Original Texts in the Classroom

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Abstract

In this paper we make a survey of a 3-hour workshop based on historical material that has been adapted for use in the teaching of geometry and algebra. The first part of the workshop was devoted to the results of our work with 16-year students of the Greek Lyceum reading original texts on Euclidean geometry proofs. The second part of the workshop was devoted to the plan of our future work with 15-year students of the Greek Gymnasium reading original texts which reveal different levels of generality in algebra. In both cases the students are given worksheets with original texts of different authors (Euclid and Proclus on geometry, Diophantos, Viète and Euler on algebra) and they are engaged in small group discussion guided by their teachers.

1 Some arguments for using original texts in the mathematics classroom

Introducing original texts in the mathematics classroom to improve students learning of mathematics and enrich their view of mathematics is a quite old idea advocated by many authors. In recent years several arguments have been put forward to support this idea. For instance Barbin has argued that original texts appropriately introduced in the mathematics classroom allow,

- . . . to study the nature of mathematical activity in its various facets: To analyze the role of problems, proof, conjecture, evidence, error in constructing mathematical knowledge;
- . . . to gain access to epistemological & philosophical concepts which permeate mathematical texts;
- . . . to study the scientific, philosophical, cultural and social context in which the mathematical knowledge was elaborated and to see the cultural aspects of mathematical knowledge by an interdisciplinary approach (Barbin 1991).

More recently, Arcavi & Bruckheimer (2000) analysed the didactical uses of original texts along the same lines and provided elaborated arguments supporting this idea. More specifically, they stressed that original texts,

- help to trace back the evolution of a subject, in a way impossible for secondary sources;
- provide alternative ways to represent mathematical ideas and algorithms, by illustrating genuine ways of creating mathematics;

- show that mathematics in the making are characterized by doubts, misunderstandings, failures, which are inevitable;
- act as a motivation of discussing often-neglected metamathematical issues; the nature of mathematical objects & mathematical activity;
- emphasize explanations and arguments close to common sense; hence they may be much simpler than modern texts;
- provide direct contact with definitions of mathematical concepts in a particular era, possibly quite different from modern ones;
- provide links to students' cultural & historical tradition and heritage.

Finally, in a recent workshop devoted to the study of original sources in mathematics education, the work that has been done so far in this area led to a more compactified form of the various arguments:

Original sources in mathematics education may be used (a) in the classroom via excerpts and worksheets based on them; (b) by the teacher only, to deepen his/her understanding of a subject and enhance his/her awareness of mathematical results and activities. In this way, both the teacher and students may be helped

(1) to see mathematics as an intellectual activity, rather than just a corpus of knowledge, or a set of techniques;

(2) to place mathematics in the scientific, technological, philosophical and cultural context of a particular time in the history of ideas and societies;

(3) to participate in activities oriented more to processes of understanding, than to final results;

(4) to appreciate the role and importance of the different languages involved; those of the source, of modern mathematics and of everyday life;

(5) to see what is supposed to be "familiar", becoming "unfamiliar"; (Jahnke et al. 2006).

Integrating original texts at various levels of mathematics education has been implemented in various ways for various mathematical subjects. Pioneering work in this direction has been done by Arcavi (1986), who developed educational material based on historical texts in the form of worksheets and used this material for teachers' education. Another attempt has been made by Harper (1981, 1987), who used the results of a historical analysis and historical problems as the basis for an empirical research with secondary school students. A comprehensive review of the theoretical background and possible implementations can be found in Jahnke et al. 2000 (and reference therein).

The present paper concerns the implementation of these ideas in two cases: (a) to present the cross-curricular work that has been done with 16-year students of the Greek Lyceum reading original texts on Euclidean geometry proofs; (b) to give a brief account of the design of our future work with 15-year students of the Greek high school reading original texts, which reveal different levels of generality in algebra. Due to space limitations, the text focuses on (a).

2 Ancient Greek mathematical texts in the teaching of Euclidean geometry in the Greek lyceum: a cross — curricular approach[1]

The specific aims of this teaching experiment were to integrate original texts in teaching Euclidean Geometry for 16-years old students in the context of a cross-curricular approach and to create a new didactical environment and accordingly explore the realisation of specific aims of teaching mathematics: "initiation in mathematical proof", and "development of critical thinking".[2]

The experiment took place during the 2002–2003 & 2003–2004 school years in Thessalonica, Greece with students in the 1st year of the Lyceum. It consisted of 10 two-hour cross-curricular sessions in Euclidean Geometry, Ancient Greek Language and History.

As didactical material we made use of 4 worksheets with excerpts of geometrical propositions from Euclid's *Elements* (c. 300 BC) and Proclus' *Commentary* (5th century AD) on ancient philosophers' criticism against Euclid.

The teaching approach, in which teachers of mathematics, ancient Greek language and history participated with alternating interventions, aimed at students' guided work to analyse ancient texts mathematically, linguistically and historically. The focus was on formulating mathematical, linguistic and historical questions emerging from the analysis of texts, and classroom discussion of students' point of view on them.

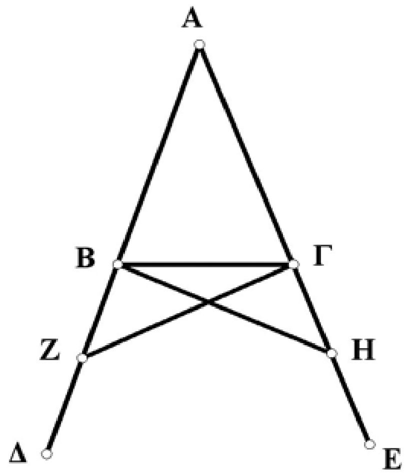
Every worksheet contained ancient Greek mathematical texts, requesting its reading and translation as well as answering questions on the text (2 to 3) and doing some relevant homework (1 or 2 assignments). As a sample we present the contents of the worksheet No 1.

2.1 Worksheet No 1

FIRST TEXT: Euclid's *Elements*, Book I, Proposition 5

In isosceles triangles the angles at the base are equal to one another, and, if the equal straight lines be produced further, the angles under the base will be equal to one another.

Let $AB\Gamma$ be an isosceles triangle having the side AB equal to the side $A\Gamma$; and let the straight lines $B\Delta$, ΓE be produced further in a straight line with AB, $A\Gamma$. I say that the angle $AB\Gamma$ is equal to the angle $A\Gamma B$, and the angle $\Gamma B\Delta$ to the angle $B\Gamma E$. Let a point Z be taken at random on $B\Delta$; from AE the greater let AH be cut off equal to AZ the less; and let the straight lines $Z\Gamma$, HB be joined. Then, since AZ is equal to AH and AB to $A\Gamma$, the two sides ZA, $A\Gamma$ are equal to the two sides ΓA, AB, respectively; and they contain a common angle, the angle ZAH. Therefore the base $Z\Gamma$ is equal to the base HB, and the triangle $AZ\Gamma$ is equal to the triangle AHB, and the remaining angles will be equal to the remaining angles respectively, namely those which the equal sides subtend, that is, the angle $A\Gamma Z$ to the angle ABH and the angle $AZ\Gamma$ to the angle AHB.



And, since the whole AZ is equal to the whole AH, and in these AB is equal to $A\Gamma$, the remainder BZ is equal to the remainder ΓH. But $Z\Gamma$ was also proved equal to HB;

[1] Research in collaboration with Y. Petrakis, S. Stafylidou, K. Touloumis, of the Experimantal School of University of Macedonia.

[2] These aims are strongly related to a long tradition of teaching Euclidean geometry in Greek secondary education. The course, which is of course a modern version of Euclidean geometry, is taught in the first two years of Lyceum (age: 16–17) and its main aims are to familiarize the students with the process of deductive reasoning and develop their critical thinking.

therefore the two sides BZ, $Z\Gamma$ are equal to the two sides ΓH, HB respectively; and the angle $BZ\Gamma$ is equal to the angle ΓHB, while the base $B\Gamma$ is common to them; therefore the triangle $BZ\Gamma$ is also equal to the triangle ΓHB, and the remaining angles will be equal to the remaining angles respectively, namely those which the equal sides subtend; therefore the angle $ZB\Gamma$ is equal to the angle $H\Gamma B$, and the angle $B\Gamma Z$ to the angle ΓBH.

Accordingly, since the whole angle ABH was proved wqual to the angle $A\Gamma Z$, nd in these the angle ΓBH is equal to the angle $B\Gamma Z$, the remaining angle $AB\Gamma$ is equal to the remaining angle $A\Gamma B$; and they are at the base of the triangle $AB\Gamma$. But the angle $ZB\Gamma$ was also proved equal to the angle $H\Gamma B$; and they are under the base.

Therefore, in isosceles triangles the angles at the base are equal to one another, and, if the equal straight lines be produced further, the angles under the base will be equal to one another; (being) what it was required to prove.

The above text is the formulation and the proof of a well known geometrical theorem, as it appears in Euclid's Elements (ca 300 BC). After reading carefully and making a rough translation of the text, try to answer the following questions:

QUESTIONS

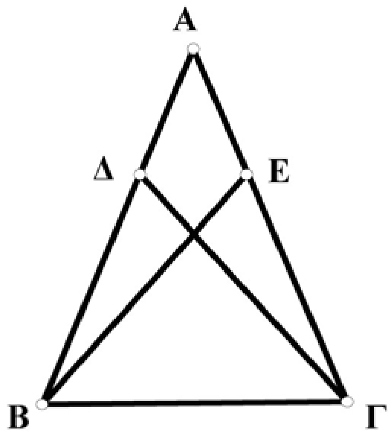
(1) Find the corresponding theorem in the geometry textbook.

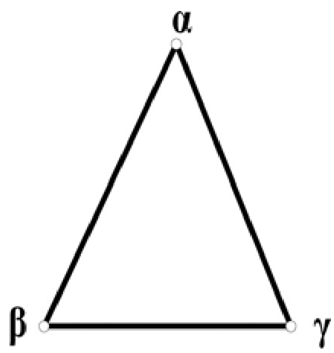
(2) Find similarities & differences between Euclid's and the textbook's proofs.

HOMEWORK

(1) Translate the ancient text keeping to Euclid's spirit as close as possible (e.g. do not use terminology and notation not used by Euclid).

(2) Find information on Euclid and his *Elements* using Encyclopaedias or other resources.[3]





SECOND TEXT: Proclus' *Commentary on the first Book of Euclid's Elements*, 248, 250

If anyone should demand that we demonstrate the equality of the base angles of an isosceles triangle without prolonging the equal sides — for it is not necessary to demonstrate their equality through the equality of the angles under the base — we can show the proposition to be true by altering the construction slightly and putting the outer angles inside the isosceles. Pappus has given a still shorter demonstration that needs no supplementary construction, as follows. Let $\alpha\beta\gamma$ be isosceles with side $\alpha\beta$ equal to side $\alpha\gamma$. Let us think of this triangle as two triangles and reason thus; since $\alpha\beta$ is equal to $\alpha\gamma$ and $\alpha\gamma$ is equal to $\alpha\beta$, the two sides $\alpha\beta$ and $\alpha\gamma$ are equal to the two sides $\alpha\gamma$ and $\alpha\beta$, and the angle $\beta\alpha\gamma$ is equal to the angle $\gamma\alpha\beta$, for they are the same; therefore all the corresponding parts are equal, $\beta\gamma$ to $\beta\gamma$, the

[3] A participant of the workshop made the observation that an interesting assignment for students' homework would be a study of Proclus' life and work, for which many facts are known.

triangle $\alpha\beta\gamma$ to he triangle $\alpha\beta\gamma$, the angle $\alpha\beta\gamma$ to the angle $\alpha\gamma\beta$ and angle $\alpha\gamma\beta$ to angle $\alpha\beta\gamma$; for these are angles subtended by the equal sides $\alpha\beta$ and $\alpha\gamma$; hence the angles at the base of an isosceles are equal.

It looks as if he discovered this method of proof when he noted that in the fourth theorem it was by uniting the two triangles so that they coincide with each other, thus making them one instead of two, that the author of the *Elements* perceived their equality in all respects.

In the same way, then, it is possible for us, by assumption, to see two triangles in this single one and so prove the equality of the angles at the base.

The above text is an excerpt from the commentary written for Euclid's *Elements* by the philosopher Proclus (ca 450 AD). Proclus cites here two different proofs of the theorem you have studied previously, one given by Proclus himself and one given by Pappus (ca 300 AD). After reading carefully the text, make the following homework.

HOMEWORK

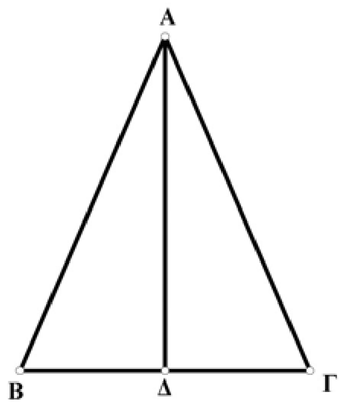
(3) Translate Proclus' text to modern Greek.

(4) Find similarities and differences among Euclid's, Proclus' and Pappus' proofs.

(5) Try to explain why all ancient proofs are different from the textbook proof.

2.1.1 THE STUDY OF WORKSHEET No 1 IN THE CLASSROOM

The proofs given by Euclid, Proclus and Pappus became the object of study in the classroom and compared with the one given in the students' official textbook of Euclidean geometry. This is a different, rather simple, proof which makes use of the bisector $A\Delta$ of the angle between the equal sides of the triangle $AB\Gamma$ and the equality of the triangles $AB\Delta$ and $A\Gamma\Delta$.



The comparison of the proofs provoked extensive classroom discussion on the following questions:

Q1. In your opinion, why did Euclid give a complicated proof?

Q2. Why did the ancients avoid using the bisector of the angle at the top vertex? How it can be ensured that the usual construction (by ruler and compass) of the bisector of an angle does indeed bisect the angle?[4]

Q3. Comment on Proclus' and Pappus' proofs.[5]

Some of the students' responses in classroom discussion were the following:

On Q1, Q2:

- Euclid wanted to impress his readers, because when scientists do complicated things, their authority increases.

[4] As a participant of the workshop observed, the specific formulation of these questions may influence and even canalize the students' answers. However, the formulations emerged during the discussion, as for example in the first question, which we posed to the students after their general agreement that Euclid's proof is a rather complicated one.

[5] A participant of the workshop remarked that the study of different proofs for the same theorem in historical texts is of great importance to modern curricula, which aim at bringing to light the factors related to the production of a proof (a reference is made to the new French mathematics curricula).

- Euclid wanted to show how to use the triangles' equality criteria.
- Euclid wants a theoretical, not a practical proof. Bisecting an angle is a practical issue and is not accurate. This construction is naïve, possible for all people, because it is like folding in two a piece of paper.
- Euclid could not draw the bisector accurately; he could not prove that the two angles are equal. The bisector concept had not been discovered yet.
- Euclid wanted to exploit that particular proof in order to prove other properties that exist in that particular figure.

On Q3 (for Pappus' proof):

- It looks like proofs that we gave at the elementary school.
- It is a proof appropriate for babies(!)
- It is more difficult; it requires more thinking (it is more probable that we do make a mistake).
- It is adapted to practice, whereas, Proclus' and Euclid's proofs have elements of logic and scientific reasoning.[6]

We proceed now to the brief description of two other worksheets, which were studied and discussed in the classroom.

2.2 Worksheet No 2

Excerpts:

(i) Euclid's *Elements*, Book I, proposition 9: To bisect a given angle.

(ii) Proclus' *Commentary*, 273–274: Refuting objections against Euclid's proof.

Questions:

(1) Find the corresponding problem in the geometry textbook.

(2) Find similarities & differences between Euclid's and the textbook's solutions

Homework:

(1) Translate the ancient text keeping to Euclid's spirit as close as possible (e.g. don't use terminology or notation not used by Euclid)

(2) Find information on Euclid and his *Elements* using Encyclopaedias or other resources.

(3) Translate Proclus' text to modern Greek.

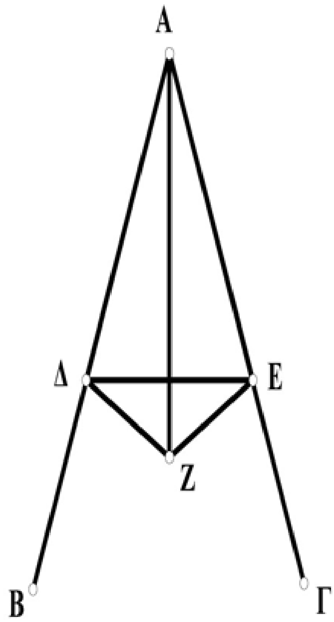
(4) Write your own opinion about the arguments against Euclid's solution and about Proclus' arguments.

(5) Examine whether similar objections can be put forward against the textbook solution of the same problem.

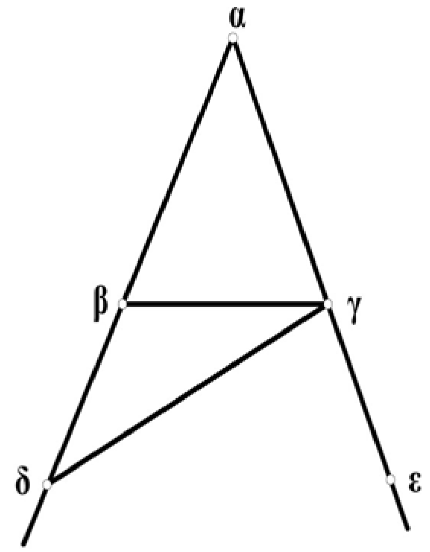
[6] Pappus peculiar proof stimulated also some discussion among the participants of the workshop, especially on the compatibility of the Euclidean axioms with the method of superposition of figures and its applicability as a method of proof.

Classroom discussion on Worksheet No 2 was centered initially (and rather unexpectedly!), on students' confusion with the term "rectilinear angle" used by Euclid. When the teacher explained that there are "curved" angles (e.g. on a spherical surface), a student was wondering ironically whether there are "curved straight lines" as well.

The teacher explained briefly that on a spherical surface a different geometry holds.



Euclid's construction of the bisector AZ of an angle $BA\Gamma$, after taking $A\Delta = AE$ and constructing the equilateral triangle ΔEZ.



Ancient geometers objections (according to Proclus) against Euclid's construction of the bisector. How can one be sure that the vertex δ of the equilateral triangle $\beta\gamma\delta$ lies always inside the angle?

Further classroom discussion was carried out on the following questions:

Q1. Compare Euclid's construction of the bisector of an angle with that given in the school textbook.

Q2. What do you think about the objections against Euclid's construction?

Q3. How could Proclus prove that the argument put forward against Euclid's proof is not valid?

Students' responses in these questions can be summarized as follows:

On Q1:

- Students confessed that there are no essential differences but Euclid's proof is easier to understand, for two reasons:

The segment obtained by using the compass is not taken arbitrarily.

The proof is based on the comparison of triangles and not by reference to the median of a circle's chord used in the school textbook

- Who and for what reason did change Euclid's construction and proof?
- Euclid's does not call bisector the segment that bisects the angle

On Q2:

Most students consider that questioning of Euclid's proof as justified. At that time what Euclid suggested were unknown, hence could not be accepted; as it happens nowadays for something that appears for the first time. People are convinced later, after the arguments and justifications are given.

On Q3:

- Many students said: “By reduction ad absurdum”.
- Analysing the proof, it became clear that Euclid was very careful to include the equality of the exterior angles of an isosceles triangle (in the enunciation of proposition I, 5 as it is stated in Worksheet No 1).
- There was a discussion on the issue of “geometrical order”, further extended to the issue of the discourse among scientists and philosophers in antiquity and modern times.

2.3 Worksheet No 3

Excerpts:

(i) Euclid’s *Elements*, Book I, proposition 20: The triangle inequality.

(ii) Proclus’ *Commentary*, 322, 323: Refuting the Epicureans’ objections against the necessity of proving this proposition.

Questions:

(1) Find the corresponding theorem in the geometry textbook.

(2) Find similarities & differences between Euclid’s and the textbook’s proofs

Homework:

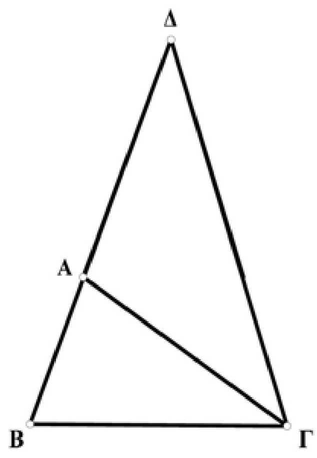
(1) Translate the ancient text keeping to Euclid’s spirit as close as possible (e.g. do not use terminology and notation not used by Euclid)

(2) Find information on Euclid and his *Elements* using Encyclopaedias or other resources.

(3) Translate Proclus’ text to modern Greek.

(4) Comment on the arguments of Euclid’s critics and on Proclus’ answer. What is your own opinion?

(5) Prove the proposition in the way suggested by Proclus, by drawing the bisector of one angle (as in the figure in the worksheet)



Euclid’s proof of the triangle inequality

$$AB + A\Gamma > B\Gamma,$$

after constructing the triangle $A\Gamma\Delta$ with

$$A\Delta = A\Gamma.$$

The Epicureans are wont to ridicule this theorem, saying that it is evident even to an ass and needs no proof; it is as much the mark of an ignorant man, they say, to require persuasion of evident truths as to believe what is obscure without question. Now whoever lumps these things together is clearly unaware of the difference between what is and what is not demonstrated. That the present theorem is known to an ass they make out from the observation that, if straw is placed at one extremity of the sides, an ass in quest of provender will make his way along the one side and not by way of the two others.
(From Proclus’ commentary)

Classroom discussion on Worksheet No 3 led to the following questions:

Q1. Do you agree with Euclid's approach to prove in detail even obvious properties of geometrical figures?

Q2. Why certain ancient philosophers questioned, or even ridiculed Euclid's geometrical proof?

Q3. Does the debate of Epicureans and Euclid indicate significantly divergent points of view between science and philosophy? What is your opinion?

Students' responses in these questions can be summarized as follows:

On Q1:

- Euclid should convince those who doubted, those who use geometry for practical reasons.
- The necessity to classify in a system geometrical knowledge requires the proof of all propositions, even the most evident ones.
- The necessity of the existence of propositions that are used as the basis for the proof of other propositions is fundamental.
- Scientists should be sure as they proceed further in their research.
- Every science should found its results on logic and theory.

On Q2:

- In general, philosophers opposed to scientists, who were favoured by the kings and had a lot of privileges.
- The Epicureans' objections express the opposition against authority, because the absolute knowledge provoked by science fits well with the characteristics of an absolute monarch.
- The criticism of the Epicureans stems from their philosophical beliefs, according to which knowledge is originally founded on sensations and not on the logical causes of the phenomena.

2.4 Some remarks on methodological issues concerning cross-curricular activities

- A cross-curricular approach to original texts helped to face important issues concerning translation and interpretation and placed original texts in the appropriate historical context.
- The original texts and the translation process led to etymological comments on the origin, meaning and accurateness of mathematical terminology.
- The clarity and conciseness of ancient Greek mathematical language was revealed by connecting two apparently disjoint disciplines: study of ancient Greek language and mathematics.

Some results

- Studying original texts created a new didactical environment, in which students actively participated in the classroom discourse and exhibited a positive attitude towards the subject under consideration.
- Students' commented that the whole activity led them to a more global understanding of what Euclidean geometry really is.
- The variety of students' answers and contradictions among them, that were produced by studying original texts reveal the number of factors that influence the understanding of metamathematical concepts, like the concept of proof.
- Critical thinking not only requires the technical ability to formulate particular proofs, but also more general abilities to globally conceive notions, to formulate correct assertions etc.
- Such requirements brought up by studying original texts, link the specific didactical aims of learning particular mathematical concepts and theories, with the wider pedagogical aims of teaching mathematics (raising metamathematical issues, access to philosophical & epistemological concepts, links to the historical & cultural tradition etc).

3 Original texts in the teaching of algebra: reading how Diophantos, Viète and Euler solved the same problem

In the second part of the workshop we dealt with the integration of original texts in the teaching of elementary algebra to 15–16 year-old secondary school students. It is frequently stated in the literature that the majority of secondary school students, who have been taught basic algebra (powers, equations, functions, transformation of polynomial and rational expressions, (linear) system of equations), face important difficulties in using algebraic tools for solving problems and expressing general results in abstract form. Our work is motivated by the often-cited work by Harper (1981, 1987). More specifically, Harper used the results of a historical analysis as the basis for an empirical research, which registered secondary school students' methods for solving the following problem:

If you are given the sum and the difference of any two numbers, show that you can always find out what the numbers are. Make your answer as general as you can.

This problem has been solved by Diophantos (ca. 250AD) in his *Arithmetica*, by Viète (1540–1603) in his *Zeteticorum Libri Quinque*, and by Euler in his *Vollständige Anleitung zur Algebra* in different ways that reveal different stages of the evolution of Algebra.

Harper's research indicated that despite the extended teaching of algebra, most students use concrete numbers to solve a problem stated in general terms, or face great difficulties to manipulate the variables that are necessary for giving a general algebraic solution. The problems of learning basic algebraic concepts and methods are related to fundamental issues of cognitive development and understanding; given the particular epistemological nature of algebra, these problems are also related to important meta-cognitive issues on the nature of mathematical concepts and methods and the procedures followed to solve problems. Therefore, coping with these problems appears to be a complicated didactical step that requires a combination of different approaches and reveals the role of teacher to a key factor:

…there is no possible entrance to the world of algebra without a strong push and guidance from the teacher because there is no natural passage from the *problématique* accessible from the child's world to the mathematical *problématique* (Balacheff, 2001, p. 259).

The historical analysis and the integration of historical elements of algebra's development in teaching constitute one of the tools that may be used in this context:

The "potentiality" of theoretical concepts is also gained in the process of historically reconstructing the development of a mathematical concept or a mathematical idea. History provides us with the insight that there is not one mathematics, and this insight might encourage and strengthen the learner with respect to her or his own personality and approach to knowledge. ... Mathematics education has to take into account that there is no knowledge without metaknowledge, that one cannot learn a theoretical concept without learning something about concepts, in order to understand what kind of entities those are. This metaknowledge can, however, be developed by means of historical studies (Otte & Seeger, 1994, p. 353).

To realize this, the study of original sources in the classroom is a basic tool, because it reveals in the most direct way the fact hidden in modern teaching, namely, the historical nature of mathematical knowledge. Therefore, we have chosen texts of Diophantos, Viète, and Euler, which unfold the way they faced the problem used in Harper's research. The basic characteristic of these texts is that they present the solution of the same problem by using basic algebraic concepts, like the unknown and equation, in a different stage of their conceptual development and symbolic representation. These texts are included in worksheets to be given to students who have just finished high school and are entering the Greek Lyceum (15–16 years old) and have been taught the basic algebraic concepts and methods (use of unknowns and variables, solution of equations and their use to solve problems, transformations of algebraic expressions) for two years. However, their majority is very weak in treating algebraic calculations and expressing general results, which is a basic characteristic of symbolic algebra. The worksheets will be studied during classroom activities under the supervision of the mathematics' teacher and students will be asked to answer the questions that follow the original texts and participate in the follow-up discourse. These activities are under implementation. Here we simply sketch them, due to space limitations. Empirical results will be presented in a future paper.

In the light of the theoretical discussion above, the aims of these activities are: (a) To integrate original texts in teaching Algebra for 15-years old students in the context of review lessons; (b) to follow the gradual development of basic algebraic concepts and means of their representation; (c) to develop metacognitive skills concerning the nature of basic algebraic concepts and the procedures followed to solve problems.

The problem appears as follows:

Diophantos: *To divide a given number into two [numbers] having a given difference.*

Viète: *Given the difference between two roots and their sum, to find the roots.*

Euler: *It is required to divide α into two parts, so that the greater may exceed the less by b*; or

It is required to find two numbers, whose sum may be α, and the difference b.

The content and structure of the worksheets are as follows:

3.1 DIOPHANTOS

1. Information about Diophantos.
2. Basic elements of Diophantos' method, in particular his terminology, concept of "unknown" and algebraic symbolism, with examples for the students to get acquainted with.
3. Excerpts from Diophantos' *Arithmetika*:
   (a) Introduction: Comments on issues of teaching and learning.
   (b) Introduction: Didactic guidelines on some basic rules for solving equations.
   (c) Book I: Problem 1.

4. Questions on these excerpts for the students to work in the classroom and at home; e.g., for (a) "How Diophantos expresses the difficulty of the subject he is going to present?"; for (b) "What mathematical process does Diophantos describe in the above excerpt?"; for (c) "If you solve this problem today, what would you write differently?"

3.2 Viéte

1. Information about Viéte and his books.

2. Basic elements of Viéte's algebraic method ("the art of analysis"), which involves three stages: *zetetics*, i.e. asking for; *poristics*, i.e. providing;, *exegetics*, i.e. explaining; as well as his notation based on the systematic use of letters for representing the unknown and the data of each problem.

3. Excerpts from Viéte's work:

   (a) *In Artem Analyticem Isagoge*, Chapter II: On the Fundamental Rules of Equations and Proportions.

   (b) *In Artem Analyticem Isagoge*, Chapter V: On the Rules of Zetetics. Chapter VIII: On the nomenclature of Equations, and an Epilogue to the Art.

   (c) *Zeteticorum Libri Quinque*, First Book: Zetetic I.

4. Questions on these excerpts for the students to work in the classroom and at home, e.g., for (a) "By using modern notation, explain the rules of equations and proportions mentioned by Viète in the previous excerpt from Chapter II"; for (b) "By giving examples, explain the meaning of the rules called by Viète 'antithesis', 'hypovivasmos' and 'paravolismos'"; for (c) "Compare Viète's solution above with that given to the same problem by Diophantos".

3.3 Euler

1. Information about Euler's *Vollständige Anleitung zur Algebra* (Complete Introduction to Algebra); in particular, on the unique conditions under which it was written, its modern character as far as notation is concerned, and the variety of problems treated.

2. Excerpts from Euler's algebra:

   (a) Chapter I: Of the Solution of Problems in general.
   Chapter II: Of the Resolution of Simple Equations, or Equations of the First Degree.

   (b) Chapter III: Of the Solution of Equations relating to the preceding Chapter.
   Chapter IV: Of the Resolution of two or more Equations of the First Degree.

3. Questions on these excerpts for the students to work in the classroom and at home, e.g., for (a) "Write in detail the transformation rules of equations described by Euler in paragraph 571"; for (b) "How many different solutions of the problem solved by Diophantos and Viète are given by Euler in the above excerpts?".

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