Mathématiques ici et ailleurs

"Baccalauréat Australien"

Un nouveau type d'épreuves au Baccalauréat ?

La Commission Second Cycle, par l'intermédiaire de notre collègue Jacques VERDIER de Nancy, a pu se procurer les sujets du "Baccalauréat Australien" de l'Etat de Victoria. Nous avons pensé qu'il était intéressant de les publier et d'ouvrir un débat au sein de l'A.P.M.E.P. : nos épreuves restent faites de deux exercices et d'un problème, sans formulaire mais avec la possibilité pour certains candidats d'intégrer des formules dans leur calculatrice, alors que dans d'autres pays, elles se présentent sous forme de séries de questions-tests avec fréquemment un formulaire intégré à l'énoncé.

Nous manquons de renseignements précis à propos de ces épreuves, en particulier, nous ignorons à quelle classe d'âge elles s'adressent et à quel type d'enseignement elles correspondent. Nous ne savons pas non plus si ces épreuves ont été mises en place récemment, s'il s'agit d'une expérience ou si cela existe depuis plusieurs années. Enfin, peut-être existe-t-il d'autres pays où il y a de semblables épreuves ?

Pensez-vous qu'une expérience de ce genre pourrait être tentée pour le baccalauréat en France ? Dans quelle série ? Autant de questions auxquelles nous souhaitons répondre.

Toute réponse de votre part sera la bienvenue.

Jean CAPRON

Victorian Certificate of Education VCE(HSC)
Core Examination 1989

MATHEMATICS A GROUP 1

Friday, 17 November 1989 : 9.30 am to 12.30 pm
Total time allowed for examination : 3 hours

QUESTION BOOKLET

Number of questions	*Number of questions which may be answered*	*Percentage of examination total*
20	20	100

Directions to candidates

- You may attempt to answer ALL questions.
- All questions must be answered in the script book provided.
it is not necessary to begin an answer to each question on a separate page.
- However, a space of at least three lines should be left between answers to individual questions.
- The marks allotted to each question are iudicated beside the question. There is a total of 120 marks available for the examination.
- A data sheet consisting of miscellaneous formulae will be provided.
- An approved calculator may be used.
- Candidates need not give answers as decimals unless instructed to do so. For example, answers may iuvolve π or e, may be given in surd form, as a fraction, or as a decimal, unless instructed otherwise.
- At the end of the examination, place all other used script books inside the back cover of one of the used script books and hand them in.
- You may retrain this question booklet.
- Please ensure that you write your candidate number on all used script books.

Question 1

Solve for x

i. $(x - 3)(x - 4) = x$

ii. $\log_{10}(4 + 3x) = 2$

4 marks

Question 2

A random variable X has a probability distribution which is given by the following table :

x	0	1	2	3
$\Pr(X=x)$	0.3	0.1	0.2	0.4

i. Find $E(X)$

ii. Find the probability that X is greater than $E(X)$.

4 marks

Question 3

Find $f'(x)$ if

i. $f(x) = x^2 \log_e x$

ii. $f(x) = \sqrt{\sin x}$

4 marks

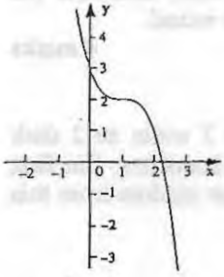
Question 4

Four functions are given by the following equations :

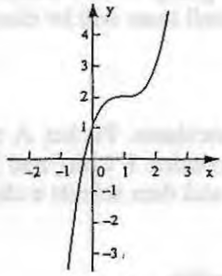
A. $y = (x+1)^3 - 2$ B. $y = (x-1)^3 + 2$

C. $y = 2 - (x+1)^3$ D $y = 2 - (x-1)^3$

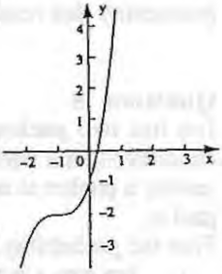
The graphs of three of these functions are shown below :



Graph 1



Graph 2



Graph 3

i. Which of A, B, C or D corresponds to Graph 1 ?
ii. Which of A, B, C or D corresponds to Graph 2 ?
iii. Which of A, B, C or D corresponds to Graph 3 ?

3 marks

Question 5

i. Write down the series expansion of e^{x} up and including the term in x^3
ii. Use this expansion to find an approximate value for $e^{-0.5}$, giving your answer correct to three decimal places.

3 marks

Question 6

On average one out of ten packets of Bongo chewing gum contains a photo of Katia's favorite rock group. Katia decides to keep on buying packets of Bongo until she gets one which contains a photo of the group. Let X be the number of packets of Bongo that Katia buys before she gets one which contains such a photo.

i. Name the distribution of X .
ii. What is the mean of X ?
iii. If a packet of Bongo costs 60 cents, find the expected cost to Katia of getting the photo.

4 marks

Question 7

A netball team of 7 players is classified as *mixed* if it consists either 3 women and 4 men or of 4 women and 3 men. If a netball team were to be chosen at random from a group of 5 women and 4 men, calculate the probability that resulting neball team will be classified as *mixed*.

4 marks

Question 8

Jim has two packets of chocolates. Packet A contains 2 white an 2 dark chocolates, and packet B contains 1 white and 3 dark chocolates. Jim first selects a packet at random, and then selects a chocolate at random from this packet.

Find the probability that

i. Jim gets a white chocolate
ii. Jim chose packet A, given that he got a white chocolate.

4 marks

Question 9

The line $y = 4x - 1$ is a tangent to the curve $y = x^4 + c$.
Find the value of c.

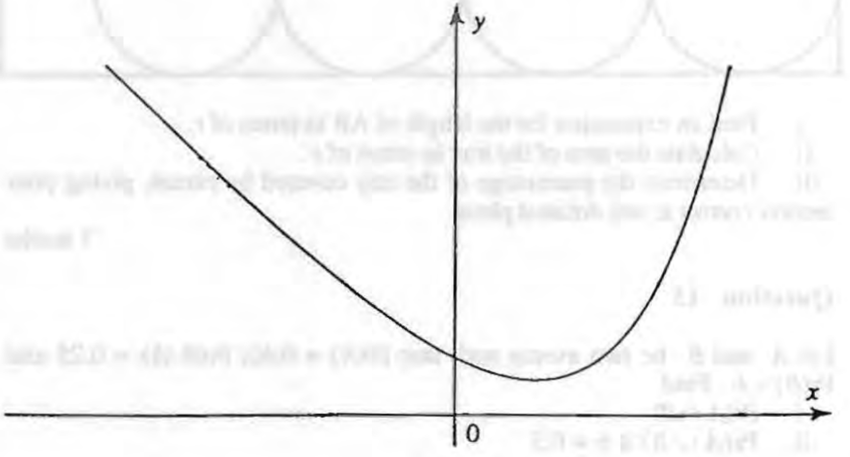
5 marks

Question 10
Find the derivative of $x \sin x + \cos x$.

Hence calculate $\int_0^1 x \cos x \, dx$, giving your answer correct to three decimal places.

5 marks

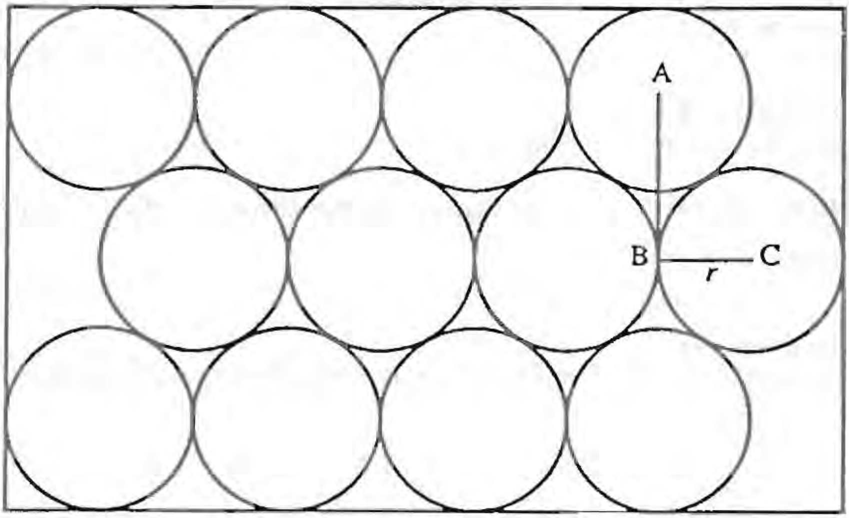
Question 11
The graph of the function $y = e^{2x} - 4x$ is shown in the following diagram.



i. Find the coordinates of the turning point, and justify that it is a minimum by considering the sign of the first derivative.
ii. Find the area of the region enclosed by the graph of $y = e^{2x} - 4x$, the line $x = 1$ and the coordinate axes.

7 marks

Question 12
Twelve pizzas, each in the shape of a circle of radius r, are packed on a tray as illustrated in the diagram below.



i. Find an expression for the length of AB in terms of r.
ii. Calculate the area of the tray in terms of r.
iii. Determine the percentage of the tray covered by pizzas, giving your answer correct to one decimal place.

7 marks

Question 13

Let A and B be two events such that $\Pr(A) = 0.40$, $\Pr(B \mid A) = 0.25$ and $\Pr(B) = b$. Find
i. $\Pr(A \cap B)$
ii. $\Pr(A \cup B)$ if $b = 0,5$
iii. the smallest possible value of b.
iv. the largest possible value of b.

7 marks

Question 14

Assume that there is a probability of p that any day is windy, independent of any other day. Consider a period of five consecutive days.
i. If $p = 0.4$, calculate the probability that
  a. at least four out of the five days are windy
  b. windy and non-windy days alternate during the five day period.

Give your answers correct to three decimal places.

ii. If during the five day period the probability of three windy days is the same as the probability of four windy days, find p.

9 marks

Question 15

Consider the function f given by

$$f(x) = \log_e x - \log_e (1 - x)$$

Find

i. the values of x for which f is defined

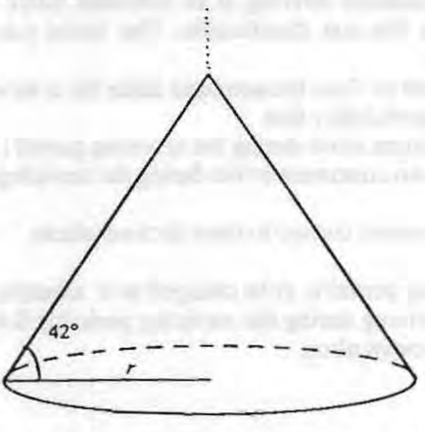
ii. $\dfrac{1}{f(0.8)}$, giving your answer correct to two decimal places

iii. $f^{-1}(0.8)$, giving your answer correct to two decimal places.

7 marks

Question 16

Sand is poured on to the floor of a storage area at the rate of 2 cubic metres per minute. The sand forms a conical pile with a constant angle of 42° at its base. See diagram below.



i. If V cubic metres is the volume of sand in the pile when the radius of the base is r metres, find an expression for V in term of r.

ii. Find correct to the nearest minute, the time which it takes for the radius of the base of the pile to increase from 1 metre to 3 metres.

iii. Find the rate at which the radius of the base of the pile is increasing when the radius is 3 metres, giving your answer correct to three decimal places.

8 marks

Question 17

The temperature A °C inside a house at time t hours *after 4.00 am* is given by:

$$A = 21 - 3\cos(\frac{\pi t}{12}) \qquad \text{for } 0 \leq t \leq 24$$

and the temperature B °C outside the house at the same time is given by

$$B = 22 - 5\cos(\frac{\pi t}{12}) \qquad \text{for } 0 \leq t \leq 24$$

i. Find the temperature inside the house at 8.00 am.
ii. Write down an expression for $D = A - B$, the difference between the inside and outside temperatures.
iii. Sketch the graph od D for $0 \leq t \leq 24$.
iv. Determine when the inside temperature is less thau the outside temperature..

8 marks

Question 18

The number of customers arriving at an autobank teller in any given time interval follows a Poisson distribution. The mean rate of arrival is 15 customers per hour

i. It is proposed to close the autobank teller for a servicing period of 10 minutes. Find the probability that

a. no customers arrive during the servicing period ;
b. at least two customers arrive during the servicing period.

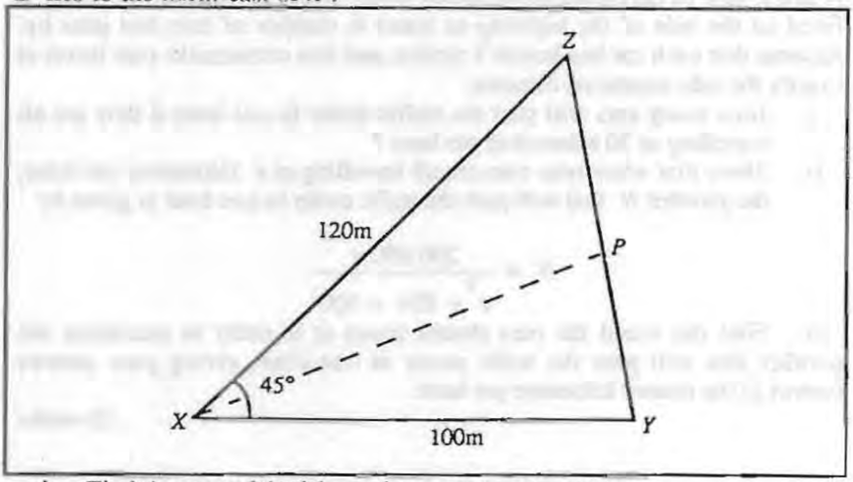
Give your answers correct to three decimal places.

ii. The servicing period is to be changed to k minutes. If the probability of no customers arriviug during the servicing period is 0.4, find the value of k correct to one decimal place.

8 marks

Question 19

The shape of a lake is a triangle XYZ whose sides XY en XZ have lengths 100m and 120m respectively. The point Y lies to te east of X and the point Z lies to the north-east of X.



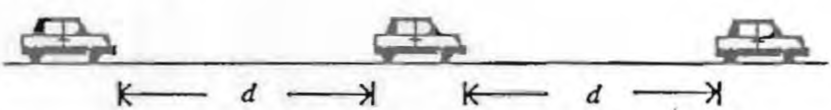
i. Find the area of the lake to the nearest square metre.

ii. A thick forest beside YZ makes it difficult to measure the length of YZ directly. Calculate the length of YZ, giving your answer correct to the nearest metre.

iii. A boat sets out from X and steers a course which bisects the angle YXZ. When the boat reaches the point P on the bank YZ, find how far the boat has travelled, giving your answer correct to the nearest metre.

9 marks

Question 20



Road safety experts believe that when a single lane of cars is travelling at a speed of v kilometres per hour, the safe separation distance between consecutive cars is d metres (see above), where d is given by the formula

$$d = \frac{v\,(50 + v\,)}{200}$$

A single lane of cars is travelling along a busy highway and a traffic meter is fixed on the side of the highway to count te number of cars that pass by. Assume that each car has length 3 metres, and that consecutive cars travel at exactly the safe separation distance.

i. How many cars will pass the traffic meter in one hour if they are all travelling at 30 kilometres per hour ?

ii. Show that when tyhe cars are all travelling at v kilometres per hour, the number N that will pass the trafic meter in one hour is given by

$$N = \frac{200\,000\,v}{v^2 + 50v + 600}.$$

iii. Find the speed the cars should travel at in order to maximize the number that will pass the trafic meter in one hour, giving your answer correct to the nearest kilometre per hour.

10 marks

END OF QUESTION BOOKLET

Victorian Certificate of Education VCE(HSC)
Core Examination 1989

MATHEMATICS B GROUP 1

Wednesday 22 November 1989 : 9.30 am to 12.30 pm
Total time allowed for examination : 3 hours

QUESTION BOOKLET

Number of questions	*Number of questions which may be answered*	*Percentage of examination total*
20	20	100

Directions to candidates

- You may attempt to answer ALL questions.
- It is not necessary to began an answer to each question on a separate page.
- The marks alloted to each question are indicated beside the question. There is a total of 120 marks available for the examination.
- You are advised not to work too hurriedly as marks are given not only for correct answers but also for correct reasoning and proper expression.
- To obtain full marks, exact answers should be given unless a question indicates otherwise.
- A data sheet of miscellaneous formulae will be provided.
- An approved calculator may be used.
- In this paper,

Tan^{-1} is the inverse function of $f : (-\frac{\pi}{2}, \frac{\pi}{2}) \to R$ where $f(x) = Tan\ x$,

$Arg\ z$ denotes the argument of the complex number z, where $-\pi < Arg\ z \leq \pi$

$\bar{z}$ is the complex conjugate of z.

....

Question 1

Let $A = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & 4 \\ 0 & 1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 3 & -3 \\ -6 & 3 & 2 \\ 2 & -1 & 1 \end{bmatrix}$

Find :

i. $A + B$;

ii. AB , and hence find A^{-1}

3 marks

Question 2

In the quadrilateral $OABC$, O is the origin, $\overrightarrow{OA} = \mathrm{i} + 2\mathrm{j} - 2\mathrm{k}$, $\overrightarrow{OB} = 2\mathrm{i} - 2\mathrm{j} + 2\mathrm{k}$ and $\overrightarrow{OC} = m\mathrm{i} - 2\mathrm{j} + \mathrm{k}$ where m is real.

Find :

i. $\left|\overrightarrow{OA}\right|$

ii. $\overrightarrow{AB}$

iii. $\overrightarrow{OA} . \overrightarrow{OC}$ (in terms of m).

iv. the value of m such that $\overrightarrow{OC}$ is perpendicular to $\overrightarrow{OA}$.

4 marks

Question 3

A curve has equation $x^2y - 5x + 3y^2 = 9$.

Find the gradient of the tangent to the curve at the point (1,2).

3 marks

Question 4

Let $z = 1 - i\sqrt{3}$.

Find

i. $|z|$;

ii. $Arg\ z$;

iii. $|z^5|$;

iv. $Arg\ (z^5)$;

v. z^5 in the form $a + ib$ where a et b are real ;

vi. the imaginary part of $z\,\overline{w}$ where $w = 1 + i$.

7 marks

Question 5

On separate Argand diagrams, sketch the following sets of complex numbers :

i. $\{ z : |z - (1 + i)| = 1\}$

ii. $\{z : Arg\ z > \frac{\pi}{4}\}$, clearly indicating which boundaries are included.

4 marks

Question 6
Find the volume of the solid of revolution formed by rotating about the x-axis the region enclosed by the curve with equation $y = x^{-\frac{3}{2}}$, the x-axis and the lines $x = 2$ and $x = 3$.

3 marks

Question 7
$P\ (4u, 2u^2)$ and $Q\ (4v, 2v^2)$ are points on the curve with equation $x^2 = 8y$ such that the tangents at P and Q are perpendicular.
Find u in term of v.

3 marks

Question 8
For the conic whose equation is : $\frac{(x-3)^2}{25} + \frac{y^2}{16} = 1$

i. sketch the graph, making intercepts on the axes ;
ii. find the co-ordinates of the foci.

5 marks

Question 9
i. Find an antiderivative of a. $\frac{1}{\sqrt{(1-9x^2)}}$

b. $\cos^2 x \sin^3 x$

ii. Evaluate $\int_3^4 \frac{x^2}{x^2-1}dx$,

giving your answer in the form $a + b\log_e c$

4 marks

Question 10
A body moves from rest at the origin so that after t seconds its acceleration is $10\mathrm{i} - e^{0.1t}\,\mathrm{j}$ metres/second^2.
Find
i. its velocity after t seconds ;
ii. its position vector when $t = 10$.

5 marks

Question 11
i. Find the three roots of the equation $z^3 - z^2 + 2 = 0,\ z \in C$.

ii. Find the remainder when $z^3 - z^2 + 2$ is divided by $(z - 1)$.

5 marks

Question 12

For the set of linear equations

$$\begin{aligned} x + y - z &= 0 \\ 3x + y + z &= 6 \\ 2x + 2y + pz &= p + 6 \end{aligned}$$

where p is a constant, find the value or values of p for which the system has

i. a unique solution

ii. infinitely many solutions.

In case **i.** only, find the solution of the system.

6 marks

Question 13

A partly filled tank contains 200 litres of water in which 1500 grams of salt have been dissolved. Water is poured into the tank at a rate of 6 litres/minute. The mixture, which is kept uniform by stirring, leaves the tank through a hole at the rate of 5 litres/minute.

Set up a differential equation whose solution gives the amount of salt, x grams, in the tank after t minutes.

Do not attempt to solve this differential equation.

3 marks

Question 14

A linear transformation maps (1,0) to (2,1) and (4,-1) to (9,5).

i. find the matrix of the linear transformation.

ii. Sketch the image of the square with vertices O (0,0), A (1,0), B (1,1), and C (0,1) under the linear transformation.

iii. Show that the image of the hyperbola with equation $\frac{x^2}{2} - \frac{y^2}{4} = 1$ under the linear transformation is another hyperbola.

8 marks

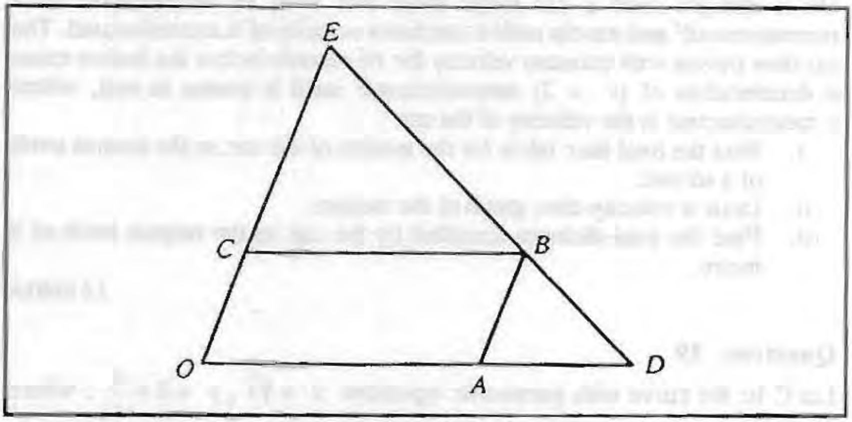
Question 15

In the diagram (next page), $OABC$ is a parallelogram and $OA = 2AD$.

Let $\overrightarrow{AD} = a$ and $\overrightarrow{OC} = c$

i. Write down an expression for $\overrightarrow{DB}$ in term of $\underset{\sim}{a}$ and $\underset{\sim}{c}$

ii. Use a vector method to show that $OE = 3OC$.



5 marks

Question 16

Let S and T be subsets of the Argand plane where

$$S = \{z : |z| \leq 2\};$$
$$T = \{z : Im\, z + re\, z \geq 4\}.$$

i. On the same diagram sketch S and T, clearly indicating which boundary points are included.

ii. Let $d = |z_1 - z_2|$, where $z_1 \in S$ and $z_2 \in T$. Find the miuimum value of d.

4 marks

Question 17

Let $f : R \to R$ where $f(x) = xTan^{-1}x$.

i. Fiud $f'(x)$ and hence express $Tan^{-1}x$ in terms of $f'(x)$ and x.

ii. Using **i.** or otherwise, find $\int_0^1 Tan^{-1} x\, dx$

iii. Find the area of the region bounded by the curve with equation $y = Tan^{-1} x\, dx.$, the line $y = \frac{\pi}{4}$ and the y- axis.

8 marks

Question 18

On a straight road a car starts from rest with an acceleration of 2 metres/second^2 and travels until it reaches a velocity of 6 metres/second. The car then travels with constant velocity for 10 seconds before the brakes cause a deceleration of (v + 2) metres/second^2 until it comes to rest, where v metres/second is the velocity of the car.

i. Find the total time taken for the motion of the car, to the nearest tenth of a second.
ii. Draw a velocity-time graph of the motion.
iii. Find the total distance travelled by the car, to the nearest tenth of a metre.

12 marks

Question 19

Let C be the curve with parametric equations $x = \sqrt{t}$, $y = 2 + \frac{k}{t}$, where k is a real constant.

i. On separate sets of axes, sketch graphs of C for the cases $k = 0$, $k = 1$ and $k = -1$.
ii. Find the values of k for which C meets the parabola $y = 4 - x^2$ in :
   a. exactly two distinct points
   b. exactly on point.

8 marks

Question 20

Particles A and B move in the cartesian plane so that at any time $t \geq 0$, their position vectors are $\mathrm{r}_A = 2t\mathrm{i} + t\mathrm{j}$

$\mathrm{r}_B = (4 - 4\sin\alpha t)\mathrm{i} + (4\cos\alpha t)\mathrm{j}$, where α is a positive constant

i. Find the speed of B in terms of α.
ii. Find the cartesian equations of the paths of A et B.
iii. On the same set of axes, sketch the paths of A and B and indicate the directions of travel.
iv. Find the points where the paths of A and B cross.
v. Find the least value of α, correct to two decimal places, for which A and B will collide.
vi. Assuming that the collision in **v.** occurs, find the distance which has been travelled by B when it collides with A, giving your answer correct to one decimal place.

14 marks

THIS DATA SHEET IS TO ACCOMPANY THE EXTERNAL EXAMINATION PAPERS FOR MATHEMATICS A AND B

MISCELLANEOUS FORMULAE

Mensuration

Arc length : $r\theta$

Cosine rule : $a^2 = b^2 + c^2 - 2bc \cos A$

Area of triangle : $\frac{1}{2} bc \sin A$

Area of sector : $\frac{1}{2} r^2\theta$

Volume of pyramid : $\frac{1}{3} Ah$

Area of trapezium : $\frac{1}{2}(a + b)h$

Volume of sphere : $\frac{4}{3}\pi r^3$

Volume of cone : $\frac{1}{3}\pi r^2 h$

Sinus rule : $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Trigonometric formulae

$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$

$\cos(u \pm v) = \cos u \cos v \pm \sin u \sin v$

$\tan(u \pm v) = (\tan u \pm \tan v) / (1 \pm \tan u \tan v)$

$\sin 2t = 2 \sin t \cos t$

$\cos 2t = \cos^2 t - \sin^2 t = 1 - 2\sin^2 t = 2\cos^2 t - 1$

$\tan 2t = (2 \tan t) / (1 - \tan^2 t)$

Series

Arithmetic Series :

$$a + (a + d) + \ldots + [a + (n - 1)d] = \frac{1}{2} n\,[2a + (n - 1)d]$$

Geometric Series :

$$a + ar + ar^2 + \ldots + ar^{n-1} = a(1 - r^n) / (1 - r) \quad \text{for } r \neq 1$$

Binomial Series :

$$a^n + \binom{n}{1} a^{n-1} + \ldots + \binom{n}{r} a^{n-1} x^r + \ldots + x^n = (a + x)^n$$

Exponential Series :

$$1 + x + \frac{x^2}{2!} + \ldots + \frac{x^n}{n!} + \ldots = e^x$$

Coordinate geometry

Name	Cartésian equation	Foci	Equations of directrices	Other relationships
ellipse	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$	$(\pm ae,0)$	$x=\pm\frac{a}{e}$	$b^2=a^2(1-e^2)$ $0<e<1$
hyperbola	$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$	$(\pm ae,0)$	$x=\pm\frac{a}{e}$	$b^2=a^2(e^2-1)$, $e>1$
parabola	$y^2=4ax$	$(a,0)$	$x=-a$	$e=1$

Calculus

$f(x)$	x^n	$\log_e x$	e^{kx}	$\sin kx$	$\cos kx$	$\tan kx$
$f'(x)$	nx^{n-1}	x^{-1}	ke^{kx}	$k\cos kx$	$-k\sin kx$	$k\sec^2 kx$

$f(x)$	$\mathrm{Sin}^{-1}\frac{x}{a}$	$\mathrm{Cos}^{-1}\frac{x}{a}$	$\mathrm{Tan}^{-1}\frac{x}{a}$
$f'(x)$	$(a^2-b^2)^{-1/2}$	$-(a^2-b^2)^{-1/2}$	$a/(a^2+b^2)$

Product rule : $\frac{d(uv)}{dx}=u\frac{dv}{dx}+v\frac{du}{dx}$; Quotient rule : $\frac{d}{dx}\left(\frac{u}{v}\right)=\dfrac{v\frac{du}{dx}-v\frac{dv}{dx}}{v^2}$

Chain rule : $\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$

Probability

$\Pr(A)=1-\Pr(A')$; $\Pr(A\cup B)=\Pr(A)+\Pr(B)-\Pr(A\cap B)$

$\Pr(A\cap B)=\Pr(B|A)\Pr(A)=\Pr(A|B)\Pr(B)$

$E(X)=\mu=\Sigma x\Pr(X=x)$; $\mathrm{var}(X)=\Sigma(x-\mu)^2\Pr(X=x)=E(X^2)-\mu^2$

Distribution	$\Pr(X=x)$	Mean	Variance
binomial	$\binom{n}{x}p^x(1-p)^{n-x}$	np	$np(1-p)$
hypergeometric	$\binom{D}{x}\binom{N-D}{n-x}/\binom{N}{n}$	$n\frac{D}{N}$	$n\frac{D}{N}\left(1-\frac{D}{N}\right)\frac{N-n}{N-1}$
Poisson	$e^{-\lambda}\lambda^x/x!$	λ	λ
geometric	$p(1-p)^x$	$\frac{1-p}{p}$	$\frac{1-p}{p^2}$