ADAM FRITACH'S *New Fortification*

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Abstract

This paper describes the usual bio/bibliographical researches you need to undertake to be able to understand the real importance of this kind of book on fortification, as well as a study of some of its mathematical contents, including geometry, trigonometry, and one of the first uses of Stevin's Tenth.

1 THE BOOK

Adam Fritach's book is well known as the first and major treatise about the "modern" Dutch theory of fortification that appeared during the EIGHTY YEARS WAR. In fact, it was not exactly the first one, if you consider Marolois's and Stevin's works, but it is certainly the most famous one of this time, as we can infer from the numerous quotations we can find in several books till the end of the 18$^{\text{th}}$ century.

Until recently, Fritach (or Freytag in German) was said to have been an architect for King Vladislaus IV of Poland, who was supposed to have sent him to Holland to study the new theories of fortification, and bring them back to Poland. We now know that this is not the case, but the reader will learn about this, little by little, and have the pleasure of discovering Fritach's life as I did. And firstly, from the book.

1.1 WHAT DOES IT TELL US ABOUT ITS AUTHOR?

Six different editions of the *New Fortification* can be found in German or French[1], all of them with the same frontispiece (see Fig. 1). The intricate engraving can be seen as a kind of prospectus and a variety of details show what the printer wanted the reader to think about Fritach's glory and the value of this book.

As the *New Fortification* deals with military matters, it's natural to find on its first folium the figure of Mars sitting upon the title as if he were on a throne, but it is surrounded by two feminine figures: on the left "Labore" and on the right "Industria", who represent the two sides of the practice of fortification: theoretical and practical.

We can read the name of the author at the end of the title (in German: Freitag; in French: Fritach), followed by his title: "der Mathematum Liebhabern" or "Mathematicien".

This is quite new in such treatises on fortification at the beginning of the 17$^{\text{th}}$ century, above all if you consider that Fritach's book is certainly not the most difficult one as far

Many thanks to Patrick Guyot for typing the French version of original texts and to my wife Karin for her patience and her help with the translations.

[1]In German in 1631, 1635(?), 1642, 1665 (Elzeviers in Leyden or Amsterdam); in French in 1635, 1640 and 1668. See the bibliography at the end of this paper.

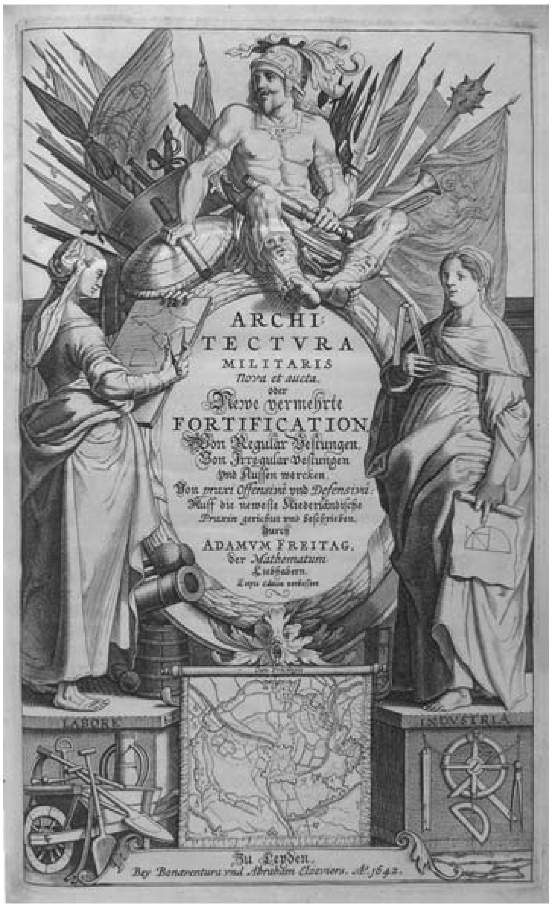


Figure 1 – Frontispiece of the 1642 edition

as mathematical content is concerned. But it stresses the fact that modern science was developing and this implied a new "noble" view on practical studies.

Miss *Industria* is holding a proportion compass (that is not mentioned once in the book!) and a trigonometry drawing (the same remark), she is standing on a pedestal, upon which different mathematical instruments have been hung: a ruler, protractor and circle, a right angle and a surveying chain. Miss *Labore* is using her compass to measure the length of a fortification line on a plan. Her pedestal holds different field instruments: a wheelbarrow, a spade, a shovel, etc. So Fritach was introduced as a mathematician, a military engineer and an architect.

Last but not least, between these two ladies there is a map, which clearly shows a fortified town in the centre, with surrounding camps and defence stockades. What town is being represented? Read on and you will learn it [it took me months to discover it, that's why it can't be revealed so early in this paper…]

1.2 The background of a 17th century military engineer

Even if the frontispiece is an important visual part of the book, its content doesn't necessarily reflect the author himself, whereas the text written inside the book shows his scientific education. Let's have a look on Fritach's different quotations concerning both books and locations.

a) Books: The most frequently mentioned author (6 times), to whom Fritach refers, is Daniel Speckle, a German architect who was working in Antwerpen in the beginning of the 17th century, but it is likely that he knew Speckle's opinions through the reading of Simon

Stevin's *Fortification*, because Stevin often quoted Speckle too.[2] As far as mathematics is concerned, the most important quotations come from Stevin and Marolois, who can be seen as the real scientific sources of Fritach: the general ideas are inspired by Stevin; and Fritach's book 1 (theory) is very similar to Marolois's book 1. The precision of the references Fritach gives shows that he was perfectly acquainted with the current editions of several other classical books, like Lorini's *Fortificationi* of 1609, or Ramelli's *Diverse et artificiose machine* of 1588.

b) Places: We can read a lot of references to various episodes of the 1600–1630 period of the eighty years war (sieges of Breda, Berg op Zoom, Rees, etc.) but the majority of them (21 quotations) concern the siege of "Bolduc". You can hardly find this place on a map today, because this name was not even really in use at the time! Fritach's *Bolduc* was actually called *Bois-le-Duc* then. The siege of Bois-le-Duc (now s'Hertogenbosch, North Brabant, Netherlands) took place in 1629 and was won by Prince Maurits of Nassau. Fritach gives a lot of anecdotes about this siege, which leads us to the question: was he actually there or did he just read about it?

2 The Man

The author himself doesn't give in the book any detail about his life and works; the dedication is the only personal matter, and for me it is misleading. Indeed, Fritach dedicated his book to Vladislas Sigismund, Prince of Poland and Sweden (in German editions from 1631), then to Vladislaus III, King of Poland (in French editions from 1635), and this never changed throughout the editions. Of course, this is the same person, elevated from Prince to King. It is this fact that has led some scholars to believe that Fritach was sent by this king to Holland. There is no actual proof of this and it seems more probable that Fritach made this dedication either to thank the sovereign for financing the publication of this book, or perhaps, because he had hoped to return to his homeland, Poland, in 1631.

2.1 Was Adam Fritach really at work in Bois-le-Duc?

There are two major contemporary witnesses of the siege of Bois-le-Duc, namely Jacques Prempart who published his *Récit ou brève description* (see bibliography) in Leuwarden in 1630, and Daniel Heinsius whose *Historia...* appeared in Leiden in 1631. However, no mention of Fritach can be found in either of them.

Nevertheless, the second one contains detailed maps of the siege and we can recognize on one of them the model of the map in the frontispiece of Fritach's book! Moreover, Heinsius is the author of the epistle to Fritach we can find in the beginning of his treatise; and finally, Heinsius is also the author of Prince Maurits of Nassau's funeral oratory. So, we can conclude that he was (and Fritach too) on Maurits's side! We can therefore deduce that our hero did really work during the siege of Bois-le-Duc. This was fortunately confirmed just before the ESU5, with the help of my colleague Janine Peblanski who translated from Polish the biography of Fritach sent by Tomasz Iwaszkiewicz, from the Association of Friends of Torun Fortifications (http://www.torun.tpf.pl), thanks to them! So now, the final episode:

2.2 All about Adam

Adam Fritach was born in 1608 in Torun, Poland (also the birthplace of Copernicus), near the German border in a wealthy family (his father was a Professor in the Academic College) with good connections (they housed a cartographers workshop). Professor Jan Brożek from the University of Cracovia noticed that the young boy was exceptionally gifted for mathematics.

[2]And we will soon discover why Fritach is likely to have shared Stevin's ideas (Hope you'll stand the suspense!)

This could be the reason why after his father's death in 1621, Fritach was granted a scholarship by the City of Torun to study in Germany (probably Leipzig) and then at the University of Frankfurt an Oder in 1625. After his studies (we don't know exactly when), he volunteered in the Army of Frederik-Hendrik of Orange and participated in the siege of s'Hertogenbosch in 1629, as we mentioned before. In July 1629, he entered the famous University of Leiden in the United Provinces (the one Stevin taught in) and received the distinction of Doctor in Medicine in 1632 (the first edition of his treatise appeared one year before).

The rest of his life was to be spent serving Duke Janusz Radziwiłł (who was to become Duke of Lithuania), as a military engineer, a fortification architect and a personal doctor, and also as a teacher of mathematics, firstly in Torun's Gymnasium and then in Radziwiłłan College, Kedainiai, Lithuania, where he died in 1650. Fritach was buried in the Evangelic Reformed Church in Kedainiai, so we can understand that his choice of volunteering in the Protestant Armies in the United Provinces was greatly due to his personal religious beliefs.

3 Some of the mathematical content of the *New Fortification*

3.1 Table of Chapters

The *New Fortification* contains three "books" (= chapters) devoted to different parts of the theory and practice of military architecture:

– Book 1: The History of Fortification, geographical considerations, the terms used and mathematics (polygons, angles, lines, including the use of tables; computations on surfaces and volumes; drawing on paper and producing in the field.)

– Book 2: Fortifying irregular places, different works of architecture and different cases (old walls, riverside, mountains, citadels)

– Book 3: Army Camps, Trenches, Attack & defence (approaches, mining, different walls, watermills.)

If Book 1 is strongly influenced by his predecessors Marolois's and Stevin's theoretical treatises, the two other ones explain the big success of Fritach's treatise, as they are real applications in the field and they update the military science to the latest inventions of the Nassau family. Nevertheless, the greatest interest for math teachers is the manner of dealing with angles and lines.

3.2 Calculating the angles: a comparison with Marolois's *Fortification*

Even if Fritach does not quote Marolois as much as Speckle, he obviously knows Marolois's book very well, as the summary of the chapters on angles and lines shows. The methods are similar, but the styles are different; we could even wonder whether Fritach doesn't just want to comment and explain the master's thought. Let's take as the first example the calculation of the angle of a polygon:

MAROLOIS ($f^\circ A, v^\circ$) gives the general rule: *In order to find the angle of the Polygon,* 2 *will be subtracted from the quantity of its angles the rest will be multiplied by* 2*; the product will be the quantity of the right angles contained in such a Polygon* [...The example of the Pentagon is then given: 5 (angles) $-$ 2 $=$ 3, and 3 $\times$ 2 $=$ 6, then 6 $\times$ 90 (degrees) $=$ 540 for the total of the angles of the pentagon; finally, 540/5 $=$ 108 degrees for each one of them.] *And with the same rule the following angles of the Polygons beginning with the square to the dodecagon will be [found].*

FRITACH (Book I, chap. V, p. 14) divides it into two parts (is it really useful?):

Rule 1*: Divide the entire circumference or* 360 *degrees by the numbers of the sides in each figure, & you shall have the angle of the centre.*

Practice. In a square are four sides that is why I divide 360 *degrees by the number* 4, *which results in* 90 *degrees for the angle of the centre in a square. In the same way, in a figure of* {V. VI. VII. VIII. IX. X. &c} *angles,* [we can find] *for the angle of the centre* {72. 60. 51, 25, 43. 45. 40. 36.}

Rule 2: This angle [of the circumference] *is the complement of* 180 *degrees of the angle previously found. Thus you subtract the angle of the centre of each figure from* 180 *degrees, & you shall have the angle of the circumference or the angle of the needed polygon.*

Practice. The angle of the centre we found in the square is 90 *degrees: I subtract then* 90 *degrees from* 180 *degrees, & the rest being* 90 *degrees will be the angle of the circumference of the square. In the same way, in a figure of* {V. VI. VII. VIII. IX. X.} *angles, the angle of the circumference will be* {108. 120. 128, 34, 17. 135. 140. 144.}

Let's take another example: the flanked angle, or angle of the bastion point.

MAROLOIS ($f^\circ B, r^\circ$) gives a general explanation, followed by a complete table (and the reader is left to make his own sense of it!):

Thus to proportionally increase the angles of the Fortresses according to the increase of their polygon angles, we shall take a half of their angles, add 15. *degrees to them, the sum will be the angle of the bulwark which we will name the flanked angle & if the flanked angle is subtracted from the Polygon angle; the rest will be double the measure of the interior flanking angle, which being subtracted from* 180. *deg: will remain the exterior flanking angle or tenaille & if to the interior flanking angle is added* 90. *degrees the sum will be the angle of the shoulder.*

	4	5	6	7	8	9	10	11	12	
	90	72	60	$51\frac{3}{7}$	45	40	36	$32\frac{8}{11}$	30	angl:of the centre
	90	108	120	$128\frac{4}{7}$	135	140	144	$147\frac{3}{11}$	150	angl: of the Polig.
	45	54	60	$64\frac{2}{7}$	$72\frac{1}{2}$	70	72	$73\frac{7}{11}$	75	half
	15	15	15	15	15	15	15	15	15	
	60	69	75	$79\frac{2}{7}$	$82\frac{1}{2}$	85	87	$88\frac{7}{11}$	90	flank. ang.
Rest	30	39	45	$49\frac{2}{7}$	$52\frac{1}{2}$	55	57	$58\frac{7}{11}$	60	doub. of the ang.
	180	180	180	180	180	180	180	180	180	interior flank:
	150	141	135	$130\frac{5}{7}$	$127\frac{1}{2}$	125	123	$121\frac{4}{11}$	120	flank. angle..
	15	$19\frac{1}{2}$	$22\frac{1}{2}$	$24\frac{9}{14}$	$26\frac{1}{4}$	$27\frac{1}{2}$	$28\frac{1}{2}$	$29\frac{7}{22}$	30	interior flank.
	90	90	90	90	90	90	90	90	90	Ang.
	105	$109\frac{1}{2}$	$112\frac{1}{2}$	$114\frac{1}{4}$	$116\frac{1}{4}$	$117\frac{1}{2}$	$118\frac{7}{22}$	$119\frac{7}{22}$	120	an. of the shoulder.

Figure 2 – Marolois's table for the angles of the polygons

FRITACH prefers the detailed solution

[1st way] *Rule. The angle of the circumference having been divided into two equal parts, add to one of these halves the ninth part of the semi-circle, that is to say* 20 *degrees, in each figure up to nine angles included (because in every figure one must take* 90 *degrees for the angle) then you will have the flanked angle.*

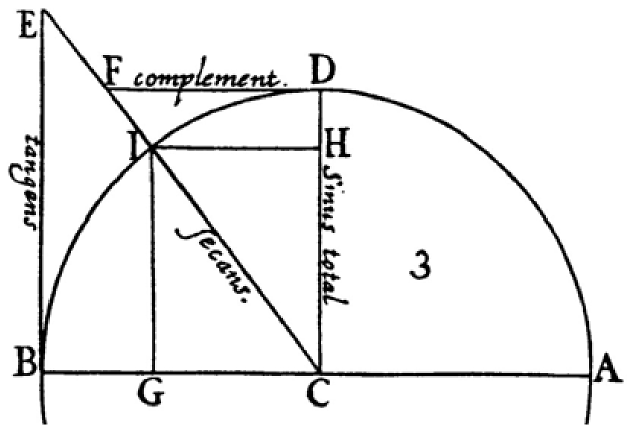
Practice. In a square the angle of the circumference is 90 *degrees, to the half of which viz.* 45, *I add* 20, *the ninth part of the semi-circle, the result is* 65 *degrees for the flanked angle CHR of the square.*

In this way in the figure of {V. VI. VII. VIII. IX. X.} *angles, the flanked angle will be* {74. 80. 84, 17, 9. 87, 30. 90. 90}.

Have you noticed the difference between the obtained values? It's 5°, which correspond to the difference between the two "formulas": $\frac{1}{2}A + 20$ instead of $\frac{1}{2}A + 15$. This difference disappears with Fritach's second way: *First of all give the smallest flanked angle of the square* 60 *degrees, the square being the first suitable figure for fortification, which also implies that the first and smallest angle of the circumference is* 90 *degrees. Subtract thus* 90 *degrees (or the smallest angle of the circumference of the figure, which you draw a bulwark on) add the half of the rest to the smallest flanked angle, then the flanked angle of the figure you desire will come.* Maybe it will be easier with the "formula" $\frac{1}{2}(A - 90) + 60$: why do things the easy way, when you can use more complicated ways? (Contemporary French proverb)

3.3 Calculating the lines: an example of 17th century trigonometry

Does trigonometry still belong to the curriculum in your country? The original texts on fortification show various uses of the theorems about right-angled triangle and trigonometric lines. Some terms are shown below:



CA = CB = [sinus total] = Radius
CH = GI = [sinus rectus] = sine
CG = HI = [sinus complement] = cosine
GB [= sinus versus]= versine
EB = tangent
CE = secant
DF = [complement of the tangent]
= cotangent

Figure 3 – From Marolois's Geometry (1616)

After the angles (*cf.* supra), Fritach shows in chapter 6 how to calculate the different lines of the fortress, taking the example of a fortified square (see fig. 4 below).

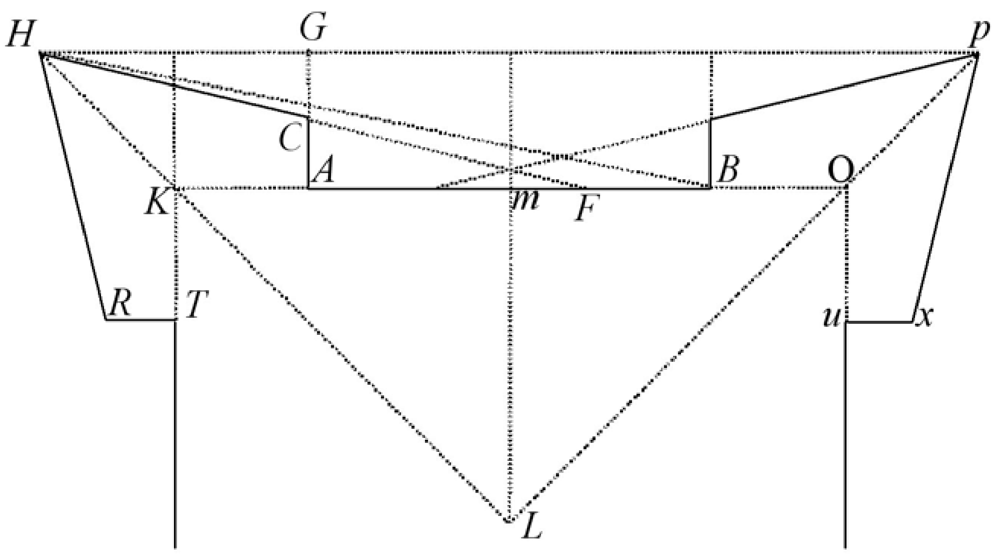


Figure 4 – Fortified square after Marolois

The curtain wall AB is 36 yards long, the face HC 24 yards and the flank AC 6 yards. The angles are also known: *Angle of the centre* $KLO = 90°$*; Angle of the circumference* $AKT = 90°$*; Flanked angle* $CHR = 65°$*; Interior flanked angle* $CFA = 12°13'$*; Angle of the flank and the grazing defence line, as* $HCG = 77°30'$. The question is: to find AF & CF (Book 1, p. 20):

Pour trouver A F & C F.

C A Radius donne C A que donne la Tangente de l'angle A C F 77 deg. 30 min.
100000 6⓪ 451071
6⓪
A F 2706426⑤

C A Radius C A La Secante de l'angle A C F 77 degr. 30 min.
100000 6⓪ 462023
6⓪
C F 2772138⑤

That is to say: *If* CA*, as the radius* $= 100\,000$*, is given the value of* 6*, what is the value of* AF *(which is the tangent of the angle* ACF*)*? ACF is of $77°30'$ because $ACF = HCG$ (corresponding angles); to fit this angle into the diagram of fig. 3, we must consider C (fig. 4) as the centre C (fig. 3) of the circle, A (fig. 4) as B (fig. 3) and F (fig. 4) as E (fig. 3); this being identified, AC is the radius of the circle, AF is the tangent and CF is the secant of the angle ACF. The Radius is taken of 100 000 (as given in the trigonometric tables and as Marolois's does), and a simple rule of three allows us to calculate: $AF = 6 \times 451\,071/100\,000$, and we find 27.06426; note the use of? meaning "units" in Stevin's style, and of? to indicate (sort of) the place of the dot (a slightly different use of Stevin's own style, probably for typographical reason?)

The second problem is solved in the manner: *If* CA*, as the radius* $= 100\,000$*, is given the value of* 6*, what is the value of* CF*, which is the secant of the angle* ACF*?*

3.4 Simon Stevin's hidden heritage

As we saw above, Stevin's conceptions of numbers can be traced inside Fritach's text, but Stevin's heritage can't be reduced to number notations; another part of the treatise shows the continuation of decimal thinking.

In fact, we haven't discussed the question of measures so far, although this question is far from easy, especially concerning the famous Rhineland yard (in French: *verges du pays de Rhin*; in German: *Reinlandische Ruthen*) popularized by Speckle and Stevin. Fritach writes down what is done in practice (according to him — Book 1, p. 30 —, in the Low Countries *workers do not use others*); it is likely that Fritach (born in 1608) never met Stevin (deceased in 1620), but Stevin's popularity was great in Leiden University and in the Army, so his ideas were still alive.

For anyone who remembers Stevin's *Tenth*, Fritach crosses a new step in his proposals for conversion: in Book 1, page 30, he suggests that we can transform the 12-foot yard into a 10-foot one "pour avoir un compte plus facile" (*to get an easier way of counting*), and he provides two conversion tables (from 10 to 12, from 12 to 10), and explains how to use them. The general tables were already given this way, and written *à la Stevin*, which could not be understood otherwise; for instance on General Table 1, page 24, we can read the half-diameter being 42.76, which must be read: "42 yards, 7 feet, 6 inches"; everyone would easily understand the problem if the yard would measure 12 feet...

4 Fortifying the square

Several examples of fortifying geometrical figures on paper are given in chapter XV [p. 53: *How a fortress project is made on paper according to the calculated tables*], which is usual in such a didactic treatise. For us it is a good way of learning how to draw one, or allowing our pupils to use their practical geometry instruments (ruler and compass).

Before writing the construction step by step, Fritach stresses the benefits of drawing previous plans: *Before starting the construction of a fortress in the country, its project first must be made on paper, according to the appropriate proportion & needed measurement, in order to have before one's eyes the size of the angles & length of the lines that we have already given and calculated in our tables; and also to see how the fortress will accommodate and defend its inhabitants well, all of which is easier to see on paper.*

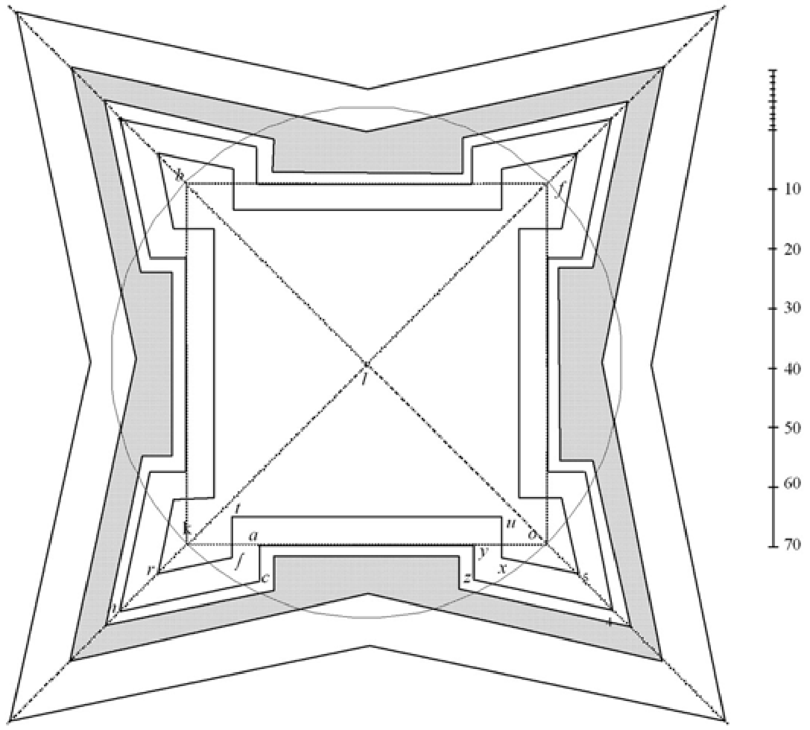


Figure 5 – Fortifying the square

Then the construction program itself starts [please take your ruler and compass first, or follow the construction on figure 5 below, which reproduces Fritach's figure 51]: *If you would like to see this in practice, consider figure* 51, *in which a square is represented; which must be portrayed according to the Grand Royal of the table calculated by the first method; I thus take in the aforesaid tables the semi-diameter of the square figure, which is marked by letters K and L, & measuring* 42 *yards* 7 *feet &* 6 *inches; whose length I take with the compass on the scale which is added to figure* 51, *& make with the same opening a hidden circumference on which I give the measurement* 60 *yards* 4 *feet &* 7 *inches to the line KO of the interior polygon, as is shown in the table; this is done four times, in such a way that the four sides KO, GF,*[3] *FB & BK can be drawn exactly within the circumference, I take* 12 *yards* 2 *feet &* 4 *inches from the table for the gorge KA, & putting a leg of the compass on the angle of the polygon in K, as you can see on the aforesaid figure* 51, *I make the mark A on the KO side with the other leg of the compass, this will cut the gorge: a perpendicular line being drawn from this point &* 6 *yards marked on it will give you the flank AC. In the same manner, another straight line drawn from the centre L through the angle K of the*

[3] An error occured (as in computers. . .): it's *OF*.

polygon, & continued to P will show you the capital line; on this line 15 *yards* 18[4] *feet &* 3 *inches are measured from K to H, whose length you can find in the table marked with the letters H, K; and a straight line drawn from H to C will end the face: & achieving this all around the figure, then the portrait will be perfect.*

Did you manage to get the right shape? Now you know how to fortify the square!

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[4] Another one! Error of reading out of the table, it is only 8 feet.