning topics. The decrease in the number of beginning topics chosen when compared with Fall 2006 may be a function of the mathematical preparedness of the students enrolled during the spring course.

## 4 Reflections for further course revision

The ability of pre-service mathematics teachers to consider the use of the history of mathematics with their future students is dependent upon their evaluation of the worth of learning mathematics from solving historical problems or investigating alternative algorithms using the historical development of a mathematical concept. Because of the lack of mathematical and pedagogical experiences connecting mathematical topics with their historical development throughout a mathematics teacher preparation program, a course such as *Using History*

Table 3 – Model Lesson Topic Choices: Content Areas and Topic Descriptors (Spring 2007)

| Content area (number of *Model Lessons* created) | Topic descriptors |
|---|---|
| Number (5) | magic squares; fractions; operations with integers |
| Beginning Algebra (3) | Cartesian plane; linear equations; quadratic equations |
| Geometry (4) | development of $\pi$; area and volume; Pythagorean Theorem |
| Advanced Algebra (3) | combinatorics; matrices; Fibonacci sequence |
| Trigonometry (4) | development of sine; development of trigonometry as a field; identities |
| Calculus (3) | L'Hospital's rule; the derivative |
| Other: beginning topic (2)<br>Other: advanced topic (2) | tessellations; building structures special constants (e); concept of infinity |

must provide pre-service teachers with a venue to experience the benefits of historical problems and investigations when learning — or as is often the case, re-learning — mathematical concepts found in secondary school mathematics. In many ways, I viewed the pre-service teachers' work on either a teaching unit or model lesson as a way for them to make sense of mathematical topics while applying an historical perspective. During this sense-making process, I wanted students to develop with respect to their own learning and to consider instances in the secondary school curriculum for which investigating a topic using an historical perspective (e.g., operations with integers) contributes to conceptual understanding. Indeed, many of the prospective teachers benefited from an historical examination of operations with integers because they were confronted with having to explain why algorithms work (e.g., "a negative times a negative is positive"). On many occasions, students revealed that they merely accepted mathematical rules they were told to apply when learning mathematics in grades K–12. Now, however, the history of mathematics provided prospective teachers with access to important pedagogical tools to emphasize conceptual understanding of such rules.

To give pre-service mathematics teachers the space to do this in the *Using History* course, it became necessary to modify the requirements of the capstone project. The *Teaching Unit Assignment* was a complicated task for students. The unit potentially covered several ideas related to one topic and required students to navigate a large number of sources in order to identify or create some number of lessons that integrated historical ideas, information, or problems. Many of the Fall 2006 students found the assignment difficult because it required researching historical information, synthesizing and applying mathematical knowledge, and planning instructional tasks. Some pre-service teachers had not developed the ability attend to each of these simultaneously to the level required for the assignment. Consequently, the one aspect that was sacrificed was being able to synthesize and apply the new mathematical knowledge that confronted the pre-service mathematics teachers as they investigated the historical development of the topic they chose. This was evidenced by the small number of teaching units that included a significant mathematics component (11 % of Fall 2006 teaching units created).

After the *Teaching Unit Assignment* was modified into the *Model Lesson Assignment*, students taking the *Using History* course could concentrate more on creating one well-considered lesson and if they chose, could highlight a significant mathematics component within their lesson. Of the 12 individuals (46 % of the Spring 2007 students) deciding to emphasize mathematics within their model lesson, two-thirds relied heavily on some form of or the actual course materials when designing their lesson. In many cases, the material students relied

upon for their model lesson content came from the *Historical Modules for the Teaching and Learning of Mathematics* (Katz and Michalowicz 2005). Thus, many students' conceptions of planning for the use of history in the teaching of mathematics were influenced by becoming familiar with existing resource materials while taking *Using History.* This observation motivates two reflections for future iterations of the *Using History* course. First, since the units of study within the course heavily influenced choices and construction of the model lessons, I will continue to refine the topics chosen for the content aspect of the *Using History* course. For example, greater attention will be given to selecting topics that strengthen pre-service teachers' understanding of topics that they will teach. And, more class sessions will be spent on each topic so that students can participate in deeper mathematical and historical inquiry. Current *Using History* plans for Fall 2007 include focusing on nine or ten secondary school topics as opposed to fourteen.

The second reflection on modification of the *Model Lesson Assignment* is to require students to incorporate a significant mathematics component and to continue to emphasize that the construction of model lessons contain evidence that students developed some aspect of the lesson on their own. In previous *Teaching Unit* and *Model Lesson* assignments, the students could draw upon the work (e.g., lesson activities, lesson plans) of others, but the entire unit or lesson could not be the work of others. Students could not merely piece together content from resources. Instead, they were encouraged to construct coherent lessons that incorporate a variety of mathematical, historical, and cultural content. The way in which students combined these elements — selected directly from or built upon the ideas of other resources — was considered as evidence of lesson development.

It is worth noting that as a result of the requirement that no entire lesson could be the work of another, an overwhelming majority of students chose the inclusion of historical information or anecdotes as the content of the unit or lesson that they created — most often in the form of a PowerPoint presentation or outline of lecture notes. In most cases, the self-designed aspect of model lessons did not include mathematical content. I anticipate that the new requirement that lessons contain significant mathematics content for the Fall 2007 *Model Lesson Assignment* will contribute to skewing the selected topics (more beginning topics than advanced). In addition, the ability to select lesson ideas and activities from a variety of resources (e.g., *Historical Modules*, the course text, authored websites) will challenge students to understand their topic and the teaching and learning of mathematics well enough in order to design a coherent lesson. This challenge will encourage pre-service mathematics teachers to acquire competence with constructing coherent curriculum and may motivate them to "develop, individually, or, in collaboration, their own material... and to make it available to a wider community" (Tzanakis and Arcavi 2000: 212). Indeed, I will continue to reflect on and revise the *Model Lesson Assignment* as a way to provide pre-service mathematics teachers with a task in which they can "benefit from both primary and (perhaps more from) secondary materials and [that] they particularly welcome... didactical source material" (Tzanakis and Arcavi 2000: 212).

In closing, I return to the students' own expression of their struggles and revelations related to considering the use of the history of mathematics in teaching. At the beginning of this paper I quoted an early entry from James's reflection journal. In his earlier view, James could not quite wrap his head around the idea that "incorporating history into a math class is necessary for a student's education." At the end of the semester, however, James shared a different perspective:

> Regarding the Pascal discussions we've been having lately, I feel that there is just so much depth behind the [arithmetical] triangle that it seems like I could spend an entire semester teaching about its properties. So how would I know what to focus on? My model lesson is basically to teach the students why the triangle is

> formed the way it is and a few of its properties. I especially want to make sure that the students can make the comparison between the triangle and binomial coefficients, but I also want to teach the students kind of the same thing we learned in class today: probability and combinations. Although it wouldn't be as advanced, you can use the triangle to determine how many different combinations it takes to reach a particular "cell." (James, April 2007)

This excerpt shows that James moved from not understanding why he should consider the inclusion of the history of mathematics to struggling to plan for just the right aspects to focus on within his model lesson on Pascal's arithmetical triangle. I considered this shift — or perhaps this struggle — a successful outcome of the course. I also found James's reflection and those of other *Using History* students as evidence for continuing to craft the best possible capstone task capable of engaging pre-service mathematics teachers in creating model lessons that influence their own learning and that convince them to share their creation with their future students and colleagues.

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