Historical Documents in Everyday Classroom Work

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Abstract

Primary sources can help to design new activities for students as well as to promote new styles of teaching. A book I edited is a collection of passages taken from original documents; activities for students are present. Participants at the workshop analysed the book with the help of some translated pages. They wrote their remarks about history and the pedagogy of mathematics by discussing teachers' reasons for not using history as well as by focusing on the potentialities for new activities in everyday classroom work using originals.

Keywords: primary sources, a book for students, teachers' training

1 A book for the class

The workshop regarded analysis of and comments on the book, of which I am editor, entitled *Fare matematica con i documenti storici* (Doing mathematics with historical documents) *Una raccolta per la scuola secondaria di primo e secondo grado* (A collection for lower and upper secondary school); presentation by Fulvia Furinghetti (Demattè, 2006 a–b).

The book brings together a collection of passages selected from primary sources. As highlighted by the words "*Fare matematica*" ("Doing mathematics") in the title, it is not so much a resource for 'reading about mathematics' but rather for working with problems and exercises. This brief anthology of documents is aimed at secondary school students (aged 12–18).

The book is the result of two years of work carried out by five in-service teachers who opted to collaborate with IPRASE — *Istituto Provinciale di Ricerca, Aggiornamento e Sperimentazione Educativi del Trentino* (Institute of the Province of Trento, Italy, for research, training and experimentation in the field of education). IPRASE does not focus specifically on research in the field of mathematics but aims to improve the quality of schooling in this alpine province with a population of 400 000. The five teachers gave consideration to the educational potential of the history of mathematics and the use of primary sources in the classroom. The cultural context becomes apparent from the documents as background. Although they will have had no previous formal teaching of the history of mathematics, students will nevertheless be able to investigate the origins of mathematical ideas.

The five teachers participated with different levels of motivation and with different roles. In preliminary discussions, the members of the group shared their previous experience as

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Figure 1 – Students' volume. A small teachers' volume is also available

teachers in lower secondary school (two of them) or upper secondary school. They then discussed the structure, of the students' volume. Three of them produced specific parts of the book, one, as the owner of quite a good library, assisted by finding sources and books, another gave her contribution discussing the structure and preparing the introductory part of the book.

The aim of this book is to provide secondary school teachers with suggested activities to integrate primary sources into everyday classroom work. This integration should promote alternative ways of teaching through text-based activities and exercises to consolidate (or sometimes even to introduce) mathematical skills; see (Arcavi & Bruckheimer, 2000; Jahnke, et al., 2000).

The teaching goals underlying the choice of topics in the book can be summarised by the motto: "One more historical document, one less repetitive exercise". However, not all teachers would agree with this motto and to take this into account several exercises have been included in the book, some from Algebra, the work by Italian mathematician Rafael Bombelli, presenting simple tasks which can be solved by equations.

The small *Volume per gli insegnanti* specifically addresses teachers and provides teaching suggestions, answer keys, topics for further study and a bibliography. Both the student and teachers' books can be used as teacher-training resources.

The main source for the work was an Italian publication, Bottazzini, Freguglia & Toti Rigatelli, *Fonti per la storia della matematica* (Sources for the history of mathematics), a

collection of documents regarding arithmetic, algebra, geometry, calculus, logic and probability. Another source was (Franci, 2005). The documents selected for inclusion in the new book include writings by important authors as well as by lesser known mathematicians whose works were representative of their time. Primary sources included in the students' volume are pictures of documents, reprinted pages in the original language, translated pages, redrawn diagrams. Reference is made to the main topics taught in Italian secondary school.

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Centro di Documentazione Scolastica — IPRASE del Trentino

http://www.iprase.tn.it/attivit%E0/documentazione/index.asp

2 Translated parts of the book

From the students' volume (Demattè, 2006, a)

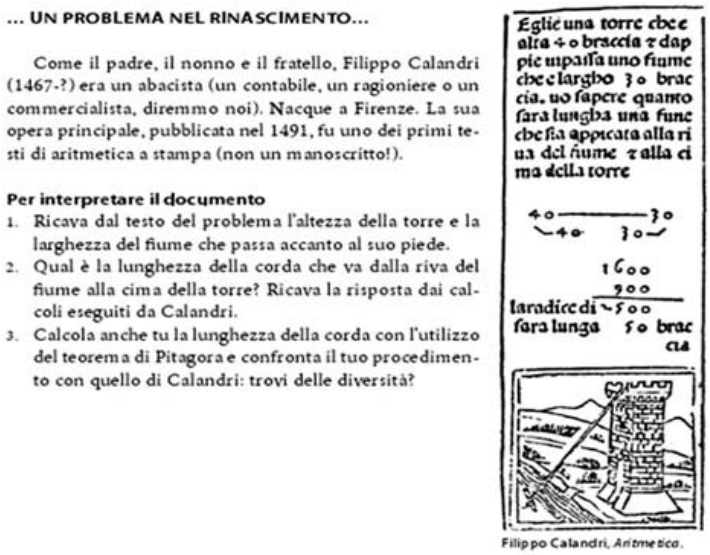
CONTENTS

Preface by Fulvia Furinghetti. **Introduction** for students.

CHAPTER 1: FROM ARITHMETIC TO ALGEBRA — Numeration: Egyptians; Babylonians; Greeks; Romans; Mayas; Indians, at last; Who invented binary numbers? — **Operations and non-negative integers**: Middle Ages and Renaissance — **Not only non-negative numbers:** Fractions in Egypt: the Horus' eye; How Egyptians wrote fractions; Decimals and Arabs; Decimals in Europe — **The arithmetic triangle:** Chinese, Arabs, Europeans... — **Curious problems:** Let's solve together; Other problems: the text; Other problems: the solutions — **"False" numbers:** In sixteenth-century Italy; A woman grapples with mathematics — **From words to symbols:** A great Arabian mathematician; Diophantus left a mark; All of them are equations; A "recipe" to solve an equation; The science of "literal calculus"; Philosopher, physician and... mathematician — **Problems and equations:** Linear and quadratic problems — **Bombelli and the number i:** Is it a number? — **Logarithms:** An ancient idea; An authoritative answer — **And more... evolution of symbols**.

CHAPTER 2 — FACES OF GEOMETRY — Arithmetic and geometry: figurate numbers: Polygonal numbers; Pythagorean terns; Ingenious ways to obtain Pythagorean terns — **Pythagorean theorem:** A walk through history: sides and squares...; ...a problem in the Renaissance...; ...problems and equations — **Far points:** About towers and other buildings; How to bore a tunnel and not come out in the wrong place — $\sqrt{2}$**:** How did they do it? — $\pi : g$ hat is the true value? — **Archimedes:** A volley of propositions; The area of the circle and the method of *exhaustion* — **Cartesian coordinates?...:** In the fourteenth century; One of the fathers — **Geometry, of Euclid and not:** An authoritative introduction, but...; The *Elements*: almost a Bible; Two millennia later — **Trigonometry:** From a sixteenth-century book — **What is topology?:** A new geometry; The problem of Königsberg's bridges; The explanation of Euler — **And more... solid numbers**.

CHAPTER 3: THEMES OF MODERN MATHEMATICS — Logic: an ancient but current science: What are logical connectives?; The art of... reasoning; Mathematics takes possession of logic — **Logic to build numbers:** Gottlob Frege and Bertrand Russell — **Let's measure uncertainty:** Galileo and a problem about the casting of three dice; Epistolary interchanges; The classical conception of probability; Other conceptions of probability — **Infinity:** Runners, arrows, hares, tortoise, ...; The whole is not greater than the part; Infinite is a source of other paradoxes; Let's arrange our knowledge — **Cantor's paradise:** Real numbers are more than integers; Cantor in Hilbert's opinion — **Infinitesimals before Newton:** The circle; The torus; The indivisibles — **Limits, derivatives, integrals** (I'm sorry if it is too little): Isaac Newton — **We don't stop... history continues...**



... UN PROBLEMA NEL RINASCIMENTO...

Come il padre, il nonno e il fratello, Filippo Calandri (1467-?) era un abacista (un contabile, un ragioniere o un commercialista, diremmo noi). Nacque a Firenze. La sua opera principale, pubblicata nel 1491, fu uno dei primi testi di aritmetica a stampa (non un manoscritto!).

Per interpretare il documento

1. Ricava dal testo del problema l'altezza della torre e la larghezza del fiume che passa accanto al suo piede.
2. Qual è la lunghezza della corda che va dalla riva del fiume alla cima della torre? Ricava la risposta dai calcoli eseguiti da Calandri.
3. Calcola anche tu la lunghezza della corda con l'utilizzo del teorema di Pitagora e confronta il tuo procedimento con quello di Calandri: trovi delle diversità?

Eglie una torre che e alta 40 braccia e dappie un passa uno fiume che e largho 30 braccia. uo sapere quanto sara lungha una fune che sia appicata alla riva del fiume e alla cima della torre

40 ——— 30
40 30

1600
900

laradice di 2500
sara lunga 50 braccia

Filippo Calandri, *Aritmetica*.

... A Problem in the Renaissance...

Like his grandfather, father and brother, Filippo Calandri (1467–?) was an abacist (a bookkeeper, an accountant or a business expert as we would say). He was born in Florence. His masterpiece published in 1491 was one of the first printed arithmetic books (not a manuscript!).

To interpret the document

1. In the text of the problem find both the tower height and the width of the river flowing near the base of the tower.
2. What is the length of the rope that starts from the riverside and ends at the top of the tower? Find the answer in Calandri's calculations.
3. Calculate the rope length by means of the Pythagorean theorem and compare your procedure with Calandri's procedure: do you find any differences?

From the teachers' volume (Dematté, 2006 b)

... A Problem in the Renaissance...

Calandri's problem about the rope length provides an opportunity to interpret a primary mathematical source to students who have a certain ability in numerical applications of the theorem. Students will be able to deduce the meaning of some words from the context. Other words may remain obscure but this shouldn't impede the analysis of the rest of the document. The figure will also help find both the data and the answers of the problem. We may give the student complementary notes on the lack of operation symbols (in the part that concludes the first chapter you can find information about when addition, subtraction, and square root symbols entered common use).

3 Activities for students

As mentioned earlier, the aim of the book is to provide secondary school teachers with suggested activities to integrate primary sources into everyday classroom work, not as "paradigmatic" experiences but as consolidation tasks (e.g., medieval algorithms for arithmetic operations) or occasionally to introduce a new idea (e.g., topology). The activities that follow each document also help the student gain a better understanding of mathematical ideas, such as ancient numeration systems, and improve their skills in critical analysis, for example, identifying inaccuracies such as Boole's repeated use of an adjective. For the most part these

activities are based on text analysis. In Italy, many students have difficulty using textbooks for mathematics. The questions and activities in our book not only help students analyse content but also introduce them to the use of a textbook. Students are sometimes asked to reflect on the causes of certain historical facts although they probably know very little about the history of the fact under investigation. An exploration of these kinds of questions would require an expertise that few historians have. Students are asked to make their hypothesis analysing reasonable answers (it could be very rewarding for the teachers if students spontaneously formulate historical questions, see Brown & Walter, 1983, p. 26).

It is significant that a group of secondary school teachers had the possibility to realize a book. Their work drew on the didactical research on history of mathematics, specifically on the use of originals. Contacts with university (specifically with Fulvia Furinghetti — University of Genoa) were particularly motivating. The awareness of working within an international research stream and tackling didactical problems that are shared by other teachers (not only Italian) was the very best stimulus to persevere; see (Furinghetti, 2005) for other works that have been produced by the same group.

In my opinion, secondary school teachers gave a significant specific contribution to the book that comes from their experience in everyday classroom practice. In planning activities they focused attention on the students and on how to involve them actively. Documents were chosen after an a priori analysis that took into account the difficulties students might have: reading mathematical texts, interpreting ancient Italian, focusing main ideas, sketching logical structure of documents, applying previous mathematical knowledge.

The Iprase Institute usually sends its publications to schools of the province. Teachers thought it was important to extend the proposal of using originals to the widest number of colleagues working in the provincial secondary schools. Therefore they chose to include in the book documents that are relevant topics for both lower and upper secondary school (6th–13th grade students). They tested some, but not all, documents in class because they weren't teaching in every year of secondary school. Thus about half of the documents have been tested in class. In any case, the remaining material is inside the book and is a proposal to colleagues. Some of them expressed their opinion directly, others are expected to send written remarks or contact some of the group members.

In Italian classrooms there is a significant number of foreign students, mainly from Eastern Europe, North Africa or South America. Sometimes students ask about the mathematical heritage of their native countries (for example, at the beginning of the last school year a girl asked me to confirm that Arabic mathematics was actually important for contemporary civilization). As the documents in the book are by both European and non-European authors, they could in my opinion, be a helpful resource for teaching in a multicultural perspective, providing an opportunity for "humanistic mathematics education" (Brown, 1996).

4 A book for teachers

During the workshop the participants were asked some questions. Main themes regarded: use of the history of mathematics in everyday classroom activities, originals as a resource for deepening mathematical concepts, the role of teachers and attention to students.

Participants confirmed that teachers generally don't use the history of mathematics in their countries either see also (Fraser & Koop, 1978; Siu, 2006) and that consequently originals are not considered a relevant teaching resource.

Furinghetti (2007) deals with the problem of teacher education through the history of mathematics. She focuses on the need to address prospective teachers' belief (Leder, Pehkonen & Törner, 2002) that they must reproduce the style of mathematics teaching seen in their school days. In my opinion, this suggests the core of future renewing of teaching (and learning). Towards this aim, she argues that the prospective teachers need a context al-

lowing them to look at the topics they will teach in a different manner. This context may be provided by the history of mathematics. She also describes some laboratory activities of mathematics education. Prospective teachers produced plans for teaching sequences, exercises, problems, reports of classroom experimentation. One report dealt with a problem from Paolo Dell'Abbaco's 14^{th} century *Trattato d'Aritmetica* (Treatise of Arithmetic); recent edition: (Dell'Abbaco, 1964).

A gentlemen asked his servant to bring him seven apples from the garden. He said: "You will meet three doorkeepers and each of them will ask you for half of all apples plus two taken from the remaining apples." How many apples must the servant pick if he wishes to have seven apples left?

This problem was also included in (Demattè, 2006 a). I used it at my school during optional activities of recreational mathematics for 10^{th} grade students. The solutions I collected contained aspects about the use of algebraic symbols that are very similar to those quoted in (Furinghetti, 2007), specifically with respect to description of the situation explained in the problem. Students used many (too many) letters, so that an algebraic solution was initially impossible, for example:

x total number of apples that the servant must pick

a the number of apples that the servant must give to the first doorkeeper

y total of apples left to the servant after the first doorkeeper

and so on.

This example from an Italian context shows that an historical problem can encourage teachers to reflect specifically on the usual approach to algebra in secondary school: insistence on algebraic manipulation, repeated solution of similar equations, lack of using letters to express both generalizations and relations among quantities. Furinghetti presented excerpts of students' writings to prospective teachers for a discussion of the way in which pupils give meaning to the concepts of unknown and, in a broader sense, of the method of algebra.

Participants in the workshop gave also their critical contribution. A remark by a researcher regarded the characteristics of a book that is quoted into References in teachers' volume. There, a short review expresses an excellent judgement, because the book is very rich in both ideas and materials for teachers despite having been written by a single author. In the researcher's opinion, the book in question contains several mistakes so that the excellent judgement is not pertinent. This episode shows that researchers and teachers sometimes have different views of the role that mathematics and its history can have in teaching. The teachers who wrote the judgement appreciated the richness and potential interest for readers, particularly teachers who are not specialists in the history of mathematics. From the researcher's point of view, therefore, academic rigour could not be set aside whereas in my opinion, in certain circumstances and contexts some mistakes could be considered *felix culpa*. At the moment, both researchers and teachers who want to enhance the connections between history and pedagogy of mathematics share a real problem, that is, increasing the number of colleagues engaged in HPM. It is a considerable achievement for an author to write a book on important aspects of the history of mathematics that is read and appreciated by many people. Rigour is surely an ideal to be pursued, but can also constitute a point of discussion, as the history of mathematics shows us. Every teacher hopes that as many students as possible might aspire to this ideal but, in my opinion, it is necessary first to stimulate their interest for mathematics.

5 Concluding remarks

In Italian schools the history of mathematics appears in almost every secondary school text book. Euclidean *Elements*, without any substantial change with respect to the original, were the text of geometry until a few decades ago. But it was a remarkable exception because

history is only seldom used as a framework for student activities and moreover its role is complementary with respect to traditional activities like training by means of exercises. Some text books propose quotations of varying lengths, but I am not aware of examples of activities that have their starting point in this sort of document. Sometimes illustrations from ancient documents are present, but their function is only aesthetic.

Our book contains documents (problems, explanations, diagrams, and images) that have a fundamental role with respect to the activities. The main goal is to bring mathematical concepts into focus by analysing different kinds of historical sources. The pedagogical proposal is discussed in the teachers' volume, which includes explanatory references. These references provide a useful starting point for teachers who wish to broaden their range of teaching materials to include the most suitable primary sources for their classes.

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