Jouer avec des casseroles (pour faire des math en anglais)

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La séance que je vais exposer ici en anglais a été présentée en classe de Première S section européenne[1] au lycée Bourdelle de Montauban. Je l'ai ensuite adaptée, puis proposée au niveau Seconde.

Pour la première fois cette année, j'ai enseigné les mathématiques en anglais en section européenne, une heure par semaine en seconde, première et terminale scientifiques. Cela m'a demandé un investissement considérable que je souhaite partager espérant ainsi être utile à d'autres.

Je cherchais un problème concret d'optimisation à presenter aux élèves et l'idée de la casserole m'a été suggérée par une collègue de mathématiques. Mon collègue de langue et l'assistante d'anglais du lycée m'ont aidée à mettre au point cette séance.

I brought two saucepans from my home. On entering the classroom the students were amazed to see my saucepans on the desk and seemed curious to know what we could do with saucepans in a maths class.

(1) Créées en 1992, les sections européennes ou de langues orientales accueillent 160 000 élèves dans 3 600 sections. Consulter le site : http://www.education.gouv.fr/bo/2003/42/MENE0302456N.htm. Pour les modalités de l'évaluation au baccalauréat, voir : http://eduscol.education.fr/index.php?./D0121/accueil.htm

Vocabulary

First, we worked on the vocabulary. I asked the students to talk about the saucepans and I wrote the sentences on the board. We came up with the following :

These saucepans are made of stainless steel.
The handle is removable.
One is bigger than the other (deeper, taller, higher, wider).
They don't have the same capacity.
Their shape is almost cylindrical.
They can be used to boil water, cook noodles or warm some soup.
The bottom is flat.

I asked the students to give the nouns matching the adjectives :

High → height
Long → length
Wide → width
Deep → depth
Broad → breadth
Thick → thickness

Capacity

I asked the students :
« Guess the capacity of each saucepan and write the answer in your notebook. »
« How much liquid can each contain ? »
Then, we checked the actual values by pouring water from the bottles that were on the desk into saucepans.
« Who would like to fill these saucepans up to the brim? »
Nathalie and Vincent were willing to do it.

« Be careful when pouring the water, don't let the saucepans overflow. »

We deduced this result : the small one was a one-litre, and the large one a three-litre saucepan.
Therefore the capacity of the larger saucepan was three times the capacity of the smaller.

Height

The next question was to compare the heights, more precisely, to give the ratio of the height of the larger saucepan over the height of the smaller.
« Could it be three ? »
It seemed obvious that three was not a good answer [could we say that the presence of the saucepans was the reason why this was so clear ?].
Simon needed some information about the saucepans, especially the diameter. We agreed on the fact that the saucepans had the same shape and therefore proportional dimensions.

A few numbers were proposed such as 1.5 ; $\sqrt{2}$; $\sqrt{3}$ and the cubic root of three was given by Gregoire.

I proposed to check. I asked Gregoire to measure the heights of the two saucepans with a metal ruler. He found that the height of the smaller saucepan was about 6.65 cm and the height of the larger one was about 9.60 cm. The fact that the saucepans were filled with water made the measurement easier.

The students were eager to see who was right, so they all used their calculators to give an approximate value of the ratio (1.44) and of the cubic root of three (1.44). [I have since realized that I was very lucky with the measurements. We could have found an approximate value closer to $\sqrt{2}$.]

At this point, everyone was convinced that Gregoire was right : If all the dimensions of a saucepan are multiplied by the cubic root of three, the volume is multiplied by three.

An optimisation problem

« Let us suppose that we want to minimise the production cost of the saucepans .
As the price of the handle is the same for all saucepans, we won't take it into account. We will assume that the cost only depends on the quantity of metal used to make them.
We take that the saucepan is a cylinder and the thickness is the same throughout.
Hence for a given volume, say one litre, the cost is minimum when the surface of the cylinder is as reduced as possible. »

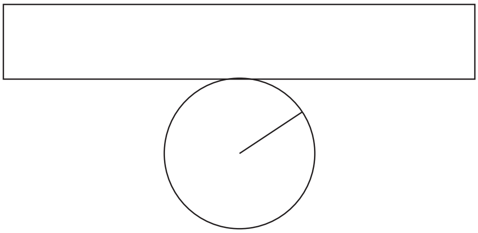
Here is the solution we gave to the problem :

Let's call R the radius of the bottom and *h* the height of a one litre saucepan.

Using the volume formula, let's express *h* in terms of R.

$V = 1$ and $V = \pi R^2 h$ therefore $h = \dfrac{1}{\pi R^2}$.

Let's now express the area A of the saucepan in terms of R and h.
For that we need to flatten the surface of the open cylinder :



$$A = \pi R^2 + 2\pi R h.$$

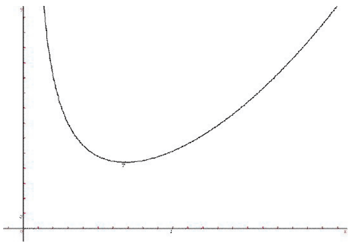
Substituting the previous value of h, we express A in terms of R :

$$A(R) = \pi R^2 + \frac{2\pi R}{\pi R^2}.$$

This leads us to :

$$A(R) = \pi R^2 + \frac{2}{R}.$$

We get the function graph on the calculator using an appropriate window.



We notice that there is a turning point(1).

When putting the pointer on this point, we get an approximate value of the minimum of A and the corresponding value of R : A is minimum for $R \approx 0.683$. As the unit of volume that we use is the cubed decimetre, the unit of length is the decimetre.

This result is accurate enough for our purpose, but for intellectual reasons, we might also want to get the exact value of R.
In order to do this, we differentiate the function A with respect to R. A is differentiable in $(0;\infty)$:

(1) A stationary point is a point on the function graph where the gradient of the function is zero. If the gradient of the function changes sign at the stationary point, then it is called a turning point, which can be a local maximum or local minimum.

$$A'(R) = 2\pi R - \frac{2}{R^2}.$$

As we want to study the sign of A', we factorise :

$$A'(R) = 2\frac{\pi R^3 - 1}{R^2}.$$

The sign of A' is that of $\pi R^3 - 1$ and as the cubed function is increasing in **R** we can set the following table :

R	0		$\frac{1}{\sqrt[3]{\pi}}$		∞
$A'(R)$	‖	$-$	0	$+$	
$A(R)$	‖	↘	$A\left(\frac{1}{\sqrt[3]{\pi}}\right)$	↗	

Therefore A is minimum for $R = \frac{1}{\sqrt[3]{\pi}}$ and, in that case, the height has the same value $\frac{1}{\sqrt[3]{\pi}}$.

Is it the case of the one litre saucepan which is on the desk ? Almost : the radius is about 7 cm.

Conclusion : In order to answer our problem the saucepans must have heights that are nearly equal to the radius of their bottom.

At the end of the lesson, I asked the students to check if this was true with the saucepans they had at home.